Functional inequalities and Hamilton-Jacobi equations in geodesic spaces

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Abstract

We study the connection between the p–Talagrand inequality and the q–logarithmic Sololev inequality for conjugate exponents $p \geq 2$, $q \leq 2$ in proper geodesic metric spaces. By means of a general Hamilton–Jacobi semigroup we prove that these are equivalent, and moreover equivalent to the hypercontractivity of the Hamilton–Jacobi semigroup. Our results generalize those of Lott and Villani. They can be applied to deduce the p-Talagrand inequality in the sub-Riemannian setting of the Heisenberg group.