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LEAVING CERTIFICATE EXAMINATION, 1994

MATHEMATICS—HIGHER LEVEL

SAMPLE PAPER II (300 marks) — $2\frac{1}{2}$ hours

Attempt **five** questions from Section A and **one** question from Section B. Each question carries 50 marks.

Marks may be lost if necessary work is not shown or if you do not indicate where a calculator has been used.

SECTION A

1. (*a*) Find the cartesian equation of the circle

$$x = 3 + 4\cos\theta \quad ; \quad y = 2 + 4\sin\theta$$

$$0 \leqslant \theta < 2\pi.$$

(*b*) The centre (α, β) of a circle S is in the line $2x - y + 3 = 0$. Show that S can be written as

$$x^2 + y^2 - (\beta - 3)x - 2\beta y + c = 0$$

If $(-3,2)$ and $(4,1)$ are points on the circle, find the equation of S.

(*c*) Find two values of k for which

$$8x + 3y + k = 0$$

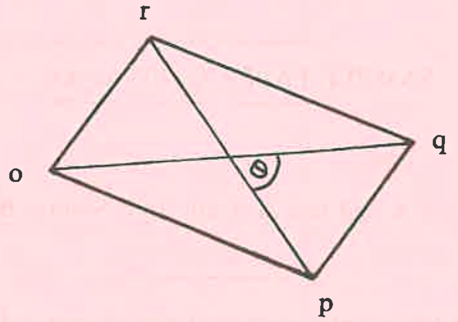
is a tangent to the circle

$$x^2 + y^2 + 4x - 3y - 12 = 0$$

2. Vertices p and r of the parallelogram $opqr$ are $\vec{p} = 4\vec{i} - \vec{j}$ and $\vec{r} = 2\vec{i} + 3\vec{j}$, where o is the origin.

Find (i) $\vec{p} + \vec{r}$ and $\vec{p} - \vec{r}$

(ii) $|\vec{p} + \vec{r}|$ and $|\vec{p} - \vec{r}|$



Show that θ, the acute angle between $\vec{oq}$ and $\vec{rp}$, is given by

$$\cos^{-1} \frac{1}{5\sqrt{2}}$$

A point d in the plane is such that $\vec{d} = \vec{r} - k\vec{p}$, $k \in \mathbb{R}$ and $\vec{d} \perp \vec{p}$. Find the value of k.

3. Verify that lines $L: \ x - 2y + 3 = 0$
$K: 2x + y = 0$

are perpendicular.

Find the equation $f(K)$, if f represents a transformation:

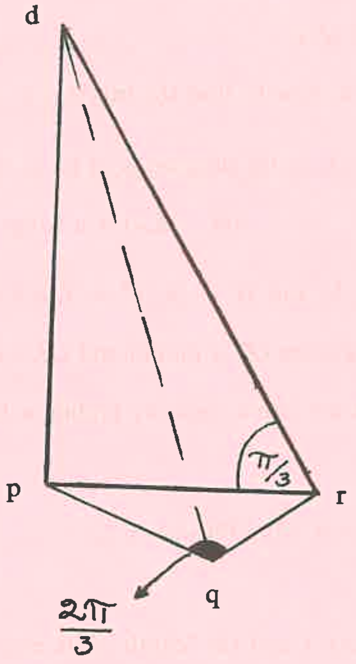
$$x' = 3x + 2y$$

$$y' = 5x + 3y$$

If $f(L)$ is $13x - 8y - 3 = 0$, investigate if $f(L) \perp f(K)$.

$P: ax + by + c = 0$ and Q is a line parallel to P. Find $f(P)$ and $f(Q)$ and investigate if $f(P) \parallel f(Q)$.

4. (*a*) Points p,q,r are on the horizontal. $|pq| = 5$, $|qr| = 3$ and $|\angle pqr| = 120°$.



(i) Calculate $|pr|$.

(ii) $[pd]$ represents a vertical mast. The angle of elevation of d from r is 60°. Find $|dq|$.

(*b*) If $\theta + \varphi = \frac{\pi}{4}$, write $\tan\theta$ in terms of $\tan\varphi$ and then prove

$$(1 + \tan\theta)(1 + \tan\varphi) = 2$$

Show $\tan 22{\cdot}5° = \sqrt{2} - 1$.

5. (*a*) If $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$, $0° \leqslant A,B \leqslant 90°$, evaluate $\cos(A + B)$.

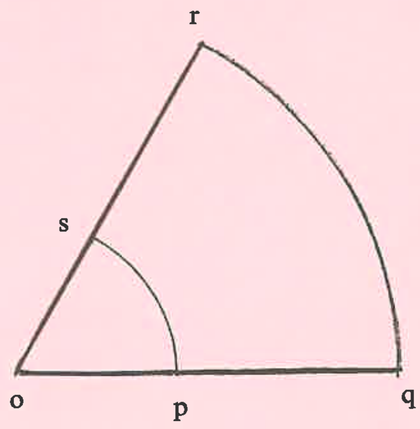
(*b*) Find, to the nearest degree, two values of θ, if

$$3\cos^2\theta + \sin\theta = 0,\ 0° \leqslant \theta \leqslant 360°.$$

(*c*) A section of roadway *spqr* has curved edges *ps* and *qr*, both of which are arcs of a circle centre *o*. If the radius of the arc *ps* is x units and the radius of the arc *qr* is *y* units where $|\angle qor| = 60°$, express in terms of x and/or y

(i) the area of the section of *pqrs*

(ii) the ratio $|pr|$: length of arc *qr*.



6. (*a*) (i) Calculate the weighted mean percentage increase for the data shown.

	Gas	Bread	Public Transport
% increase	3	7	5
Weighting	2	5	3

(ii) If the weighting for bread is doubled, calculate the change in the weighted mean percentage increase.

(*b*) Of the 100 tickets sold in a raffle, 40 were red, 30 were blue and 30 were green. The winning ticket is drawn at random.

(i) Find the probability that

— it is red

— it is not blue.

(ii) Every red ticket is even numbered, while every blue ticket is odd numbered, and of the green tickets, 20 are even numbered and 10 are odd numbered. Find the probability that the winning ticket is green or even numbered.

(iii) A ticket is drawn and then replaced. If three such tickets are drawn what is the probability that at least two are red?

7. (*a*) A subcommittee of 5 is to be chosen from a committee of 10. If one particular person is to serve on both committees, in how many ways can the subcommittee be chosen?

(*b*) Solve the difference equation

$2u_{n+2} - 11u_{n+1} + 5u_n = 0, n \geqslant 0$ if $u_0 = 2$ and $u_1 = -8$.

(*c*) A drawer contains 5 red and x blue biros. One is drawn at random and not replaced. Another is then drawn at random. If the probability that both were blue was $\frac{1}{6}$, how many blue biros were in the drawers?

SECTION B

Do **ONE** Question.

8. (*a*) Use integration by parts to find the indefinite integral $\int xe^{2x}\,dx$.

(*b*) A closed rectangular box is made of thin metal. The length of the box is three times its width. The volume of the box is 36 cm and its width is x cm.

Show that the surface area is $\left(6x^2 + \dfrac{96}{x}\right)$ cm^2.

Find the dimensions of the box with least surface area.

(*c*) Find the Maclaurin series for
$\log_e (1 + x)$.

Deduce a Maclaurin series for $\log_e (1 - x)$

and hence for $\log_e \left(\dfrac{1 + x}{1 - x}\right)$.

Use the first four non-zero terms of this latter series to find an approximation to $\log_e 2$.

9. (*a*) When a biased 6-sided die is thrown a score of 1 is twice as likely as a score of 2 and a score of 2 is twice as likely as a score of 3. Scores of 3, 4, 5 and 6 are equally likely. Find the probability of a score of

(i) 6

(ii) 1.

If, then, a fair die is thrown simultaneously with the biased die, calculate the probability of a score of 8.

(*b*) A sample of 100 potatoes taken from a consignment have this distribution of mass

Mass (kg)	·2	·25	·3	·35	·4
Frequency	11	21	38	17	13

from which the mean and standard deviation are calculated as 0·3 and 0·05788, respectively. Assuming that the distribution is approximately normal, find the range of mass for packing purposes, if 5% at either extreme are rejected as being too small or too large.

10. (*a*) $(Z,*)$ is a group, $Z = \{\ldots -3,-2,-1,0,1,2,\ldots]$ and $a * b = a + b - 5$.

(i) Find the identity of $(Z,*)$

(ii) For any $x \in Z$, show how to find the inverse, x^{-1}.

(*b*) Prove that if $(H,*)$ and $(K,*)$ are subgroups of $(G,*)$, then

$$(H \cap K,*) \text{ is a subgroup of } (G,*).$$

(*c*) Let $Z_6 = \{0, 1, 2, 3, 4, 5\}$ and $H = \{h \mid h^6 = 1, h \in C\}$.

Show that each of the groups $(Z_6, +\text{mod}6)$ and $(H, \times)$ is a finite cyclic group, where $\times$ denotes multiplication.

Show that these groups are isomorphic by finding a function $\phi : Z_6 \rightarrow H$

and proving that

$$\phi\,(z_1 + z_2) = \phi(z_1) \times \phi(z_2) \text{ for } z_1,z_2 \in z_6.$$

11. (*a*) An ellipse has eccentricity $\frac{1}{2}$ and the length of its major axis is 4. Find its equation in standard form.

(*b*) Show that under each transformation of the form

$x' = ax + by + k_1$

$y' = cx + dy + k_2$

the ratio of lengths of segments on parallel lines is invariant.

(*c*) Given a unit circle C centre the origin, show that there is a transformation f of the above type such that $f(C) = E$, where E is an ellipse with equation in standard form.

Prove that the locus of midpoints of parallel chords of E is a diameter of E.