Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2013
Sample Paper

Mathematics
(Project Maths – Phase 2)

Paper 1

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	100 marks	4 questions
Section B	Contexts and Applications	100 marks	2 questions
Section C	Functions and Calculus (old syllabus)	100 marks	2 questions

Answer all eight questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

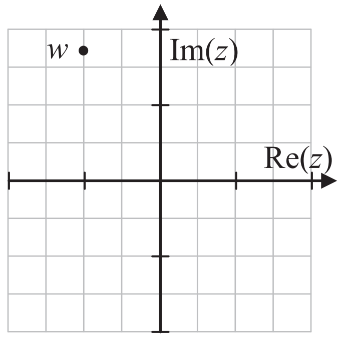
Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	100 marks

Answer **all four** questions from this section.

Question 1 **(25 marks)**

(a) $w = -1 + \sqrt{3}i$ is a complex number, where $i^2 = -1$.



(i) Write w in polar form.

(ii) Use De Moivre's theorem to solve the equation $z^2 = -1 + \sqrt{3}i$, giving your answer(s) in rectangular form.

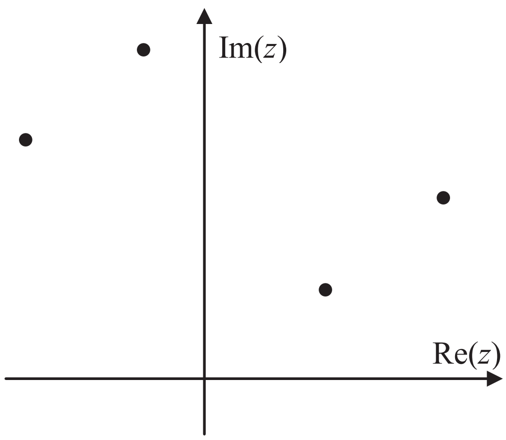
(b) Four complex numbers z_1, z_2, z_3 and z_4 are shown on the Argand diagram. They satisfy the following conditions:

$z_2 = iz_1$

$z_3 = kz_1$, where $k \in \mathbb{R}$

$z_4 = z_2 + z_3$.

The same scale is used on both axes.



(i) Identify which number is which, by labelling the points on the diagram.

(ii) Write down the approximate value of k.

Answer: ______________

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Question 2 **(25 marks)**

(a) Prove by induction that $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$, for any $n \in \mathbb{N}$.

(b) State the range of values of x for which the series $\sum_{r=2}^{\infty} (4x-1)^r$ is convergent, and write the infinite sum in terms of x.

Question 3 **(25 marks)**

A cubic function f is defined for $x \in \mathbb{R}$ as

$$f : x \mapsto x^3 + (1 - k^2)x + k\,, \quad \text{where } k \text{ is a constant.}$$

(a) Show that $-k$ is a root of f.

(b) Find, in terms of k, the other two roots of f.

(c) Find the set of values of k for which f has exactly one real root.

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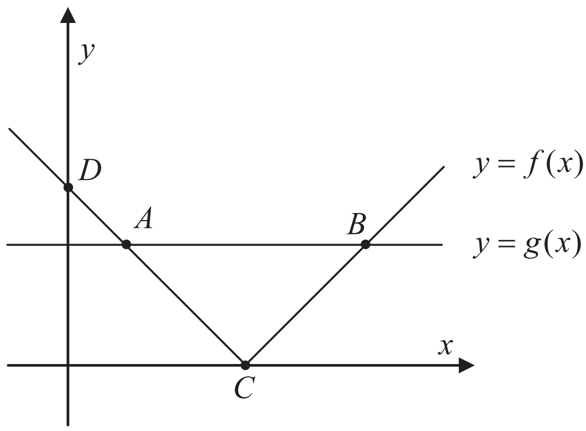
Question 4 **(25 marks)**

(a) Solve the simultaneous equations,

$$2x + 8y - 3z = -1$$
$$2x - 3y + 2z = 2$$
$$2x + y + z = 5.$$

(b) The graphs of the functions $f : x \mapsto |x-3|$ and $g : x \mapsto 2$ are shown in the diagram.

(i) Find the co-ordinates of the points A, B, C and D.



$A = (\quad , \quad)$ $B = (\quad , \quad)$

$C = (\quad , \quad)$ $D = (\quad , \quad)$

(ii) Hence, or otherwise, solve the inequality $|x-3| < 2$.

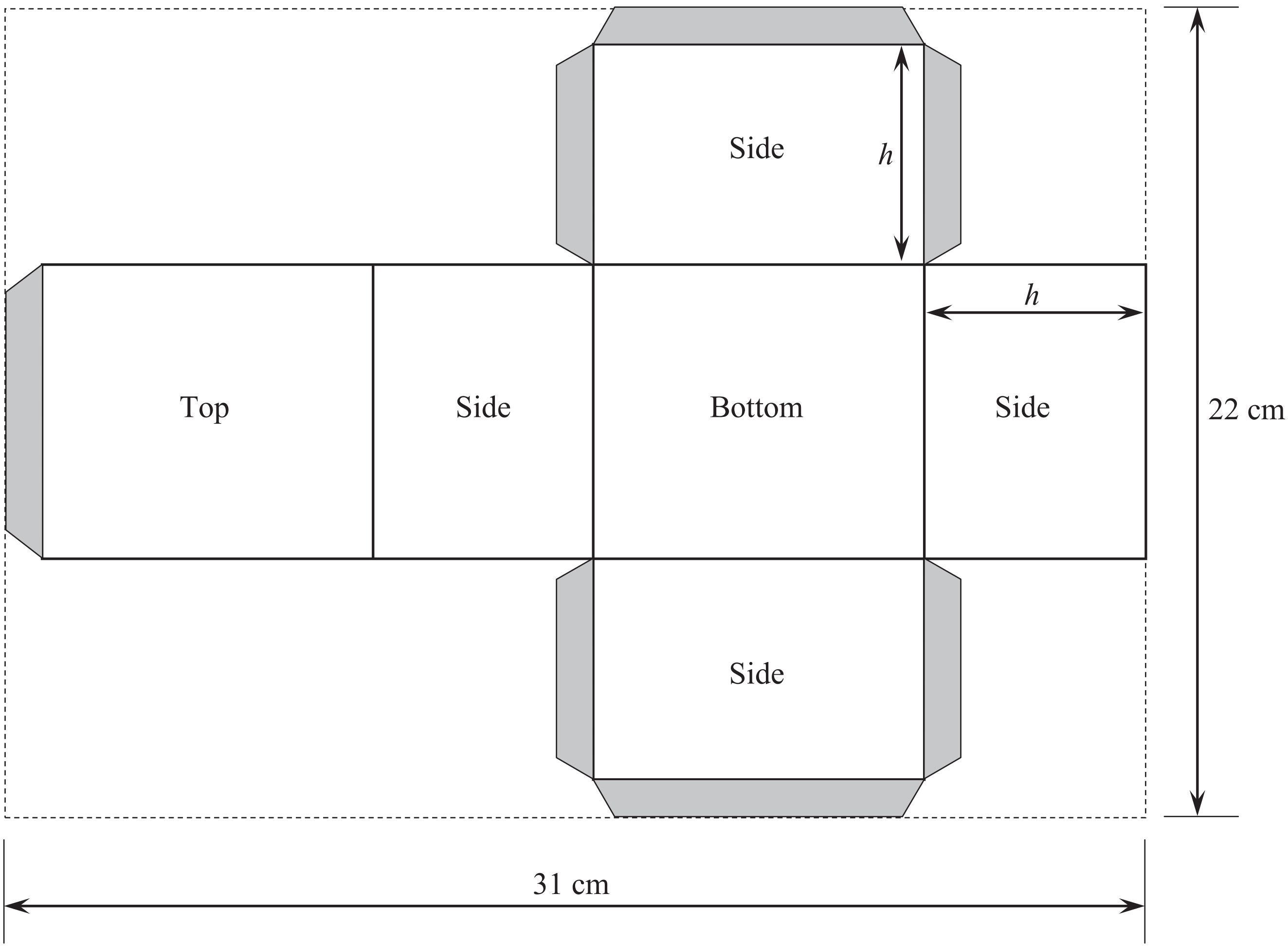
Section B	Contexts and Applications	100 marks

Answer **both** Question 5 **and** Question 6.

Question 5 **(50 marks)**

A company has to design a rectangular box for a new range of jellybeans. The box is to be assembled from a single piece of cardboard, cut from a rectangular sheet measuring 31 cm by 22 cm. The box is to have a capacity (volume) of 500 cm^3.

The net for the box is shown below. The company is going to use the full length and width of the rectangular piece of cardboard. The shaded areas are flaps of width 1 cm which are needed for assembly. The height of the box is h cm, as shown on the diagram.



(a) Write the dimensions of the box, in centimetres, in terms of h.

height = h cm

length = ________________ cm

width = ________________ cm

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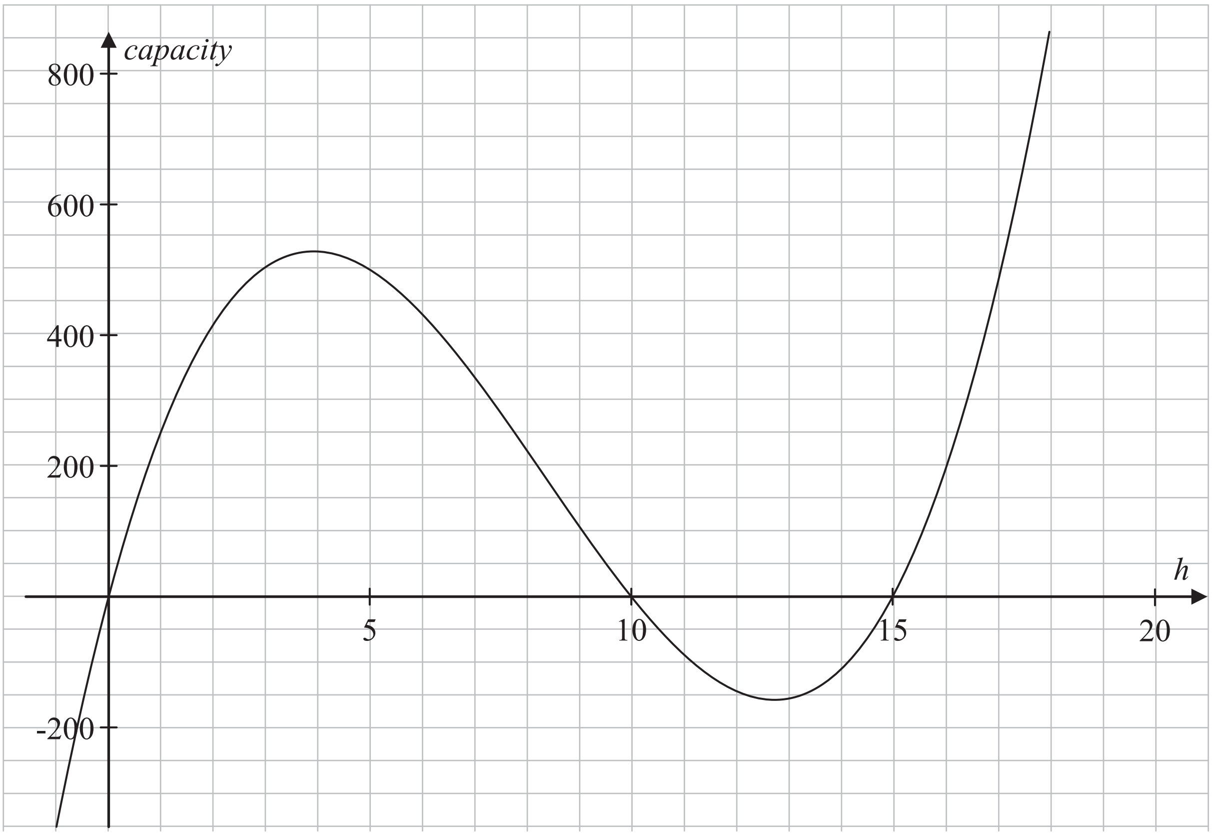
(b) Write an expression for the capacity of the box in cubic centimetres, in terms of h.

(c) Show that the value of h that gives a box with a square bottom will give the correct capacity.

(d) Find, correct to one decimal place, the other value of h that gives a box of the correct capacity.

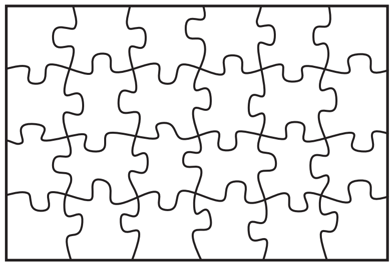
(e) The client is planning a special "10% extra free" promotion and needs to increase the capacity of the box by 10%. The company is checking whether they can make this new box from a piece of cardboard the same size as the original one (31 cm × 22 cm). A graph of the box's capacity as a function of h is shown below. Use the graph to explain why it is *not* possible to make the larger box from such a piece of cardboard.



Explanation:

Question 6 **(50 marks)**

A rectangular jigsaw puzzle has pieces arranged in rows. Each row has the same number of pieces. For example, the picture on the right shows a 4×6 jigsaw puzzle – there are four rows with 6 pieces in each row.



Every piece of the puzzle is either an *edge piece* or an *interior piece*. The puzzle shown has 16 edge pieces and 8 interior pieces.

Investigate the number of edge pieces and the number of interior pieces in an $m \times n$ jigsaw puzzle, for different values of m and n. Start by exploring some particular cases, and then attempt to answer the questions that follow, with justification.

Initial exploration:

(a) How do the number of edge pieces and the number of interior pieces compare in cases where either $m \le 4$ or $n \le 4$?

(b) Show that if the number of edge pieces is equal to the number of interior pieces, then

$$m = 4 + \frac{8}{n-4}.$$

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(c) Find all cases in which number of edge pieces is equal to the number of interior pieces.

(d) Determine the circumstances in which there are *fewer* interior pieces than edge pieces. Describe fully all such cases.

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Section C   Functions and Calculus (old syllabus)   100 marks

Answer **both** Question 7 **and** Question 8.

Question 7 **(50 marks)**

(a) The equation $x^3 + x^2 - 4 = 0$ has only one real root. Taking $x_1 = \frac{3}{2}$ as the first approximation to the root, use the Newton-Raphson method to find x_2, the second approximation.

(b) Parametric equations of a curve are:

$$x = \frac{2t-1}{t+2}, \qquad y = \frac{t}{t+2}, \qquad \text{where } t \in \mathbb{R} \setminus \{-2\}.$$

(i) Find $\frac{dy}{dx}$.

(ii) What does your answer to part **(i)** tell you about the shape of the graph?

(c) The function $f(x) = (1+x)\log_e(1+x)$ is defined for $x > -1$.

(i) Show that the curve $y = f(x)$ has a turning point at $\left(\dfrac{1-e}{e},\ -\dfrac{1}{e}\right)$.

(ii) Determine whether the turning point is a local maximum or a local minimum.

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Question 8 **(50 marks)**

(a) Find $\int \left(\sin 2x + e^{4x} \right) dx$.

(b) Let $y = \dfrac{\cos x + \sin x}{\cos x - \sin x}$.

(i) Find $\dfrac{dy}{dx}$.

(ii) Show that $\dfrac{dy}{dx} = 1 + y^2$.

(c) **(i)** Find, in terms of a and b,

$$I = \int_a^b \frac{\cos x}{1+\sin x}\, dx$$

(ii) Find in terms of a and b,

$$J = \int_a^b \frac{\sin x}{1+\cos x}\, dx .$$

(iii) Show that if $a+b=\frac{\pi}{2}$, then $I = J$.

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You may use this page for extra work

You may use this page for extra work

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Note to readers of this document:

This sample paper is intended to help teachers and candidates prepare for the June 2013 examination in *Mathematics* under Phase 2 of *Project Maths*. The content and structure do not necessarily reflect the 2014 or subsequent examinations.

In the 2013 examination, Questions 7 and 8 in Section C on Paper 1 will be similar in content and style to those that have appeared as Questions 6, 7, and 8 on the examination in previous years. On this sample paper, material from the 2010 examination has been inserted to illustrate.

Leaving Certificate 2013 – Higher Level

Mathematics (Project Maths – Phase 2) – Paper 1

Sample Paper

Time: 2 hours 30 minutes