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SPECIMEN PAPER

LEAVING CERTIFICATE EXAMINATION

MATHEMATICS — HIGHER COURSE — PAPER II — SET B

1. (a) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, show that $\alpha + \beta + \gamma = -p$; $\alpha\beta + \beta\gamma + \gamma\alpha = q$; $\alpha\beta\gamma = -r$.
   (b) If $2 + i\sqrt{3}$ is a root of $x^4 - 4x^2 + 8x + 35 = 0$ find the other roots.

2. Write out the first 4 terms of the following binomial expansions.
   (i) $(1 - x)^{-1}$; (ii) $(1 + 3x)^{\frac{1}{2}}$; (iii) $(4 + x)^{-\frac{3}{2}}$.
   If x is so small that its square and higher powers may be neglected, write
   $$\frac{\sqrt{1 + 3x}}{(1 - x)\sqrt{(4 + x)^3}}$$ in its simplest form.

3. Find the sum to n terms of the series whose n^{th} term is $\frac{1}{n(n+1)}$.
   Prove that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is convergent. State the comparison test.
   Prove that the following series are convergent
   (i) $\sum_{n=1}^{\infty} \frac{1}{n^2}$; (ii) $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n(n^3 - 1)}$.

4. State the ratio test and apply it to the following series for convergence.
   (i) $1^2x^0 + 2^2x^1 + 3^2x^2 + - - - - - - - - + n^2x^{n-1} + - - - - - - -$
   (ii) $\sum_{n=1}^{\infty} \frac{n^4x^n}{n!}$.

5. Sketch the graph $x \to \log x$. State the domain and range and prove that it is an increasing function.
   Prove formally that $\underset{n \to \infty}{\text{Lt}}\, r^n = 0$, $0 < r < 1$.

6. Differentiate, from first principles, $\cos 2x$ with respect to x.
   Differentiate (i) $x\tan 3x$; (ii) $x\sin 2x\tan 3x$; (iii) $x^2e^{3x^2}$; (iv) $ln(ax^2 + bx + c)$;
   (v) $e^{3x^2}\, ln(ax^2 + bx + c)$ [Note: $ln\, x = \log_e x$].

7. A piece of countryside is in the form of a square $a\ b\ c\ d$ of side 60 furlongs, and a path runs along the side ab. In a motor-cycle test a competitor has to go from corner a to the opposite corner c. If he can travel along the path at the rate of 5 furlongs per minute and across the country at the rate of 3 furlongs per minute, what is the shortest time in which he can go from a to c ?

8. (a) Evaluate (i) $\int_{-1}^{+1} (x + 1)^3 dx$. (ii) $\int_0^{\frac{\pi}{2}} \sin^3\theta\, d\theta$; (iii) $\int_0^1 x^2 e^{4x^3} dx$;
   (iv) $\int_0^1 \frac{x\,dx}{1 + x^2}$.
   (b) If the area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is divided into two equal parts by the line $y = a$, show that $a^3 = 16$.

9. Trace the curve $y^2 = (x - 1)(2x - 5)^2$.
   Find the volume generated by the revolution of the loop about the x axis.

10. A trader has a supply of lamps for sale. If he marks the price at 5/- each he can sell a certain number. He estimates that for every penny by which he increased the price that number will be decreased by 1 per cent. What price should he charge for his lamps in order that he will obtain the greatest amount of money from his sales ?