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SPECIMEN PAPER

LEAVING CERTIFICATE EXAMINATION

MATHEMATICS — HIGHER COURSE — PAPER II — SET A

1. If α, β, γ are roots of the equation $x^3 - 3x - 4 = 0$ form the equations whose roots are (i) $\alpha - 2$, $\beta - 2$, $\gamma - 2$ and (ii) 10α, 10β, 10γ.

Show that $x^3 - 3x - 4 = 0$ has a root lying between 2·1 and 2·2 and find the value of that root, correct to 3 decimal places.

2. Use the Binomial Theorem to find the first 4 terms and the general term of the expansion of $(1 - x)^{-\frac{1}{2}}$. Express $\sqrt[3]{10}$ in the form $(1 - x)^{-\frac{1}{3}}$ and then use the expansion to find the value of $\sqrt[3]{10}$ to 3 decimal places.

3. (a) State the ratio test for the convergence of a series of positive terms. Use the ratio test to prove that the series $\sum_{n=1}^{\infty} n^2 x^n$ converges for $0 < x < 1$.

(b) Show that the series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ is convergent for all values of x.

4. Investigate whether each of the following has or has not a sum to infinity.

(i) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots\ldots$

(ii) $\frac{1}{1\cdot3} + \frac{1}{2\cdot4} + \frac{1}{3\cdot5} + \ldots\ldots$

5. Give a rigorous definition of $\operatorname{Lt}_{n\to\infty} f(n) = l$.

From the definition prove that $\operatorname{Lt}_{n\to\infty} \frac{1}{n^3} = 0$.

6. Differentiate, from first principles, $\frac{1}{1 - x}$ with respect to x.

Differentiate with respect to x:

(i) $x \tan x + 2x \tan 2x$.

(ii) $x \sin 2x \cos^2 x$.

(iii) $x \sin (x^2 - 5)$.

(iv) $ln\ x^3$.

(Note $ln\ x = \log_e x$).

If $y = (A + Bx)e^{-2x}$, prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$.

7. Sketch the graph of $f : f(x) = \frac{x^2}{x - 1}$, paying special attention to the stationary points and the asymptotes.

(Hint: Write $f(x)$ in form $ax + b + \frac{c}{x - 1}$ where a, b, c are constants).

8. Evaluate (i) $\int_0^{\frac{\pi}{2}} \sin 2\theta \cos 3\theta \, d\theta$. (ii) $\int_0^{\frac{\pi}{2}} x \sin x \, dx$.

(iii) $\int_0^1 \frac{x^2 + 1}{x^3 + 3x + 1} \, dx$. (iv) $\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$.

9. Find the area enclosed by the parabola $y^2 = 3ax$ and the circle $x^2 + y^2 = 4a^2$.

10. (a) If $x \in R$ and $[x]$ denotes the integer k such that $k \leqslant x < k+1$, sketch the graph of $f : x \to x - [x]$, $-3 \leqslant x \leqslant +3$ and show that f is periodic. What is the least positive period ?

(b) A piece of wire 14 cm long is to be bent to form a rectangle which is not a square. Find the possible lengths of the shorter side if the diagonal of the rectangle is to be less than 5 cm long.