An Roinn Oideachais agus Scileanna

LEAVING CERTIFICATE

MATHEMATICS

SYLLABUS

FOUNDATION, ORDINARY & HIGHER LEVEL

For examination in 2012 only

Explanatory note

When the syllabus revision is complete, Leaving Certificate Mathematics will comprise material across 5 strands of study: Statistics and Probability, Geometry and Trigonometry, Number, Algebra, and Functions.

This syllabus, which is being introduced in September 2010 for examination in June 2012, contains three sections:

A. strand 1 (statistics and probability) and strand 2 (geometry and trigonometry)

B. the geometry course

C. material retained from the previous Leaving Certificate Mathematics syllabus.

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MATHEMATICS

Leaving Certificate Mathematics

Introduction and rationale

Mathematics is a wide-ranging subject with many aspects. Most people are familiar with the fact that mathematics is an intellectual discipline that deals with abstractions, logical arguments, deduction and calculation. But mathematics is also an expression of the human mind reflecting the active will, the contemplative reason and the desire for aesthetic perfection. It is also about pattern, the mathematics of which can be used to explain and control natural happenings and situations. Increasingly, mathematics is the key to opportunity. No longer simply the language of science, mathematics contributes in direct and fundamental ways to business, finance, health and defence. For students it opens doors to careers. For citizens it enables informed decisions. For nations it provides knowledge to compete in a technological community. Participating fully in the world of the future involves tapping into the power of mathematics.

Mathematical knowledge and skills are held in high esteem and are seen to have a significant role to play in the development of the knowledge society and the culture of enterprise and innovation associated with it. Mathematics education should be appropriate to the abilities, needs and interests of learners and should reflect the broad nature of the subject and its potential for enhancing their development. The elementary aspects of mathematics, use of arithmetic and the display of information by means of a graph are an everyday commonplace. Advanced mathematics is also widely used, but often in an unseen and unadvertised way. The mathematics of error-correcting codes is applied to CD players and to computers. The stunning pictures of far away planets and nebulae sent by Voyager II and Hubble could not have had their crispness and quality without such mathematics. In fact, Voyager's journey to the planets could not have been calculated without the mathematics of differential equations. In ecology mathematics is used when studying the laws of population change. Statistics not only provides the theory and methodology for the analysis of wide varieties of data but is essential in medicine for analysing data on the causes of illness and on the utility of new drugs. Travel by aeroplane would not be possible without the mathematics of airflow and of control systems. Body scanners are the expression of subtle mathematics discovered in the 19th century, which makes it possible to construct an image of the inside of an object from information on a number of single X-ray views of it. Thus mathematics is often involved in matters of life and death.

Aim

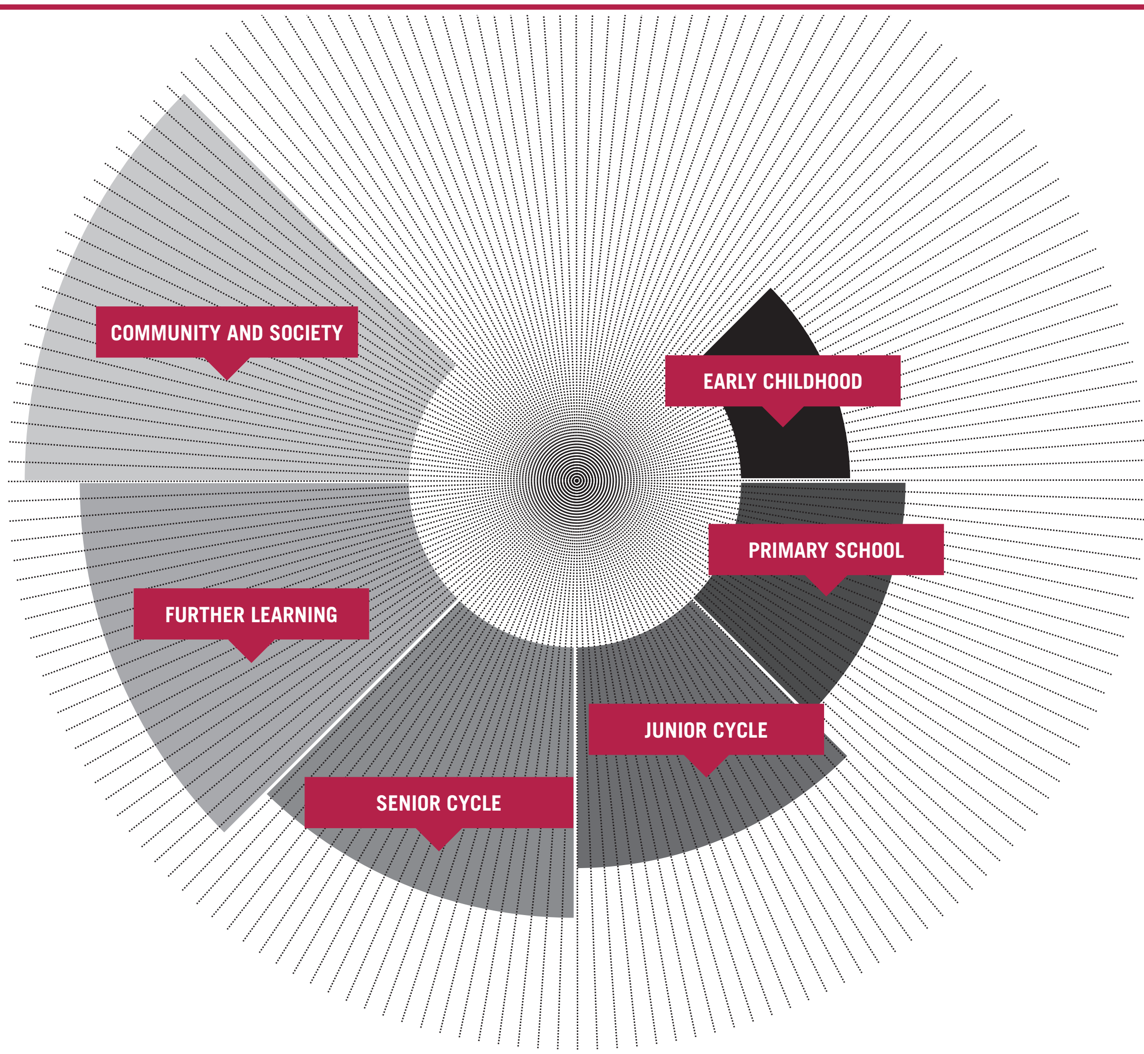
Leaving Certificate Mathematics aims to develop mathematical knowledge, skills and understanding needed for continuing education, life and work. By teaching mathematics in contexts that allow learners to see connections within mathematics, between mathematics and other subjects, and between mathematics and its applications to real life, it is envisaged that learners will develop a flexible, disciplined way of thinking and the enthusiasm to search for creative solutions.

Objectives

The objectives of Leaving Certificate Mathematics are that learners develop

- the ability to recall relevant mathematical facts
- instrumental understanding ("knowing how") and necessary psychomotor skills (skills of physical co-ordination)
- relational understanding ("knowing why")
- the ability to apply their mathematical knowledge and skill to solve problems in familiar and unfamiliar contexts
- analytical and creative powers in mathematics
- an appreciation of mathematics and its uses
- a positive disposition towards mathematics.

Related learning



The way in which mathematics learnt at different stages links is important to the overall development of mathematical understanding. The study of Leaving Certificate Mathematics encourages learners to use the numeracy and problem solving skills developed in early childhood education, primary mathematics and junior cycle mathematics. The emphasis is on building connected and integrated mathematical understanding. As learners progress through their education, mathematical skills, concepts and knowledge are developed when they work in more demanding contexts and develop more sophisticated approaches to problem solving. In this way mathematical learning is cumulative, with work at each level building on and deepening what students have learned at the previous level.

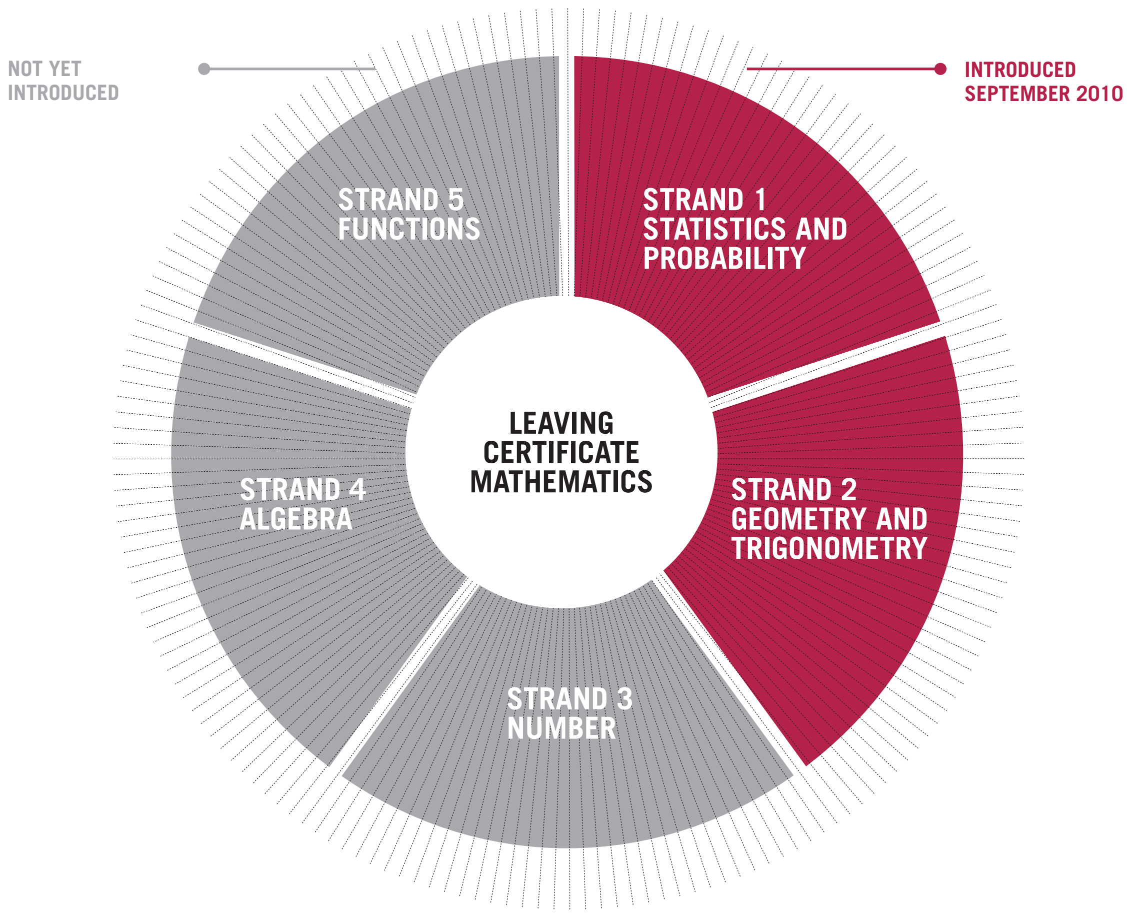
Mathematics is not learned in isolation; it has significant connections with other curriculum subjects. Many science subjects are quantitative in nature and learners are expected to be able to work with data, produce graphs and interpret patterns and trends. Design and Communication Graphics uses drawings in the analysis and solution of two- and three-dimensional problems through the rigorous application of geometric principles. In Geography learners use ratio to determine scale, and every day learners use timetables, clocks and currency conversions to make life easier. Consumers need basic financial awareness and in Home Economics learners use mathematics when budgeting and making value for money judgements. Learners use mathematics in Economics for describing human behaviour. In Business Studies learners see how mathematics can be used by business organisations in accounting, marketing, inventory management, sales forecasting and financial analysis.

Mathematics, Music and Art have a long historical relationship. As early as the fifth century B.C., Pythagoras uncovered mathematical relationships in music; many works of art are rich in mathematical structure. The modern mathematics of fractal geometry continues to inform composers and artists. Mathematics sharpens critical thinking skills, and by empowering learners to critically evaluate information and knowledge it promotes their development as statistically aware consumers.

SYLLABUS OVERVIEW

Syllabus overview

Leaving Certificate Mathematics



Structure

When complete, the Leaving Certificate Mathematics syllabus will comprise five strands:

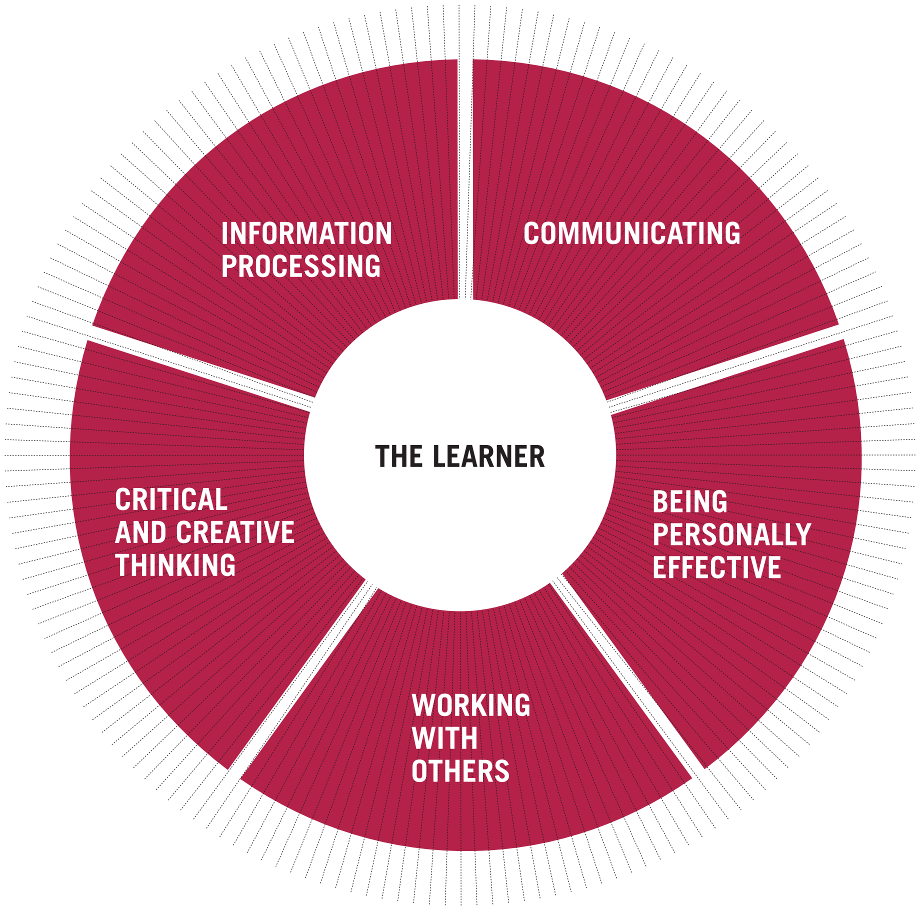
1. Statistics and probability
2. Geometry and trigonometry
3. Number
4. Algebra
5. Functions

Learning outcomes specified for strands 1 and 2 are listed. The selection of topics to be covered in the strand is presented in tabular form, with material ranging from Foundation level to Higher level. Syllabus material at each syllabus level is a sub-set of the next level.

Time allocation

The Leaving Certificate Mathematics syllabus is designed as a 180–hour course of study.

Key Skills



There are five key skills identified as central to teaching and learning across the curriculum at senior cycle. These are *information processing, being personally effective, communicating, critical and creative thinking* and *working with others*. These key skills are important for all learners to reach their full potential – both during their time in school and in the future – and to participate fully in society, including family life, the world of work and lifelong learning. By engaging with key skills learners enhance their ability to learn, broaden the scope of their learning and increase their capacity for learning.

Leaving Certificate Mathematics develops key skills in the following ways.

Information processing

Successful mathematics learning requires the efficient processing of the information that defines the mathematical tasks. Information is readily accessible from a variety of sources and information processing relates to the ways in which learners make sense of, or interpret, the information to which they are exposed.

Critical and creative thinking

There is a strong emphasis on investigation in mathematics and engaging in the investigative process requires learners to critically evaluate information and think creatively about it. Learners are encouraged to solve problems in a variety of ways and are required to evaluate methods and arguments and to justify their claims and results.

Communicating

In mathematics learners are encouraged to discuss approaches and solutions to problems and are expected to consider and listen to other viewpoints. Since mathematics emphasises investigation an important aspect of this is communicating findings to a variety of audiences in different ways.

Working with others

In mathematics learners are encouraged to work together in groups to generate ideas, problem solve and evaluate methods.

Being personally effective

Studying mathematics empowers learners to gain knowledge and skills that will benefit them directly in other aspects of their everyday lives. They participate in a learning environment that is open to new ideas and gain confidence in expressing their mathematical ideas and considering those of others.

While the Leaving Certificate Mathematics syllabus places particular emphasis on the development and use of information processing, logical thinking and problem-solving skills, the approach to teaching and learning involved gives prominence to learners being able to develop their skills in communicating and working with others. By adopting a variety of approaches and strategies for solving problems in mathematics, learners develop their self-confidence and personal effectiveness. The key skills are embedded within the learning outcomes and are assessed in the context of the learning outcomes.

In Leaving Certificate Mathematics students not only learn procedures and acquire reliable methods for producing correct solutions on paper-and-pencil exercises, but also learn mathematics with understanding. In particular, they should be able to explain why the procedures they apply are mathematically appropriate and justify why mathematical concepts have the properties that they do.

Teaching and learning

In line with the syllabus objectives and learning outcomes, the learners' experiences in the study of mathematics should contribute to the development of their problem-solving skills through the application of their mathematical knowledge and skills to appropriate contexts and situations. In each strand, at every syllabus level, emphasis is placed on appropriate contexts and applications of mathematics so that learners can appreciate its relevance to their current and future lives. The focus should be on learners understanding the concepts involved, building from the concrete to the abstract and from the informal to the formal.

Learners will build on their knowledge of mathematics constructed initially through their exploration of mathematics in the primary school and through their continuation of this exploration at junior cycle. Particular emphasis is placed on promoting learners' confidence in themselves (confidence that they can do mathematics) and in the subject (confidence that mathematics makes sense). Through the use of meaningful contexts, opportunities are presented for learners to achieve success.

Learners will integrate their knowledge and understanding of mathematics with economic and social applications of mathematics. By becoming statistically aware consumers, learners are able to critically evaluate knowledge claims and learn to interrogate and interpret data – a skill which has a value far beyond mathematics wherever data is used as evidence to support argument.

The variety of activities that learners engage in enables them to take charge of their own learning by setting goals, developing action plans and receiving and responding to assessment feedback. As well as varied teaching strategies, varied assessment strategies will provide information that can be used as feedback for teachers so that teaching and learning activities can be modified in ways which best suit individual learners. Results of assessments may also be used by teachers to reflect on their teaching practices so that instructional sequences and activities can be modified as required. Feedback to learners about their performance is critical to their learning and enables them to develop as learners. This formative assessment when matched to the intended learning outcomes helps to ensure consistency between the aim and objectives of the syllabus and its assessment. A wide range of assessment methods may be used, including investigations, class tests, investigation reports, oral explanation, etc.

Careful attention must be paid to learners who may still be experiencing difficulty with some of the material covered in the junior cycle. Nonetheless, they need to learn to cope with mathematics in everyday life and perhaps in further study. Their experience of Leaving Certificate Mathematics must therefore assist them in developing a clearer knowledge of and improved skills in, basic mathematics, and an awareness of its usefulness. Appropriate new material should also be introduced so that the learners can feel that they are progressing. At Leaving Certificate, the course followed should pay great attention to consolidating the foundation laid in the junior cycle and to addressing practical issues; but it should also cover new topics and lay a foundation for progress to the more traditional study of mathematics in the areas of algebra, geometry and functions.

Differentiation

In strands 1 and 2 the learning outcomes are set out in terms of Foundation level, Ordinary level and Higher level and each level is a subset of the next level. So, learners studying Higher level are expected to achieve the Foundation level, Ordinary level and Higher level learning outcomes. Learners studying at Ordinary level are expected to achieve the Foundation level learning outcomes as well as those at Ordinary level.

Mathematics at Higher level is geared to the needs of learners who may proceed with their study of mathematics to third level. However, not all learners are future specialists or even future users of academic mathematics. Moreover, when they start to study the material some of them are only beginning to deal with abstract concepts.

Given the diversity of ability, particular emphasis must be given to pace. Provision must be made not only for the academic student of the future, but also for the citizen of a society in which mathematics appears in, and is applied to, everyday life. The syllabus therefore focuses on material that underlies academic mathematical studies, ensuring that learners have a chance to develop their mathematical abilities and interests to a high level. It also covers the more practical and obviously applicable topics that learners are meeting in their lives outside school.

For Higher level, particular emphasis can be placed on the development of powers of abstraction and generalisation and on the idea of rigorous proof, hence giving learners a feeling for the great mathematical concepts that span many centuries and cultures. Problem-solving can be addressed in both mathematical and applied contexts.

Mathematics at Ordinary level is geared to the needs of learners who are beginning to deal with abstract ideas. However, many of them may go on to use and apply mathematics in their future careers, and all of them will meet the subject to a greater or lesser degree in their daily lives. Ordinary level Mathematics, therefore, must start by offering mathematics that is meaningful and accessible to learners at their present stage of development. It should also provide for the gradual introduction of more abstract ideas, leading the learners towards the use of academic mathematics in the context of further study.

Mathematics at Foundation level places particular emphasis on the development of mathematics as a body of knowledge and skills that makes sense, and that can be used in many different ways as an efficient system for solving problems and finding answers. Alongside this, adequate attention must be paid to the acquisition and consolidation of fundamental skills, in the absence of which the learners' development and progress will be hindered. Foundation level Mathematics is intended to equip learners with the knowledge and skills required in everyday life, and it is also intended to lay the groundwork for learners who may proceed to further studies in areas in which specialist mathematics is not required.

Learners taking Foundation level Mathematics may have limited acquaintance with abstract mathematics. Thus, their experience of mathematics at Leaving Certificate should be approached in an exploratory and reflective manner, adopting a developmental and constructivist approach which prepares them for gradual progression to abstract concepts. An appeal should be made to different interests and ways of learning, for example by paying attention to visual and spatial as well as to numerical aspects.

Differentiation will also apply in how strands 1 and 2 are assessed at Foundation, Ordinary and Higher levels. Each level is a sub-set of the next level; differentiation at the point of assessment will be reflected in the depth of treatment of the questions. It will be achieved also through the language level in the examination questions and the amount of structured support provided for examination candidates at different syllabus levels, particularly at Foundation level. Information about the general assessment criteria applying to the examination of strands 1 and 2 is set out in the assessment section (page 25).

STRANDS OF STUDY

Strand 1: Statistics and Probability

The aim of the probability unit is two-fold: it provides certain understandings intrinsic to problem solving and it underpins the statistics unit. It is expected that the conduct of experiments (including simulations), both individually and in groups, will form the primary vehicle through which the knowledge, understanding and skills in probability are developed. References should be made to the appropriate contexts and applications of probability.

It is envisaged that throughout the statistics course students will be involved in identifying problems that can be explored by the use of appropriate data, designing investigations, collecting data, exploring and using patterns and relationships in data, solving problems, and communicating findings. This strand also involves interpreting statistical information, evaluating data-based arguments, and dealing with uncertainty and variation.

Strand 1: Probability

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
1.1 Counting	– list outcomes of an experiment – apply the fundamental principle of counting	– count the arrangements of n distinct objects ($n!$) – count the number of ways of arranging r objects from n distinct objects	– count the number of ways of selecting r objects from n distinct objects
1.2 Concepts of probability	– decide whether an everyday event is likely or unlikely to happen – recognise that probability is a measure on a scale of 0-1 of how likely an event is to occur – connect with set theory; discuss experiments, outcomes, sample spaces – use the language of probability to discuss events, including those with equally likely outcomes – estimate probabilities from experimental data – recognise that, if an experiment is repeated, there will be different outcomes and that increasing the number of times an experiment is repeated generally leads to better estimates of probability – associate the probability of an event with its long run relative frequency	– discuss basic rules of probability (AND/ OR, mutually exclusive) through the use of Venn Diagrams – calculate expected value and understand that this does not need to be one of the outcomes – recognise the role of expected value in decision making and explore the issue of fair games	– extend their understanding of the basic rules of probability (AND/ OR, mutually exclusive) through the use of formulae • Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ • Multiplication Rule (Independent Events): $P(A \cap B) = P(A) \times P(B)$ • Multiplication Rule (General Case): $P(A \cap B) = P(A) \times P(B \mid A)$ – solve problems involving conditional probability in a systematic way – appreciate that in general $P(A \mid B) \neq P(B \mid A)$ – examine the implications of $P(A \mid B) \neq P(B \mid A)$ in context
1.3 Outcomes of random processes	– construct sample spaces to show all possible outcomes for two independent events – apply the principle that in the case of equally likely outcomes the probability is given by the number of outcomes of interest divided by the total number of outcomes (examples using coins, dice , spinners, urns with coloured objects, playing cards etc.)	– find the probability that two independent events both occur – apply an understanding of Bernoulli trials* – solve problems involving up to 3 Bernoulli trials – calculate the probability that the 1st success occurs on the n^{th} Bernoulli trial where n is specified	– solve problems involving calculating the probability of k successes in n repeated Bernoulli trials (normal approximation not required) – calculate the probability that the k^{th} success occurs on the n^{th} Bernoulli trial – use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions – solve problems involving reading probabilities from the normal distribution tables

*A Bernoulli trial is an experiment whose outcome is random and can be either of two possibilities: "success" or "failure".

Strand 1: Statistics

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
1.4 Statistical reasoning with an aim to becoming a statistically aware consumer	– engage in discussions about the purpose of statistics and recognise misconceptions and misuses of statistics – discuss populations and samples – decide to what extent conclusions can be generalised – work with different types of data (categorical/numerical/ordinal, discrete/continuous) in order to clarify the problem at hand	– work with different types of bivariate data	
1.5 Finding, collecting and organising data	– clarify the problem at hand – formulate one (or more) questions that can be answered with data – explore different ways of collecting data – generate data, or source data from other sources including the internet – select a sample (Simple Random Sample) – recognise the importance of randomisation and the role of the control group in studies – design a plan and collect data on the basis of above knowledge	– discuss different types of studies: sample surveys, observational studies and designed experiments – design a plan and collect data on the basis of above knowledge	– recognise the importance of representativeness so as to avoid biased samples – recognise biases, limitations and ethical issues of each type of study – select a sample (stratified, cluster, quota, etc. – no formulae required, just definitions of these) – design a plan and collect data on the basis of above knowledge

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
1.6 Representing data graphically and numerically	**Graphical** – select appropriate graphical or numerical methods to describe the sample (univariate data only) – evaluate the effectiveness of different displays in representing the findings of a statistical investigation conducted by others – use stem and leaf plots and histograms (equal intervals) to display data **Numerical** – use a variety of summary statistics to decribe the data • central tendency: mean, median, mode • variability: range	**Graphical** – describe the sample (both univariate and bivariate data) by selecting appropriate graphical or numerical methods – explore the distribution of data, including concepts of symmetry and skewness – compare data sets using back to back stem and leaf plots – determine the relationship between variables using scatterplots – recognise that correlation is a value from -1 to +1 and that it measures the extent of linear relationship between two variables – match correlation coefficient values to appropriate scatter plots **Numerical** – recognise standard deviation as a measure of variability – use a calculator to calculate standard deviation – use a stem and leaf plot to calculate quartiles and the interquartile range	**Graphical** – analyse plots of the data to explain differences in measures of centre and spread – draw the line of best fit by eye – make predictions based on the line of best fit – calculate the correlation coefficient by calculator and understand that correlation does not imply causality **Numerical** – recognise the existence and effect of outliers – use percentiles to assign relative standing – use the interquartile range appropriately when analysing data

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
1.7 Analysing, interpreting and drawing inferences from data*	– recognise how sampling variability influences the use of sample information to make statements about the population – develop appropriate tools to describe variability, drawing inferences about the population from the sample – interpret the analysis – relate the interpretation to the original question	– interpret a histogram in terms of distribution of data – make decisions based on the empirical rule	– recognise the concept of a hypothesis test – calculate the margin of error ($\frac{1}{\sqrt{n}}$) for a population proportion – conduct a hypothesis test on a population proportion using the margin of error

Students learn about	Students should be able to
1.8 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

* The final syllabus will contain additional material in this section, which has been deferred for an interim peroiod until students coming through to senior cycle have completed the relevant revised syllabus material in the junior cycle.

Strand 2: Geometry and Trigonometry

The synthetic geometry covered at Leaving Certificate is a continuation of that studied at junior cycle. It is based on the *Geometry Course for Post-primary School Mathematics*, including terms, definitions, axioms, propositions, theorems, converses and corollaries. The formal underpinning for the system of post-primary geometry is that described by Barry (2001)[1].

At each syllabus level, knowledge of geometrical results from the corresponding syllabus level at Junior Certificate is assumed. It is also envisaged that at all levels students will engage with a dynamic geometry software package.

Particularly at Foundation level and Ordinary level, the geometrical results below should first be encountered by students through investigation and discovery. Students are asked to accept these results as true for the purpose of applying them to various contextualised and abstract problems. They should come to appreciate that certain features of shapes or diagrams appear to be independent of the particular examples chosen. These apparently constant features or results can be established in a formal manner through logical proof. Even at the investigative stage, ideas involved in mathematical proof can be developed. Students should become familiar with the formal proofs of the specified theorems (some of which are examinable at Higher level). Students will be assessed by means of problems that can be solved using the theory.

1 P.D. Barry. *Geometry with Trigonometry*, Horwood, Chicester (2001)

Strand 1: Geometry and Trigonometry

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
2.1 Synthetic geometry *	– perform constructions **18, 19, 20** (see *Geometry Course for Post-primary School Mathematics*)	– perform constructions **16, 17, 21** (see *Geometry Course for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies – investigate theorems **7, 8, 11, 12, 13, 16, 17, 18, 20, 21** and **corollary 6** (see *Geometry Course for Post-primary School Mathematics*) and use them to solve problems	– perform constructions **1-15** and **22** (see *Geometry Course for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: is equivalent to, if and only if, proof by contradiction – prove theorems **11, 12, 13,** concerning ratios (see *Geometry Course for Post-primary School Mathematics*), which lay the proper foundation for the proof of the theorem of Pythagoras studied at junior cycle
2.2 Co-ordinate geometry	– use slopes to show that two lines are • parallel • perpendicular	– calculate the area of a triangle – recognise the fact that the relationships $y= mx+c$, $y-y_1 = m (x- x_1)$ and $ax + by + c = 0$ are linear – solve problems involving slopes of lines – recognise that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle centre (h, k) and radius r – solve problems involving a line and a circle with centre (0, 0)	– solve problems involving • the perpendicular distance from a point to a line • the angle between two lines – divide a line segment in a given ratio m:n – recognise that $x^2+y^2 +2gx+2fy+c = 0$ represents the relationship between the x and y co-ordinates of points on a circle centre (-g,-f) and radius r where $r = \sqrt{(g^2+f^2 - c)}$ – solve problems involving a line and a circle

* In the examination, candidates will have the option of answering a question on the synthetic geometry set out here, or answering a problem-solving question based on the geometrical results from the corresponding syllabus level at Junior Certificate. This option will apply for a three year period only, for candidates sitting the Leaving Certificate examination in 2012, 2013 and 2014. There will be no choice after that stage.

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
2.3 Trigonometry	– solve problems that involve finding heights and distances from right-angled triangles (2D only) – use of the theorem of Pythagoras to solve problems (2D only) – solve problems that involve calculating the cosine, sine and tangent of angles between 0° and 90°	– use trigonometry to calculate the area of a triangle – use the sine and cosine rules to solve problems (2D) – define sin θ and cos θ for all values of θ – define tan θ – calculate the area of a sector of a circle and the length of an arc and solve problems involving these calculations	– use trigonometry to solve problems in 3D – graph the trigonometric functions sine, cosine, tangent – graph trigonometric functions of type aSin $n\theta$, aCos $n\theta$ for $a, n \in \mathbb{N}$ – solve trigonometric equations such as Sin $n\theta = 0$ and Cos $n\theta = \frac{1}{2}$ giving all solutions – use the radian measure of angles – derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9 (see appendix) – apply the trigonometric formulae 1-24 (see appendix)
2.4 Transformation geometry	– investigate enlargements paying attention to • centre of enlargement • scale factor k, where $0<k<1$, $k>1$ $k \in \mathbb{Q}$ • area – solve problems involving enlargements		

Students learn about	Students should be able to
2.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

ASSESSMENT

Assessment

Assessment in Leaving Certificate Mathematics

Assessment for certification will be based on the aims, objectives, and learning outcomes of this syllabus. Differentiation at the point of assessment will be achieved through examinations at three levels – Foundation level, Ordinary level, and Higher level. In strands 1 and 2 of this syllabus, each level is a subset of the next level. Learners at Higher level are expected to achieve the Foundation level, Ordinary level and Higher level learning outcomes. Learners at Ordinary level are expected to achieve the Foundation level learning outcomes as well as those at Ordinary level. Differentiation will be achieved also through the language level in the examination questions, the stimulus material presented, and the amount of structured support given in the questions, especially for candidates at Foundation level.

Assessment components

There are two assessment components at each level

- Mathematics Paper 1
- Mathematics Paper 2

General assessment criteria for Strands 1 and 2

A high level of achievement in Mathematics is characterised by a demonstration of a thorough knowledge and comprehensive understanding of mathematics as described by the learning outcomes associated with each strand. The learner is able to make deductions with insight even in unfamiliar contexts and can move confidently between different forms of representation. When investigating challenging problems, the learner recognises pattern structures, describes them as relationships or general rules, draws conclusions and provides justification or proof. The learner presents a concise, reasoned justification for the method and process and, where appropriate, considers the range of approaches which could have been used, including the use of technology.

A moderate level of achievement in Mathematics is characterised by a demonstration of a broad knowledge and good understanding of mathematics as described by the learning outcomes associated with each strand. The learner is able to make deductions with some insight even in unfamiliar contexts and can move between different forms of representation in most situations. When investigating problems of moderate complexity, the learner recognises pattern structures, describes them as relationships or general rules and draws conclusions consistent with findings. The learner successfully selects and applies skills and problem solving techniques. The learner presents a reasoned justification for the method and process and provides an evaluation of the significance and reliability of findings.

A low level of achievement in Mathematics is characterised by a demonstration of limited mathematical knowledge or understanding described by the learning outcomes associated with each strand. The learner recognises simple patterns or structures when investigating problems and applies basic problem solving techniques with some success. An attempt is made to justify the method used and to evaluate the reliability of findings.

Appendix: Trigonometric Formulae

1. $\cos^2 A + \sin^2 A = 1$

2. sine formula: $\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$

3. cosine formula: $a^2 = b^2 + c^2 - 2\,bc \cos A$

4. $\cos (A-B) = \cos A \cos B + \sin A \sin B$

5. $\cos (A+B) = \cos A \cos B - \sin A \sin B$

6. $\cos 2A = \cos^2 A - \sin^2 A$

7. $\sin (A+B) = \sin A \cos B + \cos A \sin B$

8. $\sin (A-B) = \sin A \cos B - \cos A \sin B$

9. $\tan (A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

10. $\tan (A-B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

11. $\sin 2A = 2 \sin A \cos A$

12. $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$

13. $\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

14. $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

15. $\cos^2 A = \frac{1}{2} (1 + \cos 2A)$

16. $\sin^2 A = \frac{1}{2} (1 - \cos 2A)$

17. $2 \cos A \cos B = \cos (A+B) + \cos (A+B)$

18. $2 \sin A \cos B = \sin (A+B) + \sin (A+B)$

19. $2 \sin A \sin B = \cos (A+B) - \cos (A+B)$

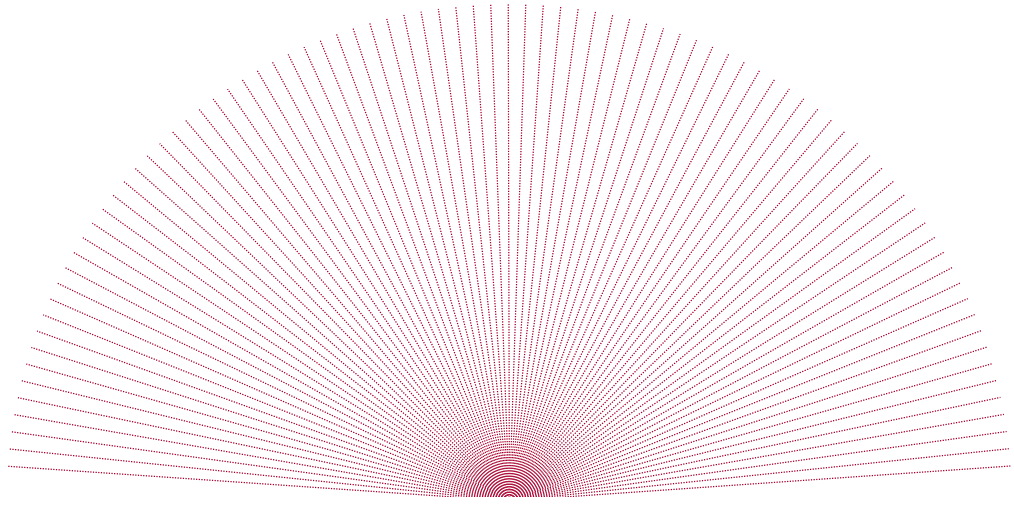
20. $2 \cos A \sin B = \sin (A+B) - \sin (A+B)$

21. $\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}$

22. $\cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}$

23. $\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}$

24. $\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}$



Section B

Geometry Course for Post-primary School Mathematics

This section sets out the agreed course in geometry for both Junior Certificate Mathematics and Leaving Certificate Mathematics. Strand 2 of the relevant syllabus document specifies the learning outcomes at the different syllabus levels.

Geometry Course for Post-primary School Mathematics

September 2010

1 Introduction

The Junior Certificate and Leaving Certificate mathematics course committees of the National Council for Curriculum and Assessment (NCCA) accepted the recommendation contained in the paper [4] to base the logical structure of post-primary school geometry on the level 1 account in Professor Barry's book [1].

To quote from [4]: We distinguish three levels:

Level 1: The fully-rigorous level, likely to be intelligible only to professional mathematicians and advanced third- and fourth-level students.

Level 2: The semiformal level, suitable for digestion by many students from (roughly) the age of 14 and upwards.

Level 3: The informal level, suitable for younger children.

This appendix sets out the agreed course in geometry for post-primary schools. It was prepared by a working group of the NCCA course committees for mathematics and, following minor amendments, was adopted by both committees for inclusion in the syllabus documents. Readers should refer to Strand 2 of the syllabus documents for Junior Certificate and Leaving Certificate mathematics for the range and depth of material to be studied at the different levels. A summary of these is given in sections 9–13 of this appendix.

The preparation and presentation of this appendix was undertaken principally by Anthony O'Farrell, with assistance from Ian Short. Helpful criticism from Stefan Bechluft-Sachs, Ann O'Shea and Richard Watson is also acknowledged.

2 The system of geometry used for the purposes of formal proofs

In the following, Geometry refers to plane geometry.

There are many formal presentations of geometry in existence, each with its own set of axioms and primitive concepts. What constitutes a valid proof in the context of one system might therefore not be valid in the context of another. Given that students will be expected to present formal proofs in the examinations, it is therefore necessary to specify the system of geometry that is to form the context for such proofs.

The formal underpinning for the system of geometry on the Junior and Leaving Certificate courses is that described by Prof. Patrick D. Barry in [1]. A properly formal presentation of such a system has the serious disadvantage that it is not readily accessible to students at this level. Accordingly, what is presented below is a necessarily simplified version that treats many concepts far more loosely than a truly formal presentation would demand. Any readers who wish to rectify this deficiency are referred to [1] for a proper scholarly treatment of the material.

Barry's system has the primitive undefined terms **plane, point, line,** $<_l$ **(precedes on a line), (open) half-plane, distance, and degree-measure**, and seven axioms: A_1: about incidence, A_2: about order on lines, A_3: about how lines separate the plane, A_4: about distance, A_5: about degree measure, A_6: about congruence of triangles, A_7: about parallels.

3 Guiding Principles

In constructing a level 2 account, we respect the principles about the relationship between the levels laid down in [4, Section 2].

The choice of material to study should be guided by applications (inside and outside Mathematics proper).

The most important reason to study synthetic geometry is to prepare the ground logically for the development of trigonometry, coordinate geometry, and vectors, which in turn have myriad applications.

We aim to keep the account as simple as possible.

We also take it as desirable that the official Irish syllabus should avoid imposing terminology that is nonstandard in international practice, or is used in a nonstandard way.

No proof should be allowed at level 2 that cannot be expanded to a complete rigorous proof at level 1, or that uses axioms or theorems that come later in the logical sequence. We aim to supply adequate proofs for all the theorems, but do not propose that only those proofs will be acceptable. It should be open to teachers and students to think about other ways to prove the results, provided they are correct and fit within the logical framework. Indeed, such activity is to be encouraged. Naturally, teachers and students will need some assurance that such variant proofs will be acceptable if presented in examination. We suggest that the discoverer of a new proof should discuss it with students and colleagues, and (if in any doubt) should refer it to the National Council for Curriculum and Assessment and/or the State Examinations Commission.

It may be helpful to note the following non-exhaustive list of salient differences between Barry's treatment and our less formal presentation.

- Whereas we may use set notation and we expect students to understand the conceptualisation of geometry in terms of sets, we more often use the language that is common when discussing geometry informally, such as "the point is/lies on the line", "the line passes through the point", etc.

- We accept and use a much lesser degree of precision in language and notation (as is apparent from some of the other items on this list).

- We state five explicit axioms, employing more informal language than Barry's, and we do not explicitly state axioms corresponding to Axioms A2 and A3 – instead we make statements without fanfare in the text.

- We accept a much looser understanding of what constitutes an **angle**, making no reference to angle-supports. We do not define the term angle. We mention reflex angles from the beginning (but make no use of them until we come to angles in circles), and quietly assume (when the time comes) that axioms that are presented by Barry in the context of wedge-angles apply also in the naturally corresponding way to reflex angles.

- When naming an angle, it is always assumed that the non-reflex angle is being referred to, unless the word "reflex" precedes or follows.

- We make no reference to results such as Pasch's property and the "crossbar theorem". (That is, we do not expect students to consider the necessity to prove such results or to have them given as axioms.)

- We refer to "the number of degrees" in an angle, whereas Barry treats this more correctly as "the degree-measure" of an angle.

- We take it that the definitions of parallelism, perpendicularity and "sidedness" are readily extended from lines to half-lines and line segments. (Hence, for example, we may refer to the opposite sides of a particular quadrilateral as being parallel, meaning that the lines of which they are subsets are parallel).

- We do not refer explicitly to triangles being **congruent** "under the correspondence $(A, B, C) \to (D, E, F)$", taking it instead that the correspondence is the one implied by the order in which the vertices are listed. That is, when we say "ΔABC is congruent to ΔDEF" we mean, using Barry's terminology, "Triangle [A,B,C] is congruent to triangle [D,E,F] under the correspondence $(A, B, C) \to (D, E, F)$".

- We do not always retain the distinction in language between an angle and its measure, relying frequently instead on the context to make the meaning clear. However, we continue the practice of distinguishing notationally between the angle $\angle ABC$ and the number $|\angle ABC|$ of degrees in the angle[1]. In the same spirit, we may refer to two angles being equal, or one being equal to the sum of two others, (when we should more precisely say that the two are equal in measure, or that the measure of one is equal to the sum of the measures of the other two). Similarly, with length, we may loosely say, for example: "opposite sides of a parallelogram are equal", or refer to "a circle of radius r". Where ambiguity does not arise, we may refer to angles using a single letter. That is, for example, if a diagram includes only two rays or segments from the point A, then the angle concerned may be referred to as $\angle A$.

Having pointed out these differences, it is perhaps worth mentioning some significant structural aspects of Barry's geometry that are retained in our less formal version:

[1]In practice, the examiners do not penalise students who leave out the bars.

- The primitive terms are almost the same, subject to the fact that their properties are conceived less formally. We treat **angle** as an extra undefined term.

- We assume that results are established in the same order as in Barry [1], up to minor local rearrangement. The exception to this is that we state all the axioms as soon as they are useful, and we bring the theorem on the angle-sum in a triangle forward to the earliest possible point (short of making it an axiom). This simplifies the proofs of a few theorems, at the expense of making it easy to see which results are theorems of so-called Neutral Geometry[2].

- **Area** is not taken to be a primitive term or a given property of regions. Rather, it is defined for triangles following the establishment of the requisite result that the products of the lengths of the sides of a triangle with their corresponding altitudes are equal, and then extended to convex quadrilaterals.

- **Isometries or other transformations** are not taken as primitive. Indeed, in our case, the treatment does not extend as far as defining them. Thus they can play no role in our proofs.

4 Outline of the Level 2 Account

We present the account by outlining:

1. A list (Section 5), of the terminology for the geometrical concepts. Each term in a theory is either undefined or defined, or at least definable. There have to be some undefined terms. (In textbooks, the undefined terms will be introduced by descriptions, and some of the defined terms will be given explicit definitions, in language appropriate to the level. We assume that previous level 3 work will have laid a foundation that will allow students to understand the undefined terms. We do not give the explicit definitions of all the definable terms. Instead we rely on the student's ordinary language, supplemented sometimes by informal remarks. For instance, we do not write out in cold blood the definition of the **side opposite** a given angle in a triangle, or the

[2] Geometry without the axiom of parallels. This is not a concern in secondary school.

definition (in terms of set membership) of what it means to say that a line **passes through** a given point. The reason why some terms **must** be given explicit definitions is that there are alternatives, and the definition specifies the starting point; the alternative descriptions of the term are then obtained as theorems.

2. A logical account (Section 6) of the synthetic geometry theory. All the material through to LC higher is presented. The individual syllabuses will identify the relevant content by referencing it by number (e.g. Theorems 1,2, 9).

3. The geometrical constructions (Section 7) that will be studied. Again, the individual syllabuses will refer to the items on this list by number when specifying what is to be studied.

4. Some guidance on teaching (Section 8).

5. Syllabus entries for each of JC-OL, JC-HL, LC-FL, LC-OL, LC-HL.

5 Terms

Undefined Terms: angle, degree, length, line, plane, point, ray, real number, set.

Most important Defined Terms: area, parallel lines, parallelogram, right angle, triangle, congruent triangles, similar triangles, tangent to a circle, area.

Other Defined terms: acute angle, alternate angles, angle bisector, arc, area of a disc, base and corresponding apex and height of triangle or parallelogram, chord, circle, circumcentre, circumcircle, circumference of a circle, circumradius, collinear points, concurrent lines, convex quadrilateral, corresponding angles, diameter, disc, distance, equilateral triangle, exterior angles of a triangle, full angle, hypotenuse, incentre, incircle, inradius, interior opposite angles, isosceles triangle, median lines, midpoint of a segment, null angle, obtuse angle, perpendicular bisector of a segment, perpendicular lines, point of contact of a tangent, polygon, quadrilateral, radius, ratio, rectangle, reflex angle ordinary angle, rhombus, right-angled triangle, scalene triangle, sector, segment, square, straight angle, subset, supplementary angles, transversal line, vertically-opposite angles.

Definable terms used without explicit definition: angles, adjacent sides, arms or sides of an angle, centre of a circle, endpoints of segment, equal angles, equal segments, line passes through point, opposite sides or angles of a quadrilateral, or vertices of triangles or quadrilaterals, point lies on line, side of a line, side of a polygon, the side opposite an angle of a triangle, vertex, vertices (of angle, triangle, polygon).

6 The Theory

Line[3] is short for straight line. Take a fixed **plane**[4], once and for all, and consider just lines that lie in it. The plane and the lines are **sets**[5] of **points**[6]. Each line is a **subset** of the plane, i.e. each element of a line is a point of the plane. Each line is endless, extending forever in both directions. Each line has infinitely-many points. The points on a line can be taken to be ordered along the line in a natural way. As a consequence, given any three distinct points on a line, exactly one of them lies **between** the other two. Points that are not on a given line can be said to be on one or other **side** of the line. The sides of a line are sometimes referred to as **half-planes**.

Notation 1. We denote points by roman capital letters A, B, C, etc., and lines by lower-case roman letters l, m, n, etc.

Axioms are statements we will accept as true[7].

Axiom 1 (Two Points Axiom)**.** *There is exactly one line through any two given points. (We denote the line through A and B by AB.)*

Definition 1. The line **segment** $[AB]$ is the part of the line AB between A and B (including the endpoints). The point A divides the line AB into two pieces, called **rays**. The point A lies between all points of one ray and all

[3]Line is undefined.
[4]Undefined term
[5]Undefined term
[6]Undefined term
[7] An **axiom** is a statement accepted without proof, as a basis for argument. A **theorem** is a statement deduced from the axioms by logical argument.

points of the other. We denote the ray that starts at A and passes through B by $[AB$. Rays are sometimes referred to as **half-lines**.

Three points usually determine three different lines.

Definition 2. If three or more points lie on a single line, we say they are **collinear**.

Definition 3. Let A, B and C be points that are not collinear. The **triangle** ΔABC is the piece of the plane enclosed by the three line segments $[AB]$, $[BC]$ and $[CA]$. The segments are called its **sides**, and the points are called its **vertices** (singular **vertex**).

6.1 Length and Distance

We denote the set of all **real numbers**[8] by $\mathbb{R}$.

Definition 4. We denote the **distance**[9] between the points A and B by $|AB|$. We define the **length** of the segment $[AB]$ to be $|AB|$.

We often denote the lengths of the three sides of a triangle by a, b, and c. The usual thing for a triangle ΔABC is to take $a = |BC|$, i.e. the length of the side opposite the vertex A, and similarly $b = |CA|$ and $c = |AB|$.

Axiom 2 (Ruler Axiom[10]). *The distance between points has the following properties:*

1. *the distance $|AB|$ is never negative;*
2. *$|AB| = |BA|$;*
3. *if C lies on AB, between A and B, then $|AB| = |AC| + |CB|$;*
4. *(marking off a distance) given any ray from A, and given any real number $k \geq 0$, there is a unique point B on the ray whose distance from A is k.*

[8] Undefined term

[9] Undefined term

[10] Teachers used to traditional treatments that follow Euclid closely should note that this axiom (and the later Protractor Axiom) guarantees the existence of various points (and lines) without appeal to postulates about constructions using straight-edge and compass. They are powerful axioms.

Definition 5. The **midpoint** of the segment $[AB]$ is the point M of the segment with [11]

$$|AM| = |MB| = \frac{|AB|}{2}.$$

6.2 Angles

Definition 6. A subset of the plane is **convex** if it contains the whole segment that connects any two of its points.

For example, one side of any line is a convex set, and triangles are convex sets.

We do not define the term angle formally. Instead we say: There are things called **angles**. To each angle is associated:

1. a unique point A, called its **vertex**;

2. two rays $[AB$ and $[AC$, both starting at the vertex, and called the **arms** of the angle;

3. a piece of the plane called the **inside** of the angle.

An angle is either a null angle, an ordinary angle, a straight angle, a reflex angle or a full angle, Unless otherwise specified, you may take it that any angle we talk about is an ordinary angle.

Definition 7. An angle is a **null angle** if its arms coincide with one another and its inside is the empty set.

Definition 8. An angle is an **ordinary angle** if its arms are not on one line, and its inside is a convex set.

Definition 9. An angle is a **straight angle** if its arms are the two halves of one line, and its inside is one of the sides of that line.

Definition 10. An angle is a **reflex angle** if its arms are not on one line, and its inside is not a convex set.

Definition 11. An angle is a **full angle** if its arms coincide with one another and its inside is the rest of the plane.

[11] Students may notice that the first equality implies the second.

Definition 12. Suppose that A, B, and C are three noncollinear points. We denote the (ordinary) angle with arms $[AB$ and $[AC$ by $\angle BAC$ (and also by $\angle CAB$). We shall also use the notation $\angle BAC$ to refer to straight angles, where A, B, C are collinear, and A lies between B and C (either side could be the inside of this angle).

Sometimes we want to refer to an angle without naming points, and in that case we use lower-case greek letters, α,β,γ, etc.

6.3 Degrees

Notation 2. We denote the number of **degrees** in an angle $\angle BAC$ or α by the symbol $|\angle BAC|$, or $|\angle\alpha|$, as the case may be.

Axiom 3 (Protractor Axiom)**.** *The number of degrees in an angle (also known as its degree-measure) is always a number between* $0°$ *and* $360°$*. The number of degrees of an ordinary angle is less than* $180°$*. It has these properties:*

1. *A straight angle has* $180°$*.*
2. *Given a ray* $[AB$*, and a number* d *between* 0 *and* 180*, there is exactly one ray from* A *on each side of the line* AB *that makes an (ordinary) angle having* d *degrees with the ray* $[AB$*.*
3. *If* D *is a point inside an angle* $\angle BAC$*, then*

$$|\angle BAC| = |\angle BAD| + |\angle DAC|.$$

Null angles are assigned $0°$, full angles $360°$, and reflex angles have more than $180°$. To be more exact, if A, B, and C are noncollinear points, then the reflex angle "outside" the angle $\angle BAC$ measures $360° - |\angle BAC|$, in degrees.

Definition 13. The ray $[AD$ is the **bisector** of the angle $\angle BAC$ if

$$|\angle BAD| = |\angle DAC| = \frac{|\angle BAC|}{2}.$$

We say that an angle is 'an angle of' (for instance) $45°$, if it has 45 degrees in it.

Definition 14. A **right angle** is an angle of exactly $90°$.

Definition 15. An angle is **acute** if it has less than 90°, and **obtuse** if it has more than 90°.

Definition 16. If $\angle BAC$ is a straight angle, and D is off the line BC, then $\angle BAD$ and $\angle DAC$ are called **supplementary angles**. They add to 180°.

Definition 17. When two lines AB and AC cross at a point A, they are **perpendicular** if $\angle BAC$ is a right angle.

Definition 18. Let A lie between B and C on the line BC, and also between D and E on the line DE. Then $\angle BAD$ and $\angle CAE$ are called **vertically-opposite angles**.

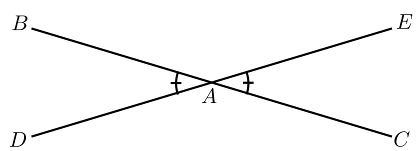


Figure 1.

Theorem 1 (Vertically-opposite Angles)**.**
Vertically opposite angles are equal in measure.

Proof. See Figure 1. The idea is to add the same supplementary angles to both, getting 180°. In detail,

$$\begin{aligned} |\angle BAD| + |\angle BAE| &= 180^\circ, \\ |\angle CAE| + |\angle BAE| &= 180^\circ, \end{aligned}$$

so subtracting gives:

$$\begin{aligned} |\angle BAD| - |\angle CAE| &= 0^\circ, \\ |\angle BAD| &= |\angle CAE|. \end{aligned}$$

□

6.4 Congruent Triangles

Definition 19. Let A, B, C and A', B', C' be triples of non-collinear points. We say that the triangles ΔABC and $\Delta A'B'C'$ are **congruent** if all the sides and angles of one are equal to the corresponding sides and angles of the other, i.e. $|AB| = |A'B'|$, $|BC| = |B'C'|$, $|CA| = |C'A'|$, $|\angle ABC| = |\angle A'B'C'|$, $|\angle BCA| = |\angle B'C'A'|$, and $|\angle CAB| = |\angle C'A'B'|$. See Figure 2.

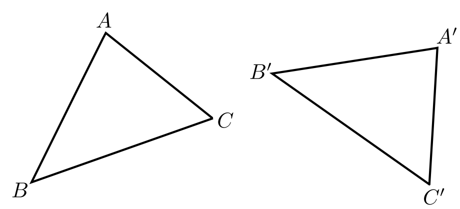


Figure 2.

Notation 3. Usually, we abbreviate the names of the angles in a triangle, by labelling them by the names of the vertices. For instance, we write $\angle A$ for $\angle CAB$.

Axiom 4 (SAS+ASA+SSS[12])**.**
If (1) $|AB| = |A'B'|$, $|AC| = |A'C'|$ *and* $|\angle A| = |\angle A'|$*,*
or
(2) $|BC| = |B'C'|$, $|\angle B| = |\angle B'|$*, and* $|\angle C| = |\angle C'|$*,*
or
(3) $|AB| = |A'B'|$, $|BC| = |B'C'|$*, and* $|CA| = |C'A'|$
then the triangles ΔABC *and* $\Delta A'B'C'$ *are congruent.*

Definition 20. A triangle is called **right-angled** if one of its angles is a right angle. The other two angles then add to 90°, by Theorem 4, so are both acute angles. The side opposite the right angle is called the **hypotenuse**.

Definition 21. A triangle is called **isosceles** if two sides are equal[13]. It is **equilateral** if all three sides are equal. It is **scalene** if no two sides are equal.

Theorem 2 (Isosceles Triangles)**.**
(1) In an isosceles triangle the angles opposite the equal sides are equal.
(2) Conversely, If two angles are equal, then the triangle is isosceles.

Proof. (1) Suppose the triangle ΔABC has $AB = AC$ (as in Figure 3). Then
ΔABC is congruent to ΔACB [SAS]
$\therefore \angle B = \angle C$.

[12] It would be possible to prove all the theorems using a weaker axiom (just SAS). We use this stronger version to shorten the course.
[13] The simple "equal" is preferred to "of equal length"

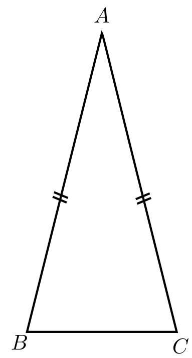


Figure 3.

(2) Suppose now that $\angle B = \angle C$. Then
ΔABC is congruent to ΔACB [ASA]
$\therefore |AB| = |AC|, \Delta ABC$ is isosceles. □

Acceptable Alternative Proof of (1). Let D be the midpoint of $[BC]$, and use SAS to show that the triangles ΔABD and ΔACD are congruent. (This proof is more complicated, but has the advantage that it yields the extra information that the angles $\angle ADB$ and $\angle ADC$ are equal, and hence both are right angles (since they add to a straight angle)). □

6.5 Parallels

Definition 22. Two lines l and m are **parallel** if they are either identical, or have no common point.

Notation 4. We write $l||m$ for "l is parallel to m".

Axiom 5 (Axiom of Parallels)**.** *Given any line l and a point P, there is exactly one line through P that is parallel to l.*

Definition 23. If l and m are lines, then a line n is called a **transversal** of l and m if it meets them both.

Definition 24. Given two lines AB and CD and a transversal BC of them, as in Figure 4, the angles $\angle ABC$ and $\angle BCD$ are called **alternate** angles.

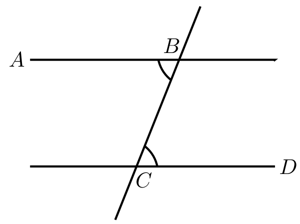


Figure 4.

Theorem 3 (Alternate Angles)**.** *Suppose that A and D are on opposite sides of the line BC.*
(1) If $|\angle ABC| = |\angle BCD|$, then $AB||CD$. In other words, if a transversal makes equal alternate angles on two lines, then the lines are parallel.
(2) Conversely, if $AB||CD$, then $|\angle ABC| = |\angle BCD|$. In other words, if two lines are parallel, then any transversal will make equal alternate angles with them.

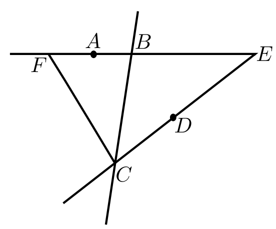


Figure 5.

Proof. (1) Suppose $|\angle ABC| = |\angle BCD|$. If the lines AB and CD do not meet, then they are parallel, by definition, and we are done. Otherwise, they meet at some point, say E. Let us assume that E is on the same side of BC as D.[14] Take F on EB, on the same side of BC as A, with $|BF| = |CE|$ (see Figure 5). [Ruler Axiom]

[14]Fuller detail: There are three cases:
1°: E lies on BC. Then (using Axiom 1) we must have $E = B = C$, and $AB = CD$.
2°: E lies on the same side of BC as D. In that case, take F on EB, on the same side of BC as A, with $|BF| = |CE|$. [Ruler Axiom]
Then ΔBCE is congruent to ΔCBF. [SAS]
Thus

$$|\angle BCF| = |\angle CBE| = 180^\circ - |\angle ABC| = 180^\circ - |\angle BCD|,$$

Then ΔBCE is congruent to ΔCBF. [SAS]
Thus

$$|\angle BCF| = |\angle CBE| = 180^\circ - |\angle ABC| = 180^\circ - |\angle BCD|,$$

so that F lies on DC. [Ruler Axiom]
Thus AB and CD both pass through E and F, and hence coincide, [Axiom 1]
Hence AB and CD are parallel. [Definition of parallel]

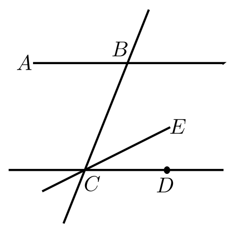


Figure 6.

(2) To prove the converse, suppose $AB||CD$. Pick a point E on the same side of BC as D with $|\angle BCE| = |\angle ABC|$. (See Figure 6.) By Part (1), the line CE is parallel to AB. By Axiom 5, there is only one line through C parallel to AB, so $CE = CD$. Thus $|\angle BCD| = |\angle BCE| = |\angle ABC|$. □

Theorem 4 (Angle Sum 180)**.** *The angles in any triangle add to* 180°.

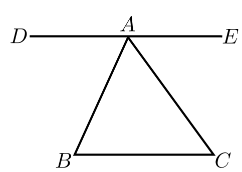


Figure 7.

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so that F lies on DC. [Protractor Axiom]
Thus AB and CD both pass through E and F, and hence coincide. [Axiom 1]
3°: E lies on the same side of BC as A. Similar to the previous case.
Thus, in all three cases, $AB = CD$, so the lines are parallel.

Proof. Let ΔABC be given. Take a segment $[DE]$ passing through A, parallel to BC, with D on the opposite side of AB from C, and E on the opposite side of AC from B (as in Figure 7). [Axiom of Parallels]
Then AB is a transversal of DE and BC, so by the Alternate Angles Theorem,

$$|\angle ABC| = |\angle DAB|.$$

Similarly, AC is a transversal of DE and BC, so

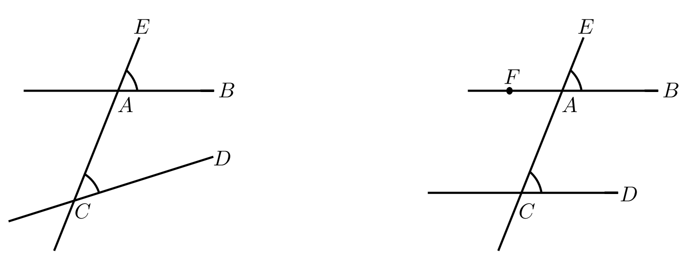
$$|\angle ACB| = |\angle CAE|.$$

Thus, using the Protractor Axiom to add the angles,

$$\begin{aligned} & |\angle ABC| + |\angle ACB| + |\angle BAC| \\ = & |\angle DAB| + |\angle CAE| + |\angle BAC| \\ = & |\angle DAE| = 180^\circ, \end{aligned}$$

since $\angle DAE$ is a straight angle. □

Definition 25. Given two lines AB and CD, and a transversal AE of them, as in Figure 8(a), the angles $\angle EAB$ and $\angle ACD$ are called **corresponding** angles[15].



(a) (b)

Figure 8.

Theorem 5 (Corresponding Angles)**.** *Two lines are parallel if and only if for any transversal, corresponding angles are equal.*

[15] with respect to the two lines and the given transversal.

Proof. See Figure 8(b). We first assume that the corresponding angles $\angle EAB$ and $\angle ACD$ are equal. Let F be a point on AB such that F and B are on opposite sides of AE. Then we have

$|\angle EAB| = |\angle FAC|$ [Vertically opposite angles]

Hence the alternate angles $\angle FAC$ and $\angle ACD$ are equal and therefore the lines $FA = AB$ and CD are parallel.

For the converse, let us assume that the lines AB and CD are parallel. Then the alternate angles $\angle FAC$ and $\angle ACD$ are equal. Since

$|\angle EAB| = |\angle FAC|$ [Vertically opposite angles]

we have that the corresponding angles $\angle EAB$ and $\angle ACD$ are equal. □

Definition 26. In Figure 9, the angle α is called an **exterior angle** of the triangle, and the angles β and γ are called (corresponding) **interior opposite angles**.[16]

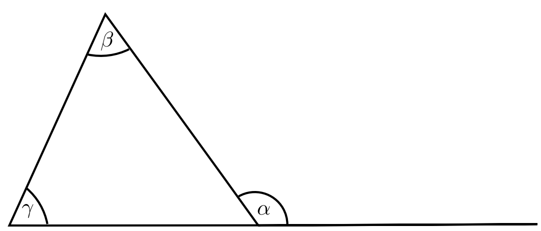


Figure 9.

Theorem 6 (Exterior Angle)**.** *Each exterior angle of a triangle is equal to the sum of the interior opposite angles.*

Proof. See Figure 10. In the triangle ΔABC let α be an exterior angle at A. Then

$|\alpha| + |\angle A| = 180°$ [Supplementary angles]

and

$|\angle B| + |\angle C| + |\angle A| = 180°$. [Angle sum 180°]

Subtracting the two equations yields $|\alpha| = |\angle B| + |\angle C|$. □

[16] The phrase **interior remote angles** is sometimes used instead of **interior opposite angles**.

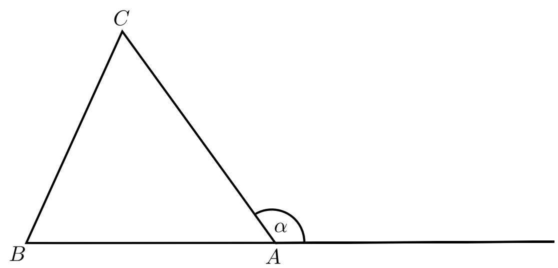


Figure 10.

Theorem 7.
(1) In ΔABC, suppose that $|AC| > |AB|$. Then $|\angle ABC| > |\angle ACB|$. In other words, the angle opposite the greater of two sides is greater than the angle opposite the lesser side.
(2) Conversely, if $|\angle ABC| > |\angle ACB|$, then $|AC| > |AB|$. In other words, the side opposite the greater of two angles is greater than the side opposite the lesser angle.

Proof.
(1) Suppose that $|AC| > |AB|$. Then take the point D on the segment $[AC]$ with
$|AD| = |AB|$. [Ruler Axiom]

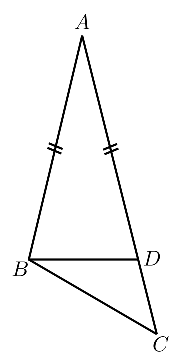


Figure 11.

See Figure 11. Then ΔABD is isosceles, so

$$\begin{aligned} |\angle ACB| &< |\angle ADB| && \text{[Exterior Angle]} \\ &= |\angle ABD| && \text{[Isosceles Triangle]} \\ &< |\angle ABC|. \end{aligned}$$

Thus $|\angle ACB| < |\angle ABC|$, as required.

(2)(This is a Proof by Contradiction!)
Suppose that $|\angle ABC| > |\angle ACB|$. See Figure 12.

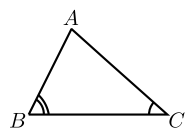


Figure 12.

If it could happen that $|AC| \leq |AB|$, then
either Case 1°: $|AC| = |AB|$, in which case ΔABC is isosceles, and then $|\angle ABC| = |\angle ACB|$, which contradicts our assumption,
or Case 2°: $|AC| < |AB|$, in which case Part (1) tells us that $|\angle ABC| < |\angle ACB|$, which also contradicts our assumption. Thus it cannot happen, and we conclude that $|AC| > |AB|$. □

Theorem 8 (Triangle Inequality)**.**
Two sides of a triangle are together greater than the third.

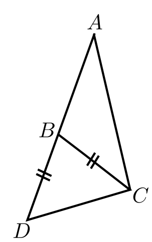


Figure 13.

Proof. Let ΔABC be an arbitrary triangle. We choose the point D on AB such that B lies in $[AD]$ and $|BD| = |BC|$ (as in Figure 13). In particular

$$|AD| = |AB| + |BD| = |AB| + |BC|.$$

Since B lies in the angle $\angle ACD$[17] we have

$$|\angle BCD| < |\angle ACD|.$$

[17] B lies in a segment whose endpoints are on the arms of $\angle ACD$. Since this angle is $< 180°$ its inside is convex.

Because of $|BD| = |BC|$ and the Theorem about Isosceles Triangles we have $|\angle BCD| = |\angle BDC|$, hence $|\angle ADC| = |\angle BDC| < |\angle ACD|$. By the previous theorem applied to ΔADC we have

$$|AC| < |AD| = |AB| + |BC|.$$

□

6.6 Perpendicular Lines

Proposition 1. [18] *Two lines perpendicular to the same line are parallel to one another.*

Proof. This is a special case of the Alternate Angles Theorem. □

Proposition 2. *There is a unique line perpendicular to a given line and passing though a given point. This applies to a point on or off the line.*

Definition 27. The **perpendicular bisector** of a segment $[AB]$ is the line through the midpoint of $[AB]$, perpendicular to AB.

6.7 Quadrilaterals and Parallelograms

Definition 28. A closed chain of line segments laid end-to-end, not crossing anywhere, and not making a straight angle at any endpoint encloses a piece of the plane called a **polygon**. The segments are called the **sides** or edges of the polygon, and the endpoints where they meet are called its **vertices**. Sides that meet are called **adjacent sides**, and the ends of a side are called **adjacent vertices**. The angles at adjacent vertices are called **adjacent angles**.

Definition 29. A **quadrilateral** is a polygon with four vertices.

Two sides of a quadrilateral that are not adjacent are called **opposite sides**. Similarly, two angles of a quadrilateral that are not adjacent are called **opposite angles**.

[18] In this document, a proposition is a useful or interesting statement that could be proved at this point, but whose proof is not stipulated as an essential part of the programme. Teachers are free to deal with them as they see fit. For instance, they might be just mentioned, or discussed without formal proof, or used to give practice in reasoning for HLC students. It is desirable that they be mentioned, at least.

Definition 30. A **rectangle** is a quadrilateral having right angles at all four vertices.

Definition 31. A **rhombus** is a quadrilateral having all four sides equal.

Definition 32. A **square** is a rectangular rhombus.

Definition 33. A polygon is **equilateral** if all its sides are equal, and **regular** if all its sides and angles are equal.

Definition 34. A **parallelogram** is a quadrilateral for which both pairs of opposite sides are parallel.

Proposition 3. *Each rectangle is a parallelogram.*

Theorem 9. *In a parallelogram, opposite sides are equal, and opposite angles are equal.*

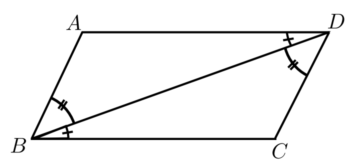


Figure 14.

Proof. See Figure 14. Idea: Use Alternate Angle Theorem, then ASA to show that a diagonal divides the parallelogram into two congruent triangles. This gives opposite sides and (one pair of) opposite angles equal.

In more detail, let $ABCD$ be a given parallelogram, $AB||CD$ and $AD||BC$. Then

$|\angle ABD| = |\angle BDC|$ [Alternate Angle Theorem]
$|\angle ADB| = |\angle DBC|$ [Alternate Angle Theorem]
ΔDAB is congruent to ΔBCD. [ASA]

$$\therefore |AB| = |CD|, \ |AD| = |CB|, \text{ and } |\angle DAB| = |\angle BCD|.$$

□

Remark 1. Sometimes it happens that the converse of a true statement is false. For example, it is true that if a quadrilateral is a rhombus, then its diagonals are perpendicular. But it is not true that a quadrilateral whose diagonals are perpendicular is always a rhombus.

It may also happen that a statement admits several valid converses. Theorem 9 has two:

Converse 1 to Theorem 9: *If the opposite angles of a convex quadrilateral are equal, then it is a parallelogram.*

Proof. First, one deduces from Theorem 4 that the angle sum in the quadrilateral is 360°. It follows that adjacent angles add to 180°. Theorem 3 then yields the result. □

Converse 2 to Theorem 9: *If the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.*

Proof. Drawing a diagonal, and using SSS, one sees that opposite angles are equal. □

Corollary 1. *A diagonal divides a parallelogram into two congruent triangles.*

Remark 2. The converse is false: It may happen that a diagonal divides a convex quadrilateral into two congruent triangles, even though the quadrilateral is not a parallelogram.

Proposition 4. *A quadrilateral in which one pair of opposite sides is equal and parallel, is a parallelogram.*

Proposition 5. *Each rhombus is a parallelogram.*

Theorem 10. *The diagonals of a parallelogram bisect one another.*

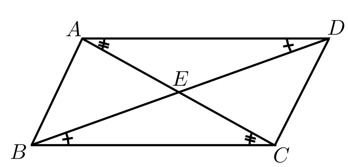


Figure 15.

Proof. See Figure 15. Idea: Use Alternate Angles and ASA to establish congruence of ΔADE and ΔCBE.

In detail: Let AC cut BD in E. Then

$$\begin{aligned} |\angle EAD| &= |\angle ECB| \text{ and} \\ |\angle EDA| &= |\angle EBC| && \text{[Alternate Angle Theorem]} \\ |AD| &= |BC|. && \text{[Theorem 9]} \end{aligned}$$

$\therefore \Delta ADE$ is congruent to ΔCBE. [ASA] □

Proposition 6 (Converse)**.** *If the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.*

Proof. Use SAS and Vertically Opposite Angles to establish congruence of ΔABE and ΔCDE. Then use Alternate Angles. □

6.8 Ratios and Similarity

Definition 35. If the three angles of one triangle are equal, respectively, to those of another, then the two triangles are said to be **similar**.

Remark 3. Obviously, two right-angled triangles are similar if they have a common angle other than the right angle.

(The angles sum to 180°, so the third angles must agree as well.)

Theorem 11. *If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.*

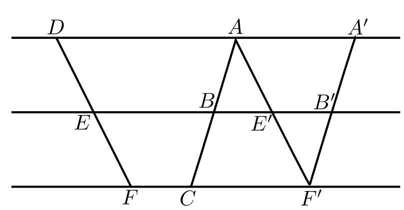


Figure 16.

Proof. Uses opposite sides of a parallelogram, AAS, Axiom of Parallels.

In more detail, suppose $AD||BE||CF$ and $|AB| = |BC|$. We wish to show that $|DE| = |EF|$.

Draw $AE'||DE$, cutting EB at E' and CF at F'.
Draw $F'B'||AB$, cutting EB at B'. See Figure 16.

Then

$\|B'F'\|$	$=$	$\|BC\|$	[Theorem 9]
	$=$	$\|AB\|$.	[by Assumption]
$\|\angle BAE'\|$	$=$	$\|\angle E'F'B'\|$.	[Alternate Angle Theorem]
$\|\angle AE'B\|$	$=$	$\|\angle F'E'B'\|$.	[Vertically Opposite Angles]
$\therefore \Delta ABE'$	is congruent to	$\Delta F'B'E'$.	[ASA]
$\therefore \|AE'\|$	$=$	$\|F'E'\|$.	

But
$|AE'| = |DE|$ and $|F'E'| = |FE|$. [Theorem 9]
$\therefore |DE| = |EF|$. □

Definition 36. Let s and t be positive real numbers. We say that a point C **divides the segment** $[AB]$ **in the ratio** $s : t$ if C lies on the line AB, and is between A and B, and

$$\frac{|AC|}{|CB|} = \frac{s}{t}.$$

We say that a line l **cuts** $[AB]$ **in the ratio** $s : t$ if it meets AB at a point C that divides $[AB]$ in the ratio $s : t$.

Remark 4. It follows from the Ruler Axiom that given two points A and B, and a ratio $s : t$, there is exactly one point that divides the segment $[AB]$ in that exact ratio.

Theorem 12. *Let ΔABC be a triangle. If a line l is parallel to BC and cuts $[AB]$ in the ratio $s : t$, then it also cuts $[AC]$ in the same ratio.*

Proof. We prove only the commensurable case.

Let l cut $[AB]$ in D in the ratio $m : n$ with natural numbers m, n. Thus there are points (Figure 17)

$$D_0 = B, D_1, D_2, \ldots, D_{m-1}, D_m = D, D_{m+1}, \ldots, D_{m+n-1}, D_{m+n} = A,$$

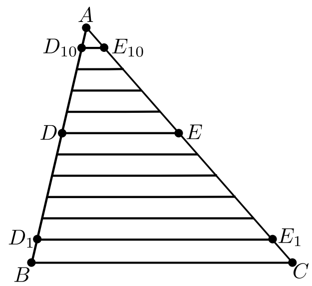


Figure 17.

equally spaced along $[BA]$, i.e. the segments

$$[D_0, D_1], [D_1, D_2], \dots [D_i, D_{i+1}], \dots [D_{m+n-1}, D_{m+n}]$$

have equal length.

Draw lines $D_1E_1, D_2E_2, \dots$ parallel to BC with $E_1, E_2, \dots$ on $[AC]$.

Then all the segments

$$[CE_1], [E_1E_2], [E_2E_3], \dots, [E_{m+n-1}A]$$

have the same length, [Theorem 11]
and $E_m = E$ is the point where l cuts $[AC]$. [Axiom of Parallels]
Hence E divides $[CA]$ in the ratio $m : n$. □

Proposition 7. *If two triangles ΔABC and $\Delta A'B'C'$ have*

$$|\angle A| = |\angle A'|, \text{ and } \frac{|A'B'|}{|AB|} = \frac{|A'C'|}{|AC|},$$

then they are similar.

Proof. Suppose $|A'B'| \leq |AB|$. If equal, use SAS. Otherwise, note that then $|A'B'| < |AB|$ and $|A'C'| < |AC|$. Pick B'' on $[AB$ and C'' on $[AC$ with $|A'B'| = |AB''|$ agus $|A'C'| = |AC''|$. [Ruler Axiom] Then by SAS, $\Delta A'B'C'$ is congruent to $\Delta AB''C''$.

Draw $[B''D$ parallel to BC [Axiom of Parallels], and let it cut AC at D. Now the last theorem and the hypothesis tell us that D and C'' divide $[AC]$ in the same ratio, and hence $D = C''$.

Thus

$$\begin{aligned} |\angle B| &= |\angle AB''C''| \text{ [Corresponding Angles]} \\ &= |\angle B'|, \end{aligned}$$

and

$$|\angle C| = |\angle AC''B''| = |\angle C'|,$$

so ΔABC is similar to $\Delta A'B'C'$. [Definition of similar]

□

Remark 5. The **Converse to Theorem 12** is true:
Let ΔABC be a triangle. If a line l cuts the sides AB and AC in the same ratio, then it is parallel to BC.

Proof. This is immediate from Proposition 7 and Theorem 5. □

Theorem 13. *If two triangles ΔABC and $\Delta A'B'C'$ are similar, then their sides are proportional, in order:*

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}.$$

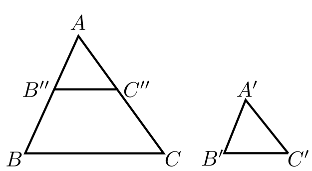


Figure 18.

Proof. We may suppose $|A'B'| \le |AB|$. Pick B'' on $[AB]$ with $|AB''| = |A'B'|$, and C'' on $[AC]$ with $|AC''| = |A'C'|$. Refer to Figure 18. Then

$\Delta AB''C''$	is congruent to	$\Delta A'B'C'$	[SAS]
$\therefore |\angle AB''C''|$	$=$	$|\angle ABC|$	
$\therefore B''C''$	$\|\|$	BC	[Corresponding Angles]
$\therefore \dfrac{|A'B'|}{|A'C'|}$	$=$	$\dfrac{|AB''|}{|AC''|}$	[Choice of B'', C'']
	$=$	$\dfrac{|AB|}{|AC|}$	[Theorem 12]
$\dfrac{|AC|}{|A'C'|}$	$=$	$\dfrac{|AB|}{|A'B'|}$	[Re-arrange]

Similarly, $\dfrac{|BC|}{|B'C'|} = \dfrac{|AB|}{|A'B'|}$ □

Proposition 8 (Converse)**.** *If*

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|},$$

then the two triangles ΔABC *agus* $\Delta A'B'C'$ *are similar.*

Proof. Refer to Figure 18. If $|A'B'| = |AB|$, then by SSS the two triangles are congruent, and therefore similar. Otherwise, assuming $|A'B'| < |AB|$, choose B'' on AB and C'' on AC with $|AB''| = |A'B'|$ and $|AC''| = |A'C'|$. Then by Proposition 7, $\Delta AB''C''$ is similar to ΔABC, so

$$|B''C''| = |AB''| \cdot \frac{|BC|}{|AB|} = |A'B'| \cdot \frac{|BC|}{|AB|} = |B'C'|.$$

Thus by SSS, $\Delta A'B'C'$ is congruent to $\Delta A''B''C''$, and hence similar to ΔABC. □

6.9 Pythagoras

Theorem 14 (Pythagoras)**.** *In a right-angle triangle the square of the hypotenuse is the sum of the squares of the other two sides.*

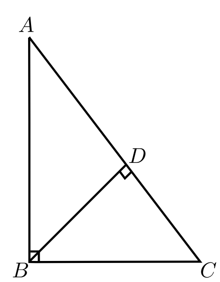


Figure 19.

Proof. Let ΔABC have a right angle at B. Draw the perpendicular BD from the vertex B to the hypotenuse AC (shown in Figure 19).

The right-angle triangles ΔABC and ΔADB have a common angle at A. $\therefore$ ΔABC is similar to ΔADB.

$$\therefore \frac{|AC|}{|AB|} = \frac{|AB|}{|AD|},$$

so

$$|AB|^2 = |AC| \cdot |AD|.$$

Similarly, ΔABC is similar to ΔBDC.

$$\therefore \frac{|AC|}{|BC|} = \frac{|BC|}{|DC|},$$

so

$$|BC|^2 = |AC| \cdot |DC|.$$

Thus

$$\begin{aligned} |AB|^2 + |BC|^2 &= |AC| \cdot |AD| + |AC| \cdot |DC| \\ &= |AC| \left(|AD| + |DC|\right) \\ &= |AC| \cdot |AC| \\ &= |AC|^2. \end{aligned}$$

□

Theorem 15 (Converse to Pythagoras)**.** *If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.*

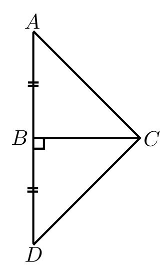


Figure 20.

Proof. (Idea: Construct a second triangle on the other side of [BC], and use Pythagoras and SSS to show it congruent to the original.)
In detail: We wish to show that $|\angle ABC| = 90°$.

Draw $BD \perp BC$ and make $|BD| = |AB|$ (as shown in Figure 20).

Then

$$\begin{aligned} |DC| &= \sqrt{|DC|^2} \\ &= \sqrt{|BD|^2 + |BC|^2} && \text{[Pythagoras]} \\ &= \sqrt{|AB|^2 + |BC|^2} && [\ |AB| = |BD|\ ] \\ &= \sqrt{|AC|^2} && \text{[Hypothesis]} \\ &= |AC|. \end{aligned}$$

$\therefore \Delta ABC$ is congruent to ΔDBC. [SSS]
$\therefore |\angle ABC| = |\angle DBC| = 90°$. □

Proposition 9 (RHS)**.** *If two right angled triangles have hypotenuse and another side equal in length, respectively, then they are congruent.*

Proof. Suppose ΔABC and $\Delta A'B'C'$ are right-angle triangles, with the right angles at B and B', and have hypotenuses of the same length, $|AC| = |A'C'|$, and also have $|AB| = |A'B'|$. Then by using Pythagoras' Theorem, we obtain $|BC| = |B'C'|$, so by SSS, the triangles are congruent. □

Proposition 10. *Each point on the perpendicular bisector of a segment* $[AB]$ *is equidistant from the ends.*

Proposition 11. *The perpendiculars from a point on an angle bisector to the arms of the angle have equal length.*

6.10 Area

Definition 37. If one side of a triangle is chosen as the base, then the opposite vertex is the **apex** corresponding to that base. The corresponding **height** is the length of the perpendicular from the apex to the base. This perpendicular segment is called an **altitude** of the triangle.

Theorem 16. *For a triangle, base times height does not depend on the choice of base.*

Proof. Let AD and BE be altitudes (shown in Figure 21). Then ΔBCE and ΔACD are right-angled triangles that share the angle C, hence they are similar. Thus

$$\frac{|AD|}{|BE|} = \frac{|AC|}{|BC|}.$$

Re-arrange to yield the result. □

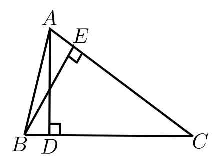


Figure 21.

Definition 38. The **area** of a triangle is half the base by the height.

Notation 5. We denote the area by "area of ΔABC"[19].

Proposition 12. *Congruent triangles have equal areas.*

Remark 6. This is another example of a proposition whose converse is false. It may happen that two triangles have equal area, but are not congruent.

Proposition 13. *If a triangle ΔABC is cut into two by a line AD from A to a point D on the segment $[BC]$, then the areas add up properly:*

$$\text{area of } \Delta ABC = \text{area of } \Delta ABD + \text{area of } \Delta ADC.$$

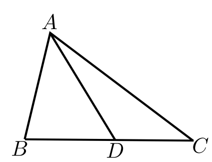


Figure 22.

Proof. See Figure 22. All three triangles have the same height, say h, so it comes down to

$$\frac{|BC| \times h}{2} = \frac{|BD| \times h}{2} + \frac{|DC| \times h}{2},$$

which is obvious, since

$$|BC| = |BD| + |DC|.$$

□

[19] $|\Delta ABC|$ will also be accepted.

If a figure can be cut up into nonoverlapping triangles (i.e. triangles that either don't meet, or meet only along an edge), then its area is taken to be the sum of the area of the triangles[20].

If figures of equal areas are added to (or subtracted from) figures of equal areas, then the resulting figures also have equal areas[21].

Proposition 14. *The area of a rectangle having sides of length a and b is ab.*

Proof. Cut it into two triangles by a diagonal. Each has area $\frac{1}{2}ab$. □

Theorem 17. *A diagonal of a parallelogram bisects the area.*

Proof. A diagonal cuts the parallelogram into two congruent triangles, by Corollary 1. □

Definition 39. Let the side AB of a parallelogram $ABCD$ be chosen as a base (Figure 23). Then the **height** of the parallelogram **corresponding to that base** is the height of the triangle ΔABC.

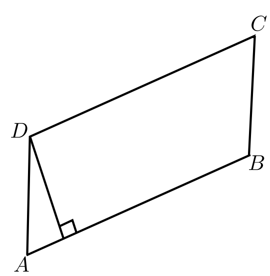


Figure 23.

Proposition 15. *This height is the same as the height of the triangle ΔABD, and as the length of the perpendicular segment from D onto AB.*

[20] If students ask, this does not lead to any ambiguity. In the case of a convex quadrilateral, $ABCD$, one can show that

$$\text{area of } \Delta ABC + \text{area of } \Delta CDA = \text{area of } \Delta ABD + \text{area of } \Delta BCD.$$

In the general case, one proves the result by showing that there is a common refinement of any two given triangulations.

[21] Follows from the previous footnote.

Theorem 18. *The area of a parallelogram is the base by the height.*

Proof. Let the parallelogram be $ABCD$. The diagonal BD divides it into two triangles, ΔABD and ΔCDB. These have equal area, [Theorem 17] and the first triangle shares a base and the corresponding height with the parallelogram. So the areas of the two triangles add to $2 \times \frac{1}{2} \times \text{base} \times \text{height}$, which gives the result. □

6.11 Circles

Definition 40. A **circle** is the set of points at a given distance (its **radius**) from a fixed point (its **centre**). Each line segment joining the centre to a point of the circle is also called **a** radius. The plural of radius is radii. A **chord** is the segment joining two points of the circle. A **diameter** is a chord through the centre. All diameters have length twice the radius. This number is also called **the** diameter of the circle.

Two points A, B on a circle cut it into two pieces, called **arcs**. You can specify an arc uniquely by giving its endpoints A and B, and one other point C that lies on it. A **sector** of a circle is the piece of the plane enclosed by an arc and the two radii to its endpoints.

The length of the whole circle is called its **circumference**. For every circle, the circumference divided by the diameter is the same. This ratio is called π.

A **semicircle** is an arc of a circle whose ends are the ends of a diameter.

Each circle divides the plane into two pieces, the inside and the outside. The piece inside is called a **disc**.

If B and C are the ends of an arc of a circle, and A is another point, not on the arc, then we say that the angle $\angle BAC$ is the angle at A **standing on the arc**. We also say that it **stands on the chord** $[BC]$.

Theorem 19. *The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.*

Proof. There are several cases for the diagram. It will be sufficient for students to examine one of these. The idea, in all cases, is to draw the line through the centre and the point on the circumference, and use the Isosceles Triangle Theorem, and then the Protractor Axiom (to add or subtract angles, as the case may be).

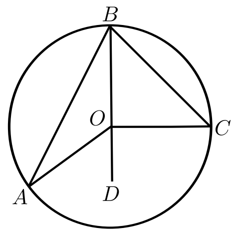


Figure 24.

In detail, for the given figure, Figure 24, we wish to show that $|\angle AOC| = 2|\angle ABC|$.

Join B to O and continue the line to D. Then

$$\begin{aligned} |OA| &= |OB|. && \text{[Definition of circle]} \\ \therefore |\angle BAO| &= |\angle ABO|. && \text{[Isosceles triangle]} \\ \therefore |\angle AOD| &= |\angle BAO| + |\angle ABO| && \text{[Exterior Angle]} \\ &= 2 \cdot |\angle ABO|. \end{aligned}$$

Similarly,

$$|\angle COD| = 2 \cdot |\angle CBO|.$$

Thus

$$\begin{aligned} |\angle AOC| &= |\angle AOD| + |\angle COD| \\ &= 2 \cdot |\angle ABO| + 2 \cdot |\angle CBO| \\ &= 2 \cdot |\angle ABC|. \end{aligned}$$

□

Corollary 2. *All angles at points of the circle, standing on the same arc, are equal. In symbols, if A, A', B and C lie on a circle, and both A and A' are on the same side of the line BC, then $\angle BAC = \angle BA'C$.*

Proof. Each is half the angle subtended at the centre. □

Remark 7. The converse is true, but one has to careful about sides of BC:

Converse to Corollary 2: *If points A and A' lie on the same side of the line BC, and if $|\angle BAC| = |\angle BA'C|$, then the four points A, A', B and C lie on a circle.*

Proof. Consider the circle s through A, B and C. If A' lies outside the circle, then take A'' to be the point where the segment $[A'B]$ meets s. We then have

$$|\angle BA'C| = |\angle BAC| = |\angle BA''C|,$$

by Corollary 2. This contradicts Theorem 6.

A similar contradiction arises if A' lies inside the circle. So it lies on the circle. ☐

Corollary 3. *Each angle in a semicircle is a right angle. In symbols, if BC is a diameter of a circle, and A is any other point of the circle, then $\angle BAC = 90°$.*

Proof. The angle at the centre is a straight angle, measuring 180°, and half of that is 90°. ☐

Corollary 4. *If the angle standing on a chord $[BC]$ at some point of the circle is a right angle, then $[BC]$ is a diameter.*

Proof. The angle at the centre is 180°, so is straight, and so the line BC passes through the centre. ☐

Definition 41. A **cyclic** quadrilateral is one whose vertices lie on some circle.

Corollary 5. *If $ABCD$ is a cyclic quadrilateral, then opposite angles sum to* 180°.

Proof. The two angles at the centre standing on the same arcs add to 360°, so the two halves add to 180°. ☐

Remark 8. The converse also holds: *If $ABCD$ is a convex quadrilateral, and opposite angles sum to* 180°, *then it is cyclic.*

Proof. This follows directly from Corollary 5 and the converse to Corollary 2. ☐

It is possible to approximate a disc by larger and smaller equilateral polygons, whose area is as close as you like to πr^2, where r is its radius. For this reason, we say that the area of the disc is πr^2.

Proposition 16. *If l is a line and s a circle, then l meets s in zero, one, or two points.*

Proof. We classify by comparing the length p of the perpendicular from the centre to the line, and the radius r of the circle. If $p > r$, there are no points. If $p = r$, there is exactly one, and if $p < r$ there are two. ☐

Definition 42. The line l is called a **tangent** to the circle s when $l \cap s$ has exactly one point. The point is called the **point of contact** of the tangent.

Theorem 20.
(1) Each tangent is perpendicular to the radius that goes to the point of contact.
(2) If P lies on the circle s, and a line l is perpendicular to the radius to P, then l is tangent to s.

Proof. (1) This proof is a proof by contradiction.

Suppose the point of contact is P and the tangent l is not perpendicular to OP.

Let the perpendicular to the tangent from the centre O meet it at Q. Pick R on PQ, on the other side of Q from P, with $|QR| = |PQ|$ (as in Figure 25).

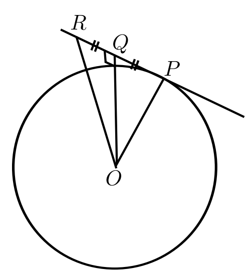


Figure 25.

Then ΔOQR is congruent to ΔOQP. [SAS]

$$\therefore |OR| = |OP|,$$

so R is a second point where l meets the circle. This contradicts the given fact that l is a tangent.

Thus l must be perpendicular to OP, as required.

(2) (Idea: Use Pythagoras. This shows directly that each other point on l is further from O than P, and hence is not on the circle.)

In detail: Let Q be any point on l, other than P. See Figure 26. Then

$$\begin{aligned} |OQ|^2 &= |OP|^2 + |PQ|^2 \quad \text{[Pythagoras]} \\ &> |OP|^2. \\ \therefore |OQ| &> |OP|. \end{aligned}$$

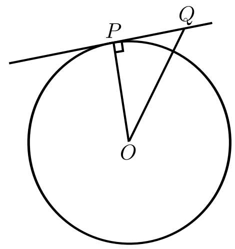


Figure 26.

$\therefore Q$ is not on the circle. [Definition of circle]
$\therefore P$ is the only point of l on the circle.
$\therefore l$ is a tangent. [Definition of tangent]
□

Corollary 6. *If two circles share a common tangent line at one point, then the two centres and that point are collinear.*

Proof. By part (1) of the theorem, both centres lie on the line passing through the point and perpendicular to the common tangent. □

The circles described in Corollary 6 are shown in Figure 27.

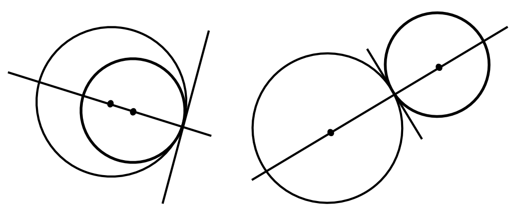


Figure 27.

Remark 9. Any two distinct circles will intersect in 0, 1, or 2 points.

If they have two points in common, then the common chord joining those two points is perpendicular to the line joining the centres.

If they have just one point of intersection, then they are said to be *touching* and this point is referred to as their *point of contact*. The centres and the point of contact are collinear, and the circles have a common tangent at that point.

Theorem 21.
(1) The perpendicular from the centre to a chord bisects the chord.
(2) The perpendicular bisector of a chord passes through the centre.

Proof. (1) (Idea: Two right-angled triangles with two pairs of sides equal.) See Figure 28.

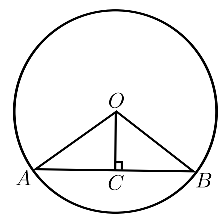


Figure 28.

In detail:

$$
\begin{aligned}
|OA| &= |OB| && \text{[Definition of circle]}\\
|OC| &= |OC| &&\\
\\
|AC| &= \sqrt{|OA|^2 - |OC|^2} && \text{[Pythagoras]}\\
&= \sqrt{|OB|^2 - |OC|^2} &&\\
&= |CB|. && \text{[Pythagoras]}
\end{aligned}
$$

$\therefore \Delta OAC$ is congruent to ΔOBC. [SSS]
$\therefore |AC| = |CB|$.

(2) This uses the Ruler Axiom, which has the consequence that a segment has exactly one midpoint.
Let C be the foot of the perpendicular from O on AB.
By Part (1), $|AC| = |CB|$, so C is the midpoint of $[AB]$.
Thus CO is the perpendicular bisector of AB.
Hence the perpendicular bisector of AB passes through O. □

6.12 Special Triangle Points

Proposition 17. *If a circle passes through three non-collinear points A, B, and C, then its centre lies on the perpendicular bisector of each side of the triangle ΔABC.*

Definition 43. The **circumcircle** of a triangle ΔABC is the circle that passes through its vertices (see Figure 29). Its centre is the **circumcentre** of the triangle, and its radius is the **circumradius**.

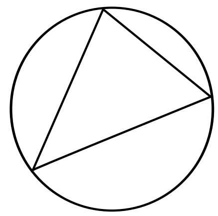


Figure 29.

Proposition 18. *If a circle lies inside the triangle ΔABC and is tangent to each of its sides, then its centre lies on the bisector of each of the angles $\angle A$, $\angle B$, and $\angle C$.*

Definition 44. The **incircle** of a triangle is the circle that lies inside the triangle and is tangent to each side (see Figure 30). Its centre is the **incentre**, and its radius is the **inradius**.

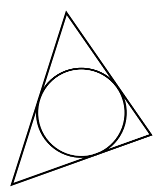


Figure 30.

Proposition 19. *The lines joining the vertices of a triangle to the centre of the opposite sides meet in one point.*

Definition 45. A line joining a vertex of a triangle to the midpoint of the opposite side is called a **median** of the triangle. The point where the three medians meet is called the **centroid**.

Proposition 20. *The perpendiculars from the vertices of a triangle to the opposite sides meet in one point.*

Definition 46. The point where the perpendiculars from the vertices to the opposite sides meet is called the **orthocentre** (see Figure 31).

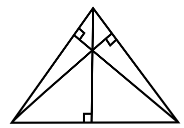


Figure 31.

7 Constructions to Study

The instruments that may be used are:

straight-edge: This may be used (together with a pencil) to draw a straight line passing through two marked points.

compass: This instrument allows you to draw a circle with a given centre, passing through a given point. It also allows you to take a given segment $[AB]$, and draw a circle centred at a given point C having radius $|AB|$.

ruler: This is a straight-edge marked with numbers. It allows you measure the length of segments, and to mark a point B on a given ray with vertex A, such that the length $|AB|$ is a given positive number. It can also be employed by sliding it along a set square, or by other methods of sliding, while keeping one or two points on one or two curves.

protractor: This allows you to measure angles, and mark points C such that the angle $\angle BAC$ made with a given ray $[AB$ has a given number of degrees. It can also be employed by sliding it along a line until some line on the protractor lies over a given point.

set-squares: You may use these to draw right angles, and angles of $30°$, $60°$, and $45°$. It can also be used by sliding it along a ruler until some coincidence occurs.

The prescribed constructions are:

1. Bisector of a given angle, using only compass and straight edge.

2. Perpendicular bisector of a segment, using only compass and straight edge.

3. Line perpendicular to a given line l, passing through a given point not on l.

4. Line perpendicular to a given line l, passing through a given point on l.
5. Line parallel to given line, through given point.
6. Division of a segment into 2, 3 equal segments, without measuring it.
7. Division of a segment into any number of equal segments, without measuring it.
8. Line segment of given length on a given ray.
9. Angle of given number of degrees with a given ray as one arm.
10. Triangle, given lengths of three sides.
11. Triangle, given SAS data.
12. Triangle, given ASA data.
13. Right-angled triangle, given the length of the hypotenuse and one other side.
14. Right-angled triangle, given one side and one of the acute angles (several cases).
15. Rectangle, given side lengths.
16. Circumcentre and circumcircle of a given triangle, using only straight-edge and compass.
17. Incentre and incircle of a given triangle, using only straight-edge and compass.
18. Angle of 60°, without using a protractor or set square.
19. Tangent to a given circle at a given point on it.
20. Parallelogram, given the length of the sides and the measure of the angles.
21. Centroid of a triangle.
22. Orthocentre of a triangle.

8 Teaching Approaches

8.1 Practical Work

Practical exercises and experiments should be undertaken before the study of theory. These should include:

1. Lessons along the lines suggested in the Guidelines for Teachers [2]. We refer especially to Section 4.6 (7 lessons on Applied Arithmetic and Measure), Section 4.9 (14 lessons on Geometry), and Section 4.10 (4 lessons on Trigonometry).

2. Lessons along the lines of Prof. Barry's memo.

3. Ideas from Technical Drawing.

4. Material in [3].

8.2 From Discovery to Proof

It is intended that all of the geometrical results on the course would first be encountered by students through investigation and discovery. As a result of various activities undertaken, students should come to appreciate that certain features of certain shapes or diagrams appear to be independent of the particular examples chosen. These apparently constant features therefore seem to be general results that we have reason to believe might always be true. At this stage in the work, we ask students to accept them as true for the purpose of applying them to various contextualised and abstract problems, but we also agree to come back later to revisit this question of their truth. Nonetheless, even at this stage, students should be asked to consider whether investigating a number of examples in this way is sufficient to be convinced that a particular result always holds, or whether a more convincing argument is required. Is a person who refuses to believe that the asserted result will always be true being unreasonable? An investigation of a statement that appears at first to be always true, but in fact is not, may be helpful, (e.g. the assertion that $n^2 + n + 41$ is prime for all $n \in \mathbb{N}$). Reference might be made to other examples of conjectures that were historically believed to be true until counterexamples were found.

Informally, the ideas involved in a mathematical proof can be developed even at this investigative stage. When students engage in activities that lead to closely related results, they may readily come to appreciate the manner in which these results are connected to each other. That is, they may see for themselves or be led to see that the result they discovered today is an inevitable logical consequence of the one they discovered yesterday. Also, it should be noted that working on problems or cuts involves logical deduction from general results.

Later, students at the relevant levels need to proceed beyond accepting a result on the basis of examples towards the idea of a more convincing logical argument. Informal justifications, such as a dissection-based proof of Pythagoras theorem, have a role to play here. Such justifications develop an argument more strongly than a set of examples. It is worth discussing what the word prove means in various contexts, such as in a criminal trial, or in a civil court, or in everyday language. What mathematicians regard as a proof is quite different from these other contexts. The logic involved in the various steps must be unassailable. One might present one or more of the readily available dissection-based proofs of fallacies and then probe a dissection-based proof of Pythagoras theorem to see what possible gaps might need to be bridged.

As these concepts of argument and proof are developed, students should be led to appreciate the need to formalise our idea of a mathematical proof to lay out the ground rules that we can all agree on. Since a formal proof only allows us to progress logically from existing results to new ones, the need for axioms is readily identified, and the students can be introduced to formal proofs.

9 Syllabus for JCOL

9.1 Concepts

Set, plane, point, line, ray, angle, real number, length, degree, triangle, right-angle, congruent triangles, similar triangles, parallel lines, parallelogram, area, tangent to a circle, subset, segment, collinear points, distance, midpoint of a segment, reflex angle, ordinary angle, straight angle, null angle, full angle, supplementary angles, vertically-opposite angles, acute angle, obtuse angle, angle bisector, perpendicular lines, perpendicular bisector of a segment, ratio, isosceles triangle, equilateral triangle, scalene triangle, right-angled triangle, exterior angles of a triangle, interior opposite angles, hypotenuse, alternate angles, corresponding angles, polygon, quadrilateral, convex quadrilateral, rectangle, square, rhombus, base and corresponding apex and height of triangle or parallelogram, transversal line, circle, radius, diameter, chord, arc, sector, circumference of a circle, disc, area of a disc, circumcircle, point of contact of a tangent, vertex, vertices (of angle, triangle, polygon), endpoints of segment, arms of an angle, equal segments, equal angles, adjacent sides, angles, or vertices of triangles or quadrilaterals, the side opposite an angle of a triangle, opposite sides or angles of a quadrilateral, centre of a circle.

9.2 Constructions

Students will study constructions 1, 2, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15.

9.3 Axioms and Proofs

The students should be exposed to some formal proofs. They will not be examined on these. They will see Axioms 1,2,3,4,5, and study the proofs of Theorems 1, 2, 3, 4, 5, 6, 9, 10, 13 (statement only), 14, 15; and direct proofs of Corollaries 3, 4.

10 Syllabus for JCHL

10.1 Concepts

Those for JCOL, and concurrent lines.

10.2 Constructions

Students will study all the constructions prescribed for JC-OL, and also constructions 3 and 7.

10.3 Logic, Axioms and Theorems

Students will be expected to understand the meaning of the following terms related to logic and deductive reasoning: **Theorem, proof, axiom, corollary, converse, implies.**

They will study Axioms 1, 2, 3, 4, 5. They will study the proofs of Theorems 1, 2, 3, 4*, 5, 6*, 9*, 10, 11, 12, 13, 14*, 15, 19*, Corollaries 1, 2, 3, 4, 5, and their converses. Those marked with a * may be asked in examination.

The formal material on area will not be studied at this level. Students will deal with area only as part of the material on arithmetic and mensuration.

11 Syllabus for LCFL

A knowledge of the theorems prescribed for JC-OL will be assumed, and questions based on them may be asked in examination. Proofs will not be required.

11.1 Constructions

A knowledge of the constructions prescribed for JC-OL will be assumed, and may be examined. In addition, students will study constructions 18, 19, 20.

12 Syllabus for LCOL

12.1 Constructions

A knowledge of the constructions prescribed for JC-OL will be assumed, and may be examined. In addition, students will study the constructions prescribed for LC-FL, and constructions 16, 17, 21.

12.2 Theorems and Proofs

Students will be expected to understand the meaning of the following terms related to logic and deductive reasoning: **Theorem, proof, axiom, corollary, converse, implies.**

A knowledge of the Axioms, concepts, Theorems and Corollaries prescribed for JC-OL will be assumed. (In the transitional period, students who have taken the discontinued JL-FL, will have to study these as well.)

Students will study proofs of Theorems 7, 8, 11, 12, 13, 16, 17, 18, 20, 21, and Corollary 6.

No proofs are examinable. Students will be examined using problems that can be attacked using the theory.

13 Syllabus for LCHL

13.1 Constructions

A knowledge of the constructions prescribed for JC-HL will be assumed, and may be examined. In addition, students will study the constructions prescribed for LC-OL, and construction 22.

13.2 Theorems and Proofs

Students will be expected to understand the meaning of the following terms related to logic and deductive reasoning: **Theorem, proof, axiom, corollary, converse, implies, is equivalent to, if and only if, proof by contradiction.**

A knowledge of the Axioms, concepts, Theorems and Corollaries prescribed for JC-HL will be assumed.

Students will study all the theorems and corollaries prescribed for LC-OL, but will not, in general, be asked to reproduce their proofs in examination.

However, they may be asked to give proofs of the Theorems 11, 12, 13, concerning ratios, which lay the proper foundation for the proof of Pythagoras studied at JC.

They will be asked to solve geometrical problems (so-called "cuts") and write reasoned accounts of the solutions. These problems will be such that they can be attacked using the given theory. The study of the propositions may be a useful way to prepare for such examination questions.

References

[1] Patrick D. Barry. *Geometry with Trigonometry.* Horwood. Chichester. 2001. ISBN 1-898563-69-1.

[2] Junior Cycle Course Committee, NCCA. *Mathematics: Junior Certificate Guidelines for Teachers.* Stationary Office, Dublin. 2002. ISBN 0-7557-1193-9.

[3] Fiacre O'Cairbre, John McKeon, and Richard O. Watson. A Resource for Transition Year Mathematics Teachers. DES. Dublin. 2006.

[4] Anthony G. O'Farrell. *School Geometry*. IMTA Newsletter 109 (2009) 21-28.

Section C

The following syllabus material is retained from the previous Leaving Certificate mathematics syllabus published in 1994.

1. INTRODUCTION

1.1 CONTEXT

Mathematics is a wide-ranging subject with many aspects. On the one hand, in its manifestations in terms of counting, measurement, pattern and geometry it permeates the natural and constructed world about us, and provides the basic language and techniques for handling many aspects of everyday and scientific life. On the other hand, it deals with abstractions, logical arguments, and fundamental ideas of truth and beauty, and so is an intellectual discipline and a source of aesthetic satisfaction. These features have caused it to be given names such as “the queen and the servant of the sciences”. Its role in education reflects this dual nature: it is both practical and theoretical – geared to applications and of intrinsic interest – with the two elements firmly interlinked.

Mathematics has traditionally formed a substantial part of the education of young people in Ireland throughout their schooldays. Its value for further and higher education, for employment, and as a component of general education has been recognised by the community at large. Accordingly, it is of particular importance that the mathematical education offered to students should be appropriate to their abilities, needs and interests, and should fully and appositely reflect the broad nature of the subject and its potential for enhancing the students’ development.

1.2 AIMS

It is intended that mathematics education would:

A. Contribute to the personal development of the students:
   - helping them to acquire the mathematical knowledge, skills and understanding necessary for personal fulfilment;
   - developing their modelling abilities, problem-solving skills, creative talents, and powers of communication;
   - extending their ability to handle abstractions and generalisations, to recognise and present logical arguments, and to deal with different mathematical systems;
   - fostering their appreciation of the creative and aesthetic aspects of mathematics, and their recognition and enjoyment of mathematics in the world around them;
   - hence, enabling them to develop a positive attitude towards mathematics as an interesting and valuable subject of study;

B. Help to provide them with the mathematical knowledge, skills and understanding needed for life and work:
   - promoting their confidence and competence in using the mathematical knowledge and skills required for everyday life, work and leisure;
   - equipping them for the study of other subjects in school;
   - preparing them for further education and vocational training;
   - in particular, providing a basis for the further study of mathematics itself.

It should be noted that in catering for the needs of the students, the courses should also be producing suitably educated and skilled young people for the requirements of the country.

1.3 GENERAL OBJECTIVES

The teaching and learning of mathematics has been described as involving facts, skills, concepts (or "conceptual structures", strategies, and – stemming from these – appreciation.

In terms of student outcomes, this can be formulated as follows:
The students should be able to recall relevant facts. They should be able to demonstrate instrumental understanding ("knowing how") and necessary psychomotor skills. They should possess relational understanding ("knowing why"). They should be able to apply their knowledge in familiar and eventually in unfamiliar contexts; and they should develop analytical and creative powers in mathematics. Hence they should develop appreciative attitudes to the subject and its uses. The aims listed in Section 1.2 can therefore be translated into general objectives as given below.

Fundamental Objectives

A. Students should be able to recall basic facts; that is, they should be able to:
- display knowledge of conventions such as terminology and notation;
- recognise basic geometrical figures and graphical displays;
- state important derived facts resulting from their studies.

(Thus, they should have fundamental information readily available to enhance understanding and aid application).

B. They should be able to demonstrate instrumental understanding; hence they should know how (and when) to:
- carry out routine computational procedures and other such algorithms;
- perform measurements and constructions to an appropriate degree of accuracy;
- present information appropriately in tabular, graphical and pictorial form, and read information presented in these forms;
- use mathematical equipment such as calculators, rulers, setsquares, protractors, and compasses, as required for the above.

(Thus, they should be equipped with the basic competencies needed for mathematical activities).

C. They should have acquired relational understanding, i.e. understanding of concepts and conceptual structures, so that they can:
- interpret mathematical statements;
- interpret information presented in tabular, graphical and pictorial form;
- recognise patterns, relationships and structures;
- follow mathematical reasoning.

(Thus, they should be able to see mathematics as an integrated, meaningful and logical discipline).

D. They should be able to apply their knowledge of facts and skills; that is, they should be able when working in familiar types of context to:
- translate information presented verbally into mathematical forms;
- select and use appropriate mathematical formulae or techniques in order to process the information;
- draw relevant conclusions.

(Thus, they should be able to use mathematics and recognise it as a powerful tool with wide ranging areas of applicability).

E. They should have developed the psychomotor and communicative skills necessary for the above.

F. They should appreciate mathematics as a result of being able to:
- use mathematical methods successfully;
- acknowledge the beauty of form, structure and pattern;
- recognise mathematics in their environment;
- apply mathematics successfully to common experience.

Other Objectives

G. They should be able to analyse information, including information presented in unfamiliar contexts:
- formulate proofs;
- form suitable mathematical models;
- hence select appropriate strategies leading to the solution of problems.

H. They should be able to create mathematics for themselves:
- explore patterns;
- formulate conjectures;
- support, communicate and explain findings.

I. They should be aware of the history of mathematics and hence of its past, present and future role as part of our culture.

Note:
Many attempts have been made to adapt the familiar Bloom taxonomy to suit mathematics education: in particular, to include a category corresponding to "carrying out routine procedures" ("doing sums" and so forth). The categories used above are intended, inter alia, to facilitate the design of suitably structured examination questions.

1.4 PRINCIPLES OF COURSE DESIGN

To implement all the aims and objectives appropriately, three courses were designed: at Higher level, Ordinary level and Foundation level.

The following principles influenced the design of all courses.

A. They should provide continuation from and development of the courses offered in the Junior Cycle.

Hence, for the cohort of students proceeding from each Junior Cycle course, there should be clear avenues of progression. These should take account of the background, likely learning style, potential for development, and future needs of the target group.

B. They should be implemented in the present circumstances and flexible as regards future development.

(a) They should be teachable, in that it should be possible to implement the courses with the resources available.

- The courses should be teachable in the time normally allocated to a subject in the Leaving Certificate programme.
  Thus, they should not be unduly long.
- Requirements as regards equipment should not go beyond that normally found in, or easily acquired by, Irish schools.
- They should be teachable by the current teaching forces.
  Hence, the aims and style of the courses should be ones that teachers support and can address with confidence, and the material should in general be familiar.

(b) They should be learnable, by virtue of being appropriate to the different cohorts of students for whom they are designed.

- Each course should start where the students in its target group are at the time, and should proceed to suitable levels of difficulty and abstraction.
- The approaches used should accommodate different abilities and learning styles.
- The material and methods should be of interest, so that students are motivated to learn.

(c) They should be adaptable – designed so that they can serve different ends and also can evolve in future.

- A measure of choice can be provided, both within courses (by providing "options" while requiring coverage of basic and important material), and between courses (by recognising the need for different types of course).
- New material (in the "options") can be tried in the classroom and maybe later moved to the core, while material with lessening relevance can be phased out gradually.
- Appropriate responses can be made as resource provision changes (for example, allowing more emphasis on use of computers).

C. They should be applicable, preparing students for further and higher education as well as for the world of work and for leisure.

Where possible, the application should be such that they can be made clear to the students (now, rather than in some undefined future), and hence ideally should be addressable at least to some extent within the course.

D. The Mathematics they contain should be sound, important, and interesting.

A broad range of appropriate aspects of mathematics should be included.

2. HIGHER LEVEL

HIGHER COURSE: RATIONALE, STYLE AND AIMS

The Higher course is aimed at the more able students. Students may choose it because it caters for their needs and aspirations as regards careers or further study, or because they have a special interest in mathematics. The course should therefore equip mathematical "specialists" – students who will pursue advanced mathematics courses; but it should also cater suitably for students who will not proceed to further study of mathematics or related subjects. Hence, material is chosen for its intrinsic interest and general applicability as well as its provision of concepts and techniques appropriate for future specialists in the field.

Students who follow the Higher course will already have shown their ability to study mathematics in an academic environment. The course offers opportunities for them to deepen their understanding of mathematical ideas, to encounter more of the powerful concepts and methods that have made mathematics important in our culture, and to enhance their enjoyment of the subject.

For the target group, particular emphasis can be given to aims concerned with problem-solving, abstracting, generalising and proving. Due attention should be given to maintenance of the more basic skills, especially in algebra (where students' shortcomings have been seen to stand in the way of their own progress). However, it may be assumed that some aims regarding the use of mathematics in everyday life and work have been achieved in the Junior Cycle; they are therefore less prominent at this level.

HIGHER COURSE: PRIOR KNOWLEDGE

Knowledge of the content of the Junior Certificate Higher course will be assumed.

HIGHER COURSE: ASSESSMENT OBJECTIVES

The assessment objectives are the fundamental objectives A, B, C, D and E (See Section 1.3), interpreted in the context of the following aims of the Higher course:

- a deepened understanding of mathematical ideas;
- an appreciation of powerful concepts and methods;
- the ability to solve problems, abstract, and generalise, and to prove the results specified in the syllabus (marked with an asterisk (*));
- competency in algebraic manipulation.

Note:
As indicated by objective E, the students should present their work comprehensively; this is especially relevant when they are using calculators.

HIGHER COURSE: TOPICS

Note for the examination:
In the case of results marked with an asterisk (*), formal proofs may be examined; in the case of other results stated in the syllabus, proofs will not be examined.

Algebra 1. Algebraic operations on polynomials and rational functions. Addition, subtraction, multiplication and division and the use of brackets and surds. Laws of indices and logarithms. *The Factor Theorem for polynomials of degree two or three. Factorisation of such polynomials (the linear and quadratic factors having integer coefficients). Solution of cubic equations with at least one integer root. Sums and products of roots of quadratic equations.		Use of Remainder theorem <u>not</u> required. Cubics <u>excluded.</u>
2. Unique solution of simultaneous linear equations with two or three unknowns Solution of one linear and one quadratic equation with two unknowns.		Sets of equations with non-unique solutions or no solutions <u>excluded</u>.
3. Inequalities: solution of inequalities of the form $g(x) < k$, $x \in$ R, where $g(x) = ax^2 + bx + c$ or $g(x) = \dfrac{ax+b}{cx+d}$ Use of notation $\|x\|$; solution of $\|x-a\| < b$		
4. Complex numbers: Argand diagram; addition, subtraction, multiplication, division; modulus; conjugate; conjugates of sums and products; conjugate root theorem. *De Moivre's theorem: proof by induction for n $\in$ Z; applications such as nth roots of unity, n $\in$ Q, and identities such as $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.		Properties of modulus <u>excluded</u>. Loci excluded. Transformations from z-plane to w-plane excluded.

5.*Proof by induction of simple identities such as the sum of the first n integers and the sum of a geometric series, simple inequalities such as $n! \geq 2^n$, $2^n \geq n^2$ ($n \geq 4$), and $(1+x)^n \geq 1+nx$ ($x>-1$), and factorisation results such as: 3 is a factor of $4^n - 1$.		
6. Matrices: dimension, 1x2, 2x1 and 2x2 matrices; addition; multiplication by a scalar; product. Properties: addition of matrices is commutative; multiplication of matrices is not necessarily commutative. Identities for addition and multiplication. Inverse of a 2 x 2 matrix. Application to solution of two linear equations in two unknowns.		Transformations <u>excluded</u> from this section.
Sequences and Series Sums of finite series of telescoping type such as $\sum_{n=1}^{m}\frac{1}{n(n+1)}$, arithmetic and geometric series, and $\sum_{n=1}^{m} n^2$. $n!$, binomial coefficients $\binom{n}{r}$, $n \in N_0$; binomial series for positive integer exponent. Informal treatment of limits of sequences; rules for sums, products, quotients; limits of sequences such as $\lim_{n\to\infty}\frac{n}{n+1}$, $\lim_{n\to\infty} r^n$ ($\lvert r \rvert < 1$). Sums of infinite series of telescoping type, such as $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$, $\sum_{n=0}^{\infty} x^n$, $\sum_{n=0}^{\infty} nx^n$ ($\lvert x \rvert < 1$). Recurring decimals as infinite geometric series.		Questions on approximations will <u>not</u> be asked. Series will be confined to those for which the sums are actually found. Limits will be found only for sequences given explicitly (not by recurrence relations). Tests for convergence <u>excluded</u>.

Functions and calculus		
1. Functions: Finding the period and range of a continuous periodic function, given its graph on scaled and labeled axis.		Range a closed interval [a, b], a, b $\in$ Z; period $\in$ N_0. Periodic graph need not necessarily be trigonometric in type: e.g. saw-tooth graph.
Informal treatment of limits of functions; rules for sums, products and quotients.		
2. Differential calculus:		
*Derivations from first principles of x^2, x^3, sin x, cos x, $\sqrt{x}$, and 1/x.		
First derivatives of: – polynomials, rational, power and trigonometric functions; – $\tan^{-1}$, $\sin^{-1}$, exponential and logarithmic functions; – *sums; *products; differences; *quotients; compositions of these.		
*Proof by induction that $\frac{d}{dx}(x^n)=n\,x^{n-1}$.		
Application to finding tangents to curves.		
Simple second derivatives.		
First derivatives of implicit and parametric functions.		
Rates of change. } Maxima and minima. }		Problems involving modelling <u>excluded</u>.
Curve sketching of polynomials and of functions of the form $\frac{a}{x+b}$ and $\frac{x}{x+b}$, with reference to turning points, points of inflections, and asymptotes.		
Newton-Raphson method for finding approximate roots of cubic equations.		

3. Integral calculus: Integration techniques (integrals of sums, multiplying constants, and substitution) applied to: a. x^n b. $\sin nx$, $\cos nx$, $\sin^2 nx$, $\cos^2 nx$; c. e^{nx} d. functions of the form: $\frac{1}{x+a}$, $\frac{1}{a^2+x^2}$, $\frac{1}{\sqrt{(a^2-x^2}}$, $\sqrt{(a^2-x^2}$. Definite integrals with applications to areas and volumes of revolution (confined to cones and spheres).		Integration by parts and partial fractions <u>excluded</u>.

3. ORDINARY LEVEL

ORDINARY COURSE: RATIONALE, STYLE AND AIMS

For many of the students for whom the Ordinary course was designed, mathematics is essentially a service subject – providing knowledge and techniques that will be needed in future for their study of scientific, economic, business and technical subjects. For other Ordinary course students, however, the Leaving Certificate may provide their last formal encounter with mathematics. In general, therefore, the course should equip students who will use mathematics in further study for the courses that they will pursue; but should also cater suitably for students who will not proceed further study of mathematics or related subjects. Hence, material is chosen for its intrinsic interest and general applicability as well as its provision of techniques useful in further education.

Students who follow the Ordinary course may have had fairly limited prior contact with abstract mathematics. The course therefore moves gradually from the relatively concrete and practical to more abstract and general concepts. As well as equipping the students with important tools, it offers opportunities for them to deepen their understanding and appreciation of mathematics and to experience some of its classical "powerful ideas".

For the target group, particular emphasis can be given to aims concerned with the use of mathematics. Due attention should be given to maintenance of the more basic skills, especially in applications of arithmetic and algebra (where students' shortcomings have been seen to stand in the way of their own progress).

ORDINARY COURSE: PRIOR KNOWLEDGE

Knowledge of the content of the Junior Certificate Ordinary course will be assumed.

ORDINARY COURSE: ASSESSMENT OBJECTIVES

The assessment objectives are the fundamental objectives A, B, C, D and E (see Section 1.3), interpreted in the context of the following aims of the Ordinary course.

- a widening of the span of the students' understanding from a relatively concrete and practical level to a more abstract and general one;
- the acquisition of mathematical techniques and their use context;
- proficiency in basic skills of arithmetic and algebra.

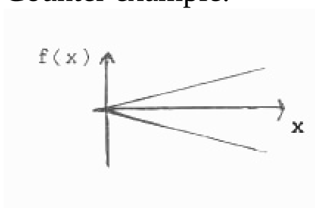
Notes:

1. It is desirable that students following the course would make intelligent and proficient use of calculators (and would carry their expertise into their lives beyond school); and it is envisaged that calculators would normally be used as a tool during the teaching, learning and examining of the course. However, the course has not been specifically designed around use of machines, and assessment of calculator skills is not a "core" or essential requirement.

2. As indicated by objective E, the students should present their work comprehensively; this is especially relevant when they are using calculators.

ORDINARY COURSE: TOPICS

Arithmetic		
1. The operations of addition, subtraction, multiplication and division of rational numbers, and the relations < and >, applied to practical problems involving counting and measurement (units {S.I. where appropriate} of length, area, volume, time, mass, temperature, and money); averages; rates; proportion; percentages; money transactions, including compound interest; taxation.		Problems may involve use of variables and knowledge of proportionality. Taxation problems may include those working from tax to original pay etc.
2. Estimation and approximation. Relative error: definition (relative error = |error/true value|, or – if the true value is unavailable – |error/estimate|); percentage error. Accumulation of error by addition or subtraction only. Simpson's method for approximating to areas of irregular figures. Use of calculators and/or tables; scientific notation.		Intelligent and accurate use of calculator required.
3. Powers and nth roots (for example as used in compound interest formulae).		
4. Areas: triangles, discs, sectors of discs; figures made from combinations of these. Volumes: sphere, hemisphere, right cone, right prism, rectangular solids; solids made from combinations of these.		
Algebra		
1. Manipulation of formulae including simple algebraic fractions. Laws of indices: $x^a x^b = x^{a+b}$; $(xy)^a = x^a y^a$; $(x^a)^b = x^{ab}$ Use of fractional and negative indices, e.g. $(-8)^{\frac{2}{3}}$, $(\frac{1}{4})^{-\frac{1}{2}}$. Solution of equations such as $5^x = \frac{1}{25}$. Solution of quadratic equations with rational coefficients.		

The Factor Theorem for polynomials of degree two or three. Factorisation of such polynomials (the linear and quadratic factors having integer coefficients). 2. Unique solution of simultaneous linear equations with two unknowns. Solution of one linear and one quadratic equation with two unknowns (e.g. $2x - y = 1$, $x^2 + y^2 = 9$). 3. Inequalities: solution of inequalities of the form $g(x) < k$, where $g(x) = ax + b$, and $a, b, k \in$ Q. 4. Complex numbers: Argand diagram, modulus, complex conjugate. Addition, subtraction, multiplication, division.		
Finite sequences and series Informal treatment of sequences. Arithmetic and geometric sequences. Sum to n terms of arithmetic and geometric series.		
Functions and Calculus 1. Functions: A function as a set of couples, no two couples having the same first element; that is, a function as a particular form of association between the elements of two sets. Function considered as specified by a formula (or rule or procedure or curve) which establishes such an association by consistently transforming input into output. Examples of functions; examples which are not functions. Use of function notation: $f(x) =$ $f: x \to$ $y =$		See Relations section of Junior Certificate Foundation course. This aspect not to be examined. Examples: (i) $f: x \to \begin{cases} 1, & x \text{ even} \\ 0, & x \text{ odd} \end{cases}$ X ∈ N (ii) alphanumeric set → ASCII code or bar code. (iii) $f: x \to 2x$, $x \in$ R Counter example: 

Graphs of functions f of linear, quadratic and cubic type and of $\frac{1}{x+a}$. Use of graphs to find approximate solutions to inequalities $f(x) \le b$ and to equalities $f(x) = cx + d$. Finding the period and range of a continuous periodic function, given its graph on scaled and labelled axis. 2. Calculus Informal treatment of limits of functions. Derivations from first principles of polynomials of degree ≤ 2. First derivatives of: – polynomials and rational functions; – sums, products, differences, quotients. Easy applications of the chain rule. Rates of change, e.g. speed, acceleration. Tangents. Calculation of maxima and minima of quadratic and cubic functions.		Range a closed interval [a,b], a, b $\in$ Z; period $\in$ N_0. Periodic graph need not necessarily be trigonometric in type: e.g. saw-tooth graph.

4. FOUNDATION LEVEL

RATIONALE

The Foundation Course is intended to equip students with the knowledge and techniques required in everyday life and in various kinds of employment. It is also intended to lay the groundwork for students who proceed to further education and training in areas in which specialist mathematics is not required. It should therefore provide students with the mathematical tools needed in their daily life and work and (where relevant) continuing study; but it should do so in a context designed to build the students' confidence, their understanding and enjoyment of mathematics, and their recognition of its role in the world around them. Hence, material is chosen for its intrinsic interest and immediate applicability as well as its usefulness beyond school.

The course is designed for students who have had only very limited acquaintance with abstract mathematics. Basic knowledge is maintained and enhanced by being approached in an exploratory and reflective manner –available of students' increasing maturity – rather than by simply repeating work done in the Junior Cycle. Concreteness is provided in particular by extensive use of the calculator; this serves as an investigative tool as well as an object of study and a readily available resource. By means of such a developmental and constructive approach, the ground is prepared for students' advance to abstract concepts via a multiplicity of carefully graded examples. Computational work is balanced by emphasis on the visual and spatial.

For the target group, particular emphasis can be given to aims concerned with the use of mathematics in everyday life and work – especially as regards intelligent and proficient use of calculators – and with the recognition of mathematics in the environment.

AIMS

In the light of the aims of mathematics education listed in Section 1.2, the aims of the Foundation course are:

- development of students' understanding of mathematical knowledge and techniques required in everyday life and employment;
- particular emphasis of meaningfulness of mathematical concepts;
- acquisition of mathematical knowledge that is of immediate applicability and usefulness.
- introduction of the students to mathematical abstraction;
- maintenance and enhancement of students' basic mathematical knowledge and skills;
- encouragement of accurate and efficient use of the calculator;
- promotion of students' confidence in working with mathematics.

ASSESSMENT OBJECTIVES

The assessment objectives are the objectives (a), (b), (c), (d) and (e) listed in Section 1.3. These objectives should be interpreted in the context of the statement of the aims of the Foundation course. Knowledge of the content of the Junior Certificate Foundation course will be assumed.

FOUNDATION LEVEL: TOPICS

Number Systems Revision of the following, using calculator for all relevant aspects: 1. Development of the systems N of natural numbers, Z of integers, Q of rational numbers and R of real numbers. The operations of addition, multiplication, subtraction and division. Representation of numbers on a line. Inequalities. Decimals Powers and roots. Scientific notation. 2. Factors, multiples, prime numbers in N. Prime factorization 3. Use of brackets. Conventions as to the order of precedence of operations.		
Arithmetic. Use of calculator for all relevant operations in the following: 1. Approximation and error; rounding off. Relative error, percentage error, tolerance. Very large and very small numbers on the calculator. Limits to accuracy of calculators. 2. Substitution in formulae. 3. Proportion, Percentage, Averages. Average rates of change (with respect to time). 4. Compound interest and depreciation formulae. $A = P\left\{1 \pm \frac{r}{100}\right\}^n$ $P = A / \left\{1 \pm \frac{r}{100}\right\}^n$		Main stages of calculation should be shown. Formula provided in examinations; **n** a natural number.

5. Value Added Tax (VAT), Rates. Income Tax (including PRSI); emergency tax; tax tables. 6. Domestic bills and charges. 7. Currency transactions, including commission. 8. Costing. Materials and labour. Wastage. 9. Metric system. Change of units. Everyday imperial units.		Conversion factors provided for imperial units.
Areas and Volumes Use of calculator for all relevant operations in the following: 1. Plane figures: disc, triangle, rectangle, square, H-figure, parallelogram, trapezium. Solid figures: right cone, rectangular block, cylinder, sphere, right prism. 2. Use of Simpson's Rule to approximate area.		See Appendix: "Engineer's Handbook". Questions will be confined to the variables given in the formulae ("Engineer's Handbook" approach).
Algebra 1. Consideration of the following, using calculator where relevant: (j) $x + a = b$; } (ii) $ax = b$; } $a, b, c \in Q$ (iii) $ax + b = c$; } (iv) $ax + b = cx$; $a, b, c \in Z$ (v) $ax + b = cx + d$; $a, b, c, d \in Z$ (vi) $ax + by = c$; } $a, b, c, d, e, f \in Z$ $dx + ey = f$; } Problems giving rise to equations of type (i) - (vi)		Cases with unique solutions only.

(vii) $x^2 = a$; $a \in Q+$ viii) $x^2 + a = b$; $b - a > 0_1$ $a, b \in Q$ ix) $ax^2 = b$; $a, b \in Q^+$ x) $ax^2 + b = c$; $a > 0$, $(c-b) > 0$ $a, b, c \in Z$ xi) $ax^2 + bx + c = 0$; $a > 0$, $b^2 \geq 4ac$, $a, b, c \in Z$ 2. Consideration of the inequalities: i) $x + a > b$; $x + a < b$; } ii) $ax > b$; $ax < b$; } iii) $ax + b > c$; $ax + b < c$) } $a, b, c \in Z$ iv) $x + ab \geq b$; $x + a \leq b$; } v) $ax \geq b$; $ax \leq b$; } vi) $ax + b \geq c$; $ax + b \leq c$; }		Use of formula (provided in examinations)
Functions and graphs 1. A function as a set of couples		A function as a special relation, hence a particular form of association between the elements of two sets.
Function considered as specified by a formula or rule. Use of function notation: $f(x) =$ $f: x \rightarrow$ $y =$		Establishment of such an association.
2. Study of the following functions and of equations of the form $f(x) = k$, $k \in Z$: $f: x \rightarrow mx$; $m \varepsilon Q$, $x \in R$		Effect on the graph of varying m
$f: x \rightarrow mx + c$; $m, c \varepsilon Q$, $x \in R$		Significance of c
$f: x \rightarrow x^2$; $x \in R$		For example, estimation of $\sqrt{2}$

$f: x \rightarrow x^2 + c;\ c, x \in R$ $f: x \rightarrow ax^2;\ a, x \in R$ $f: x \rightarrow ax^2 + bx + c;\ a, b, c, x \in R.$ Values of x for which f(x) is maximum/minimum. Intervals of x for which f(x) is increasing/decreasing. 3. Experimental results. Fitting a straight line to a set of experimental data. Prediction. 4. Interpretation of graphs in following cases: **Case 1** Cases in which information is available only at plotted points **Case 2** Continuous graphs: - distance/time - speed/time - depth of liquid/time - conversion of units		Effect of the graph of varying c. Effect of the graph of varying a. Examples: - currency fluctuations - inflation - employment/ unemployment - temperature - temperature chart (medical) - pollen count - lead levels - smog Interpretation to include: given range of values of one variable, estimate from the graph the corresponding range of values of the other.

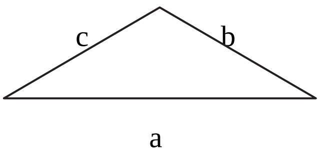
Appendix: “Engineer’s Handbook”

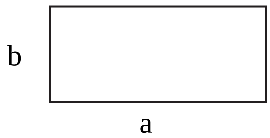
The handbook is intended to be used as follows:

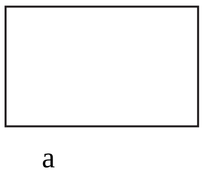
- students select the formula with the required unknown on the left-hand side and with values available for all variables on the right-hand side
- they substitute values for variables on the right-hand side of the formula;
- they evaluate the required answer, typically using a calculator.

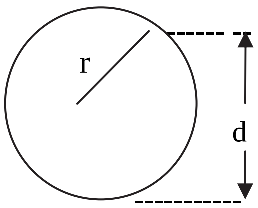
This obviates algebraic manipulation. It also provides experience of an approach widely used in practical applications.

Length

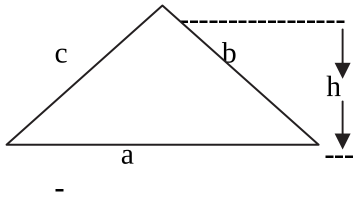
TRIANGLE	LENGTH (L)	FORMULAE
	$L = a+b+c$	$a = L - b - c$ $b = L - a - c$ $c = L - a - b$

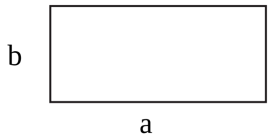
RECTANGLE	LENGTH (L)	FORMULAE
	$L = 2(a+b) = 2a + 2b$	$a = \frac{(L - 2b)}{2}$ $b = \frac{(L - 2a)}{2}$

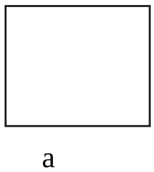
SQUARE	LENGTH (L)	FORMULAE
	$L = 4a$	$A = \frac{L}{4}$

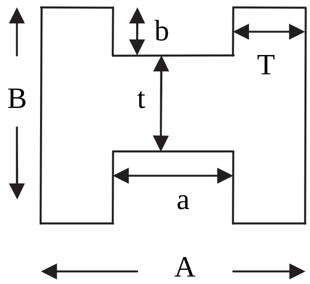
CIRCLE	LENGTH (L)	FORMULAE
	$L = 2\pi$ $L = \pi d$	$d = 2r \quad r = \frac{d}{2}$ $r = \frac{L}{2\pi}$ $d = \frac{L}{\pi}$

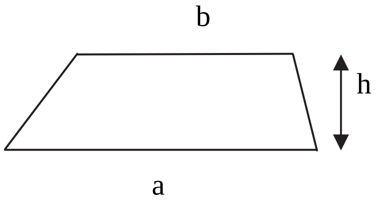
AREA

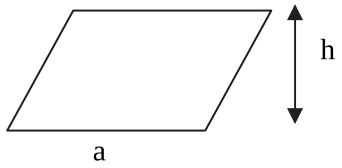
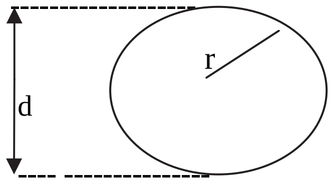
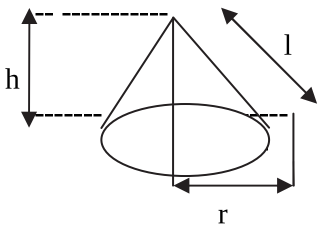
TRIANGLE	AREA	FORMULAE
	Area = $\frac{ab}{2}$	$a = \frac{2\,(area)}{h}$ $h = \frac{2\,(area)}{a}$

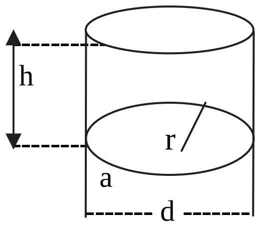
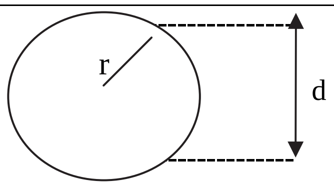
RECTANGLE	AREA	FORMULAE
	Area = ab	$a = \frac{Area}{b}$ $b = \frac{Area}{a}$

SQUARE	AREA	FORMULAE
	Area = a^2	A = $\sqrt{Area}$

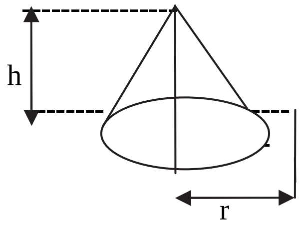
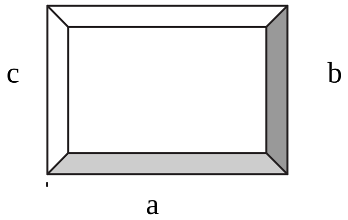
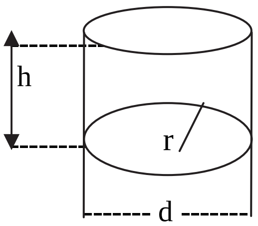
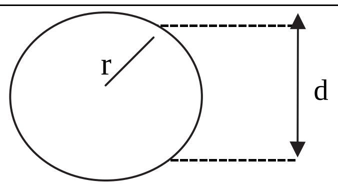
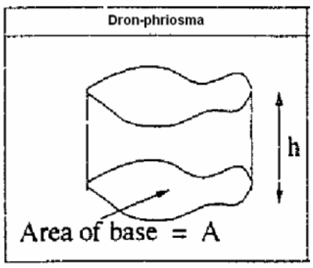
H FIGURE	AREA	FORMULAE
	Area = AB – 2ab Area = at + 2 BT Note: A = a + 2 T B = 2b + t	$A = \frac{(Area + 2ab)}{B}$ $B = \frac{(Area + 2ab)}{A}$ $a = \frac{(AB - Area)}{2b}$ $b = \frac{(AB - Area)}{2a}$

TRAPEZIUM	AREA	FORMULAE
	Area = $\frac{h(a + b)}{2}$	$a = 2\left(\frac{Area}{h}\right) - b$ $b = 2\left(\frac{Area}{h}\right) - a$ $h = \frac{2\,(Area)}{(a + b)}$

PARALLELOGRAM	AREA	FORMULAE
	Area = ah	$a = \frac{Area}{H}$ $h = \frac{Area}{a}$
DISC	**AREA**	**FORMULAE**
	$Area = \pi r^2$ $Area = \frac{\pi d^2}{4}$	$R = \sqrt{\frac{Area}{\pi}}$ $d = \sqrt{\frac{4 (Area)}{\pi}}$
RIGHT CONE	**AREA**	**FORMULAE**
	$Area = \pi r l$ Note: $1^2 = r^2 + h^2$	$R = \frac{Area}{\pi l}$ $l = \frac{Area}{\pi r}$

CYLINDER	AREA	FORMULAE
	$Area = 2 \pi r h$ $Area \ \pi d h$	$r = \frac{Area}{2 \pi h}$ $h = \frac{Area}{2 \pi r}$ $d = \frac{Area}{\pi h}$ $h = \frac{Area}{\pi d}$
SPHERE	**AREA**	**FORMULAE**
	$Area = 4 \pi r^2$ $Area = \pi d^2$	$r = \sqrt{\frac{Area}{4 \pi}}$ $d = \sqrt{\frac{Area)}{\pi}}$

VOLUME

RIGHT CONE	VOLUME (V)	FORMULAE
	$V = \frac{\pi r^2 h}{3}$	$r = \sqrt{\frac{3V}{\pi h}}$ $h = \frac{3V}{\pi r^2}$
RECTANGULAR BLOCK	**VOLUME (V)**	**FORMULAE**
	$V = abc$	$a = \frac{V}{bc}$ $b = \frac{V}{ac}$ $c = \frac{V}{ab}$
CYLINDER	**VOLUME (V)**	**FORMULAE**
	$V = \pi r^2 h$ $V = \frac{\pi h d^2}{4}$	$h = \frac{V}{\pi r^2}$ $h = \frac{4V}{\pi d^2}$ $r = \sqrt{\frac{V}{\pi h}}$ $d = \sqrt{4\frac{V}{\pi h}}$
SPHERE	**VOLUME (V)**	**FORMULAE**
	$V = \frac{4 \pi r^3}{3}$ $V = \frac{\pi d^3}{6}$	Cube roots required
RIGHT PRISM	**VOLUME (V)**	**FORMULAE**
	$V = Ah$	$A = \frac{V}{h}$ $h = \frac{V}{A}$

NCCA

An Chomhairle Náisiúnta Curaclaim agus Measúnachta
National Council for Curriculum and Assessment

AN ROINN OIDEACHAIS AGUS SCILEANNA | DEPARTMENT OF EDUCATION AND SKILLS