

A N R O I N N O I D E A C H A I S

SPECIMEN PAPER

SET B.

LEAVING CERTIFICATE EXAMINATION

MATHEMATICS — HIGHER COURSE — PAPER I.

1. The equations of two straight lines are $3x + y + 12 = 0$ and $x + 2y - 1 = 0$.
Find (i) the acute angle between them, (ii) the distance of their point of intersection from the origin, (iii) the area of the triangle formed by them and the x axis, (iv) the equations of the lines drawn perpendicular to them and containing the point $(2, 3)$.
2. Find the radius and the coordinates of the centre of each of the circles $x^2 + y^2 - 6x - 4y + 11 = 0$ and $x^2 + y^2 + 4x + 6y - 19 = 0$ and show that the circles touch externally.
Find the equation of the tangent at their point of contact.
3. (a) A chord of the parabola $y^2 = 4ax$ subtends a right angle at the vertex. Find the locus of the mid-point of the chord.
(b) Prove that the curve whose equation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has the x axis and the y axis as axes of symmetry, and has the origin as a centre of symmetry. Prove that a is the shortest distance from the origin to the curve.
4. (a) If (\vec{i}, \vec{j}) is an orthonormal basis, prove that the vector $a\vec{i} + b\vec{j}$ is orthogonal to the line whose equation is $ax + by + c = 0$.
(b) Prove that the orthogonal projection of \vec{v} on the non-zero vector \vec{x} is $\frac{\vec{v} \cdot \vec{x}}{|\vec{x}|^2} \cdot \vec{x}$.
(c) Prove that the length of the perpendicular from the point (x_1, y_1) to the line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.
5. (a) Let T be a transformation of R^2 represented by the matrix $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$.
Show that $T(\vec{o}) = \vec{o}$, $T(\vec{x} + \vec{y}) = T(\vec{x}) + T(\vec{y})$, $T(\alpha\vec{x}) = \alpha T(\vec{x})$, when $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $\vec{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ and $\vec{o} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.
(b) What is meant by a linear transformation? Show that reflection in the x axis is a linear transformation and find its matrix. Is reflection in the line $x = 1$ a linear transformation?
Why or why not?
6. (i) Indicate, with proof, which of the following are linear transformations.
(a) $F : R^2 \rightarrow R^2$ defined by $F(x, y) = (2x + y, y)$,
(b) $F : R^2 \rightarrow R^2$ defined by $F(x, y) = (x^2, y)$,
(c) $F : R^2 \rightarrow R$ defined by $F(x, y) = (x, y)$,
(d) $F : R^2 \rightarrow R^2$ defined by $F(x, y) = (x + 3, y + 2)$.
(ii) Prove that the composition of two linear transformations is a linear transformation.
7. (a) If (\vec{i}, \vec{j}) is an orthonormal basis of the pointed plane and f is the reflection in the line $R\vec{i}$, and g is the orthogonal projection on the line $R(\vec{i} + \vec{j})$ find the matrix of each of the following:
 $f, g, g \circ f, f \circ g$.
(b) By finding the images of $(1, 1)$ and $(2, 2)$ by $g \circ f$, prove that the mapping $g \circ f$ is not injective.
8. (a) Prove De Moivre's Theorem, i.e. $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$, when $n \in Z$.
Hence, or, otherwise, express $\cos 5\theta$ as a polynomial in $\cos \theta$.
(b) Sketch the graph of each of the following functions:
 $x \rightarrow |x|$; $x \rightarrow |\sin x|$; $x \rightarrow \frac{1}{2}(\sin x + |\sin x|)$ where $x \in R$.
9. If Z_1, Z_2 and Z_3 are complex numbers and if R means "the real part of" prove
(i) $|R(Z)| \leq |Z|$;
(ii) $|Z_1 + Z_2|^2 = |Z_1|^2 + |Z_2|^2 + 2R(Z_1 \bar{Z}_2)$;
(iii) $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$.
(Geometrical arguments will not be accepted.)
10. Define a group.
(i) Prove that the set of the 3 cube roots of unity is a group under multiplication.
(ii) If $G, *$ is a group prove that $\forall a, b \in G : (a * b)^{-1} = b^{-1} * a^{-1}$.
(x^{-1} denotes the inverse of x .)

OR

10. (a) Two dice are thrown together. What is the probability of a score of 8?
(b) In a large constituency 80% of the electorate voted for party A. If 10 people, chosen at random in the constituency, were asked which way they voted, what would be the probability that exactly 8 voted for party A?