Dyadic Martingales and some applications to Analysis

Artur Nicolau

Universitat Autònoma de Barcelona

Abstract

The main purpose of the course is to study certain problems in Classical Analysis using Dyadic Martingale techniques.

We will present Fatou type results, L^p estimates and versions of the Law of the Iterated Logarithm (LIL), relating the growth of a dyadic martingale and the size of its quadratic variation. These results tell that the asymptotic behaviour of a dyadic martingale is governed by the size of its quadratic variation. The continuous versions of these results are classical Theorems due to A. Calderón, E. Stein, A. Zygmund and others as well as some more recent results by R. Bauelos, I. Klemes and C. Moore. In the eighties N. Makarov proved a series of deep results on boundary behaviour of conformal maps and metric properties of harmonic measure as consequences of results on dyadic martingales. We will review some of these results.

We will also apply these discrete techniques to study three other questions in Geometric Function Theory:

- To what extent can a doubling measure be singular with respect to Lebesgue measure?
- What is the analogue of the Lusin Area Function for analytic self mappings of the disc?
- How large is the set of points where a function in the Zygmund class is differentiable?