Universal reversibility of JC*-triples

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Joint work with L. Bunce continuing earlier work with B. Feely

JC^*-triples

A (concrete) JC^*-triple is a closed $E \subseteq A$ (A C^*-algebra) such that

$$a, b, c \in E \Rightarrow \{a, b, c\} \stackrel{def}{=} \frac{1}{2}(ab^*c + cb^*a) \in E$$

Examples: $E = A$,

$$E = \left\{ \begin{pmatrix} z_2 & z_2 & \cdots & z_d \\ 0 & 0 & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & 0 \end{pmatrix} \in M_d(\mathbb{C}) = \mathcal{B}(\ell_d^2) \right\} \equiv \ell_d^2$$

row Hilbert space of dimension d. In this case $\{\xi, \eta, \zeta\} = \frac{1}{2}(\langle \xi, \eta \rangle \zeta + \langle \zeta, \eta \rangle \xi)$.

Proposition

If E is a JC^*-triple then $\mathrm{Aut}(B_E)$ acts transitively on B_E. (Note: For $\dim E = 1$, $B_E = \mathbb{D}$ and $\mathrm{Aut}(B_E)$ is the Möbius group of $\mathbb{D}$.)

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JC^*-triples (2)

Theorem (Kaup's Riemann mapping Th. 1983)

If F is a Banach space and $\mathcal{D} \subset F$ is a domain biholomorphic to bounded symmetric domain ($\Rightarrow \mathrm{Aut}(\mathcal{D})$ acts transitively) then $\mathcal{D}$ is biholomorphic to B_E for E a JB^*-triple.

More Examples: Previous examples were all TROs
$(a, b, c \in E \Rightarrow [a, b, c] \stackrel{def}{=} ab^*c \in E)$.
$E = \{x \in M_d(\mathbb{C}) : x^t = x\}$.
More generally E a JC^*-algebra $(a, b \in E \Rightarrow a^*, (ab + ba)/2 \in E)$.

Theorem (W. Kaup 1984)

$P \colon A \to A$ a linear projection of norm 1 $\Rightarrow E = P(A)$ is (isometric to) a JB^*-triple with triple product $\{a, b, c\}_P = P(\{a, b, c\})$.

Arazy-Friedman (1978): For $A = \mathcal{K}(H)$, described $P(A)$

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Reversible subtriples

Recall: a (concrete) JC^*-triple is a closed $E \subseteq \mathcal{B}(H)$ such that

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E is called reversible if

$$n \geq 2, a_1, \ldots, a_{2n+1} \in E \Rightarrow a^1 a_2^* a_3 a_4^* \cdots a_{2n+1} + a_{2n+1} a_{2n}^* \cdots a_2^* a_1 \in E$$

or equivalently if

$$n \geq 2, a_1, \ldots, a_{2n+1} \in E \Rightarrow [a_1, \cdots, a_{2n+1}] + [a_{2n+1}, \cdots, a_1] \in E$$

where

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Reversible Jordan algebras

Jordan product (on $\mathcal{B}(H)$) is $a \circ b = \frac{1}{2}(ab + ba)$.

A (concrete) JC-algebra is a closed Jordan subalgebra of $\mathcal{B}(H)_{sa} = \{x \in \mathcal{B}(H) : x = x^*\}$.

JC^*-algebra A is a closed Jordan *-subalgebra of $\mathcal{B}(H)$ ($\iff A_{sa}$ is a JC-algebra).

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A JC^*-subalgebra $A \subseteq \mathcal{B}(H)$ is reversible $\iff A_{sa}$ is reversible $\iff A$ is reversible as a JC^*-triple.

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Example

Spin factor V_n is (the JC^*-algebra) spanned by the identity and n 'spins' $s_1, \ldots, s_n$ (which satisfy $s_i^2 = 1$, $s_i \circ s_j = 0$ for $i \neq j$).
Pauli matrices are:

$$\sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

Define $s_1, \ldots, s_4 \in M_4(\mathbb{C}) \cong M_2(\mathbb{C}) \otimes M_2(\mathbb{C})$

$$s_1 = \sigma_1 \otimes I_2, s_2 = \sigma_2 \otimes I_2, s_3 = \sigma_3 \otimes \sigma_1, s_4 = \sigma_3 \otimes \sigma_2.$$

$$s_1 s_2 s_3 s_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = -\sigma_3 \otimes \sigma_3 = s_4 s_3 s_2 s_1 \notin V_4$$

V_4 is not reversible.

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V_4 is not reversible.

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By a triple homomorphism $\pi\colon E \to F$ between JC^*-triples we mean a linear map with

$$\pi\{a, b, c\} = \{\pi(a), \pi(b), \pi(c)\}$$

A TRO is a closed $T \subset \mathcal{B}(H)$ with $a, b, c \in T \Rightarrow [a, b, c] = ab^*c \in T$.

We speak of TRO homomorphisms, Jordan homomorphisms and Jordan *-homomorphisms

TRO homoms and Jordan *-homoms are triple homoms.

A JC^*-triple E is called universally reversible if $\pi(E)$ is reversible $\forall \pi\colon E \to \mathcal{B}(K)$ a triple homom.

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Example (Hilbert)

Hilbert space $H = \ell^2_d$ is a JC^*-triple in several ways, such as

$$H_{\text{row}} = \left\{ \begin{pmatrix} z_2 & z_2 & \cdots & z_d \\ 0 & 0 & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & 0 \end{pmatrix} \right\}, \quad H_{\text{col}} = \left\{ \begin{pmatrix} z_2 & 0 & \cdots & 0 \\ z_2 & 0 & & 0 \\ \vdots & & \ddots & \\ z_d & 0 & & 0 \end{pmatrix} \right\}$$

H_{row} and H_{col} are TROs — hence reversible.
3rd reversible representation:

$$\pi \colon H_{\text{row}} \to \mathcal{B}(H) \oplus \mathcal{B}(H) \subset \mathcal{B}(H \oplus H)$$
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Intrinsic versus extrinsic JC^*-triple

As in the example $E = H$, we may abstract from from a specific concrete JC^*-subtriple $E \subseteq \mathcal{B}(H)$ to features common to all triple isomorphic copies $\pi\colon E \to \pi(E) \subseteq \mathcal{B}(K)$.

If E and F are (concrete) JC^*-triples, $\pi\colon E \to F$ is a triple isomorphism $\iff$ it is a surjective linear isometry.

Essential then: E with norm or E with $\{\cdot,\cdot,\cdot\}$

Reversibility is extrinsic.

$$n \geq 2, a_1, \ldots, a_{2n+1} \in E \Rightarrow [a_1, \cdots, a_{2n+1}] + [a_{2n+1}, \cdots, a_1] \in E$$

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Universal TRO of a JC^*-triple

Theorem

Given an abstract JC^*-triple E, there exists a universal (largest) TRO $T^*(E)$ generated by E.

More precisely, there exists an isometric embedding $E \overset{\alpha_E}{\hookrightarrow} T^*(E)$, a triple homom. into a TRO $T^*(E)$ with the universal property

where $\pi\colon E \to T$ is any given triple homomorphism with values in a TRO T, and $\tilde{\pi}\colon T^*(E) \to T$ is a TRO homomorphism $\tilde{\pi}$ unique given π.

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For a JC^*-algebra A, $(T^*(A), \alpha_A) \equiv (C_J^*(A), \beta_A)$

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Theorem

For a JC^*-algebra A, $(T^*(A), \alpha_A) \equiv (C^*_J(A), \beta_A)$

Universal TRO of a JC^*-triple

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Given an abstract JC^*-triple E, there exists a universal (largest) TRO $T^*(E)$ generated by E.

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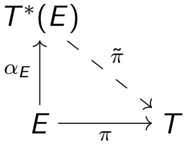

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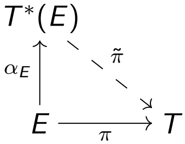

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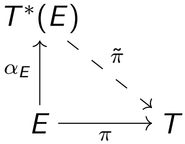

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$$
\begin{array}{ccc}
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{\scriptstyle \alpha_E}\uparrow & \searrow{\scriptstyle \tilde{\pi}} & \\
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\end{array}
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Canonical involution

Given a TRO $T \subseteq \mathcal{B}(H)$, the opposite TRO T^{op} of T is the same space T with new ternary product

$$[x, y, z]^{\mathrm{op}} = zy^*x$$

Represented as a TRO by $x \mapsto x^t$

Example. $T = H_{\mathrm{row}} \Rightarrow T^{\mathrm{op}} \equiv H_{\mathrm{col}}$

T and T^{op} are isometric, identical as JC^*-triples

$$\{x, y, z\} = ([x, y, z] + [z, y, x])/2$$

For $T^*(E)$, we must have $T^*(E)^{\mathrm{op}} \equiv T^*(E)$. Implies a TRO anti-automorphism $\Phi \colon T^*(E) \to T^*(E)$ fixing all points in $\alpha_E(E)$.

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Universal TRO of universally reversible E

Theorem

If E is universally reversible and embedded in $\mathcal{B}(H)$ such that there exists a TRO anti-automorphism of $TRO(E)$ fixing all points of E, then $T^*(E) = TRO(E)$.

Check the hypothesis?

Theorem (Hanche-Olsen 1983)

*If A is a JC^*-algebra, then A is universally reversible if and only if there are no surjective Jordan *-homomorphism $A \to V_n$ with $4 \leq n \leq \infty$.*

Theorem

A JC^*-triple E is universally reversible if and only if E has no triple representations of E onto any spin factor of dimension ≥ 5 or any Hilbert space of dimension ≥ 3.

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Corollary

A TRO T is universally reversible (as a JC^*-triple) $\iff$ there are no representations of T onto Hilbert space of dimension ≥ 3.

Corollary

Let T be a TRO with no nonzero representations onto a Hilbert space of any dimension other than 2. Then

$$T^*(T) = T \oplus_\infty T^t \equiv T \oplus_\infty T^{\mathrm{op}}$$

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Operator space

Banach spaces E are the closed linear subspaces of [commutative] C^*-algebras

A closed subspace $E \subset \mathcal{B}(H)$ has an additional feature arising from its position. $M_n(\mathcal{B}(H)) \cong \mathcal{B}(H^n)$.

An (abstract) operator space is a Banach space E together with norms $\|\cdot\|_n$ induced on matrix spaces $M_n(E)$ $(n = 1, 2, \ldots)$ from an isometric embedding $E \overset{\phi}{\hookrightarrow} \mathcal{B}(H)$ and

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Operator space structures of a JC^*-triple

Definition

Given a JC^*-triple E, a JC-operator space structure on E is an operator space structure induced by an isometric embedding $\pi\colon E \to \mathcal{B}(H)$ onto a concrete JC^*-triple $\pi(E) \subseteq \mathcal{B}(H)$.

Example

A C^*-algebra A has a canonical operator space structure as a C^*-algebra. (C^*-norm on $M_n(A)$). Same for a TRO T.
Note T and T^{op} are the same as JC^*-triples, but matrix norms on $M_n(T)$ and $M_n(T^{\mathrm{op}})$ don't usually match.
A third option is to consider
$\{x \oplus x^t : x \in T\} \subset T \oplus T^t \equiv T \oplus T^{\mathrm{op}}$

Remark: TRO isomorphisms $\phi\colon T_1 \to T_2$ are complete isometries.
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Note T and T^{op} are the same as JC^*-triples, but matrix norms on $M_n(T)$ and $M_n(T^{\mathrm{op}})$ don't usually match.
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Remark: TRO isomorphisms $\phi\colon T_1 \to T_2$ are complete isometries.
Triple isomorphisms $\phi\colon E_1 \to E_2$ are isometries.

Operator space structures of a JC^*-triple

Definition

Given a JC^*-triple E, a JC-operator space structure on E is an operator space structure induced by an isometric embedding $\pi\colon E \to \mathcal{B}(H)$ onto a concrete JC^*-triple $\pi(E) \subseteq \mathcal{B}(H)$.

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Corollary (of existence of $T^*(E)$)

For each JC-operator space structure on E, there exists a TRO ideal $\mathcal{I} \subset T^*(E)$ with $\mathcal{I} \cap \alpha_E(E) = \{0\}$ such that E is completely isometric to $E_{\mathcal{I}}$, the operator space structure on E determined by the isometric embedding $E \to T^*(E)/\mathcal{I}$ ($x \mapsto \alpha_E(x) + \mathcal{I}$)

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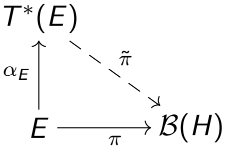

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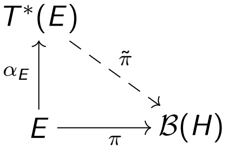

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JC-operator space structure determines algebraic structure?

Theorem

Suppose E is a universally reversible abstract JC^*-triple [linear isometric class].
Then $\mathcal{I} \mapsto E_{\mathcal{I}}$ is a bijective correspondence between the operator space ideals of $T^*(E)$ and the JC-operator space structures of E

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Example

$E = \mathcal{B}(H)$. $T^*(E) = \mathcal{B}(H) \oplus \mathcal{B}(H)$. $\alpha_E(x) = x \oplus x^t$.

Ideals of $\mathcal{B}(H)$ (for H separable) are $\{0\}, \mathcal{K}(H), \mathcal{B}(H)$.

Ideals of $T^*(E)$ are
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2. If E is universally reversible, then $T = T^*(E)$ is universally reversible and $\exists \Phi \colon T \to T$ a TRO antiautomorphism with $E = \{x \in T : \Phi(x) = x\}$. Any converse statement?

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