Hyperbolic convexity and conformal reflections

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Abstract

A classical result of Jorgensen from the 1950s says that a Euclidean disc is a hyperbolically convex subset of any simply-connected hyperbolic plane domain that contains it. This result has been generalized in various directions by Minda and Solynin. We give an application of these ideas to the problem of finding the best constant in the Hayman-Wu theorem. Jorgensen's theorem prompted us to look for a conformally invariant characterization of a Euclidean disc in a hyperbolic plane domain. We give a criterion in terms of the existence of a conformal reflection; a related criterion for hyperbolic convexity follows. If time allows, we will discuss multiply-connected and quasiconformal analogues of the criterion.