Higher Level Question

(Suggested maximum time: 10 minutes)

Below is a photograph of an island. The highest point on the island is 916 metres above sea level. Using this information, and the photograph, estimate as accurately as possible the volume of the island that is above sea level. Give your answer in the form $a \times 10^n$, where $n \in \mathbb{Z}$ and $1 \leq a < 10$. State clearly any assumptions that you make in finding your answer.
Higher Level Question

(Suggested maximum time: 10 minutes)

\( n \) is a natural number.

(a) Write down the next 3 natural numbers, in terms of \( n \).

Hence, or otherwise, complete the following.

(b) Show that the sum of any 3 consecutive natural numbers is divisible by 3.

(c) Prove or disprove the following statement:

“The sum of any 4 consecutive natural numbers is \textbf{never} divisible by 4.”
Higher Level Question  

(Suggested maximum time: 15 minutes)

*Noughts and Crosses* is a two-person game played on a $3 \times 3$ grid, made up of 9 small squares. We call each of the 3 rows, 3 columns, and 2 diagonals a *line*. An example of one type of line is shaded in each of the $3 \times 3$ grids below.

(a) In the $3 \times 3$ grid below, write in each small square the number of different lines to which it belongs. One small square is already filled in for you – it belongs to 3 different lines.

Imagine *Noughts and Crosses* played on an $n \times n$ grid, made up of $n^2$ small squares, where $n \geq 3$, $n \in \mathbb{N}$. A *line* of this grid is one of its rows, one of its columns, or one of its 2 diagonals.

(b) What is the **minimum** number of lines to which each small square of the $n \times n$ grid must belong? Justify your answer.
(c) For certain values of \( n \), the maximum number of different lines to which a small square can belong is 4, while for other values of \( n \) this maximum number is 3.

State for which values of \( n \) this maximum number is 4, and for which values of \( n \) this maximum number is 3. Justify your answer.
Higher Level Question  
(Suggested maximum time: 10 minutes)

In a shop, the selling price of each item includes VAT at a fixed rate.
Show how the shopkeeper could calculate the amount of VAT charged on an item, if she knows the selling price of the item and the rate of VAT. Give an example if necessary.

Higher Level Question  
(Suggested maximum time: 5 minutes)

Maisy writes down the following theorem:
“if a triangle has sides of length 3 cm, 4 cm, and 5 cm, then it is a right-angled triangle.”

(a) State the **converse** of Maisy’s theorem.
(b) Is the converse of Maisy’s theorem true or false? Justify your answer.

Higher Level Question  
(Suggested maximum time: 10 minutes)

(a) Factorise $5x - 15$ and $6 - 2x$.

If $A$ and $B$ are variable quantities, we say that $A$ is proportional to $B$ if the fraction $\frac{A}{B}$ is a constant.

(b) Using your answers to part (a) above, show that $5x - 15$ is proportional to $6 - 2x$.

(c) Is $x^2 + 3x + 2$ proportional to $2x + 2$? Justify your answer.
Higher Level Question  
(Suggested maximum time: 10 minutes)

Mark works two jobs – he works in Bob’s Bakery and in Ciara’s Café. He is paid €11·50 an hour for his work in Bob’s Bakery, and €9·30 an hour for his work in Ciara’s Café.

In one week he worked a total of 34 hours and was paid a total of €362·40.

Find how many hours he worked in Bob’s Bakery in this week.

Higher Level Question  
(Suggested maximum time: 5 minutes)

(a) Give an example of a data set where this statement is false:

“minimum < mean < maximum”.
(b) **Describe** for what kind of data sets this statement is false:

“minimum < **mean** < maximum”.

**Higher Level Question**  
(Suggested maximum time: 10 minutes)

A rectangular television screen has a diagonal of length 42 inches. The sides of the television screen are in the ratio 16:9.

Find the area of the television screen, correct to the nearest whole number.
Ordinary Level Question

The photograph on the right shows an American Football stadium.

The pitch is outlined in white in the centre of the photograph.

The width of the pitch is 160 feet (marked W in the photograph).

(a) By measuring the photograph and using an appropriate scale, estimate the length of the pitch (in feet) and the area of the pitch (in feet²) as accurately as you can.

(b) Sports pitches in Ireland are often laid out in metres rather than in feet.

If one metre is 3.28 feet, find the area of this pitch in square metres.
Give your answer correct to the nearest whole number.

Length = ____________ feet
Area = _____________ feet²
Ordinary Level Question  
(Suggested maximum time: 5 minutes)  
(a) Factorise $5x - 15$ and $2x - 6$.  
\[
\begin{array}{c|c}
5x - 15 &= \\
2x - 6 &= 
\end{array}
\]

If $A$ and $B$ are variable quantities, we say that $A$ is proportional to $B$ if the fraction $\frac{A}{B}$ is a constant.

(b) Using your answers to part (a) above, show that $5x - 15$ is proportional to $2x - 6$.  

Ordinary Level Question  
(Suggested maximum time: 5 minutes)  
In a shop, the VAT rate for each item is 10%.

The shopkeeper knows the selling price of an item, before VAT is added.

Give an example to show how she could calculate what the selling price of this item should be, after VAT is added.
Ordinary Level Question  

*Suggested maximum time: 15 minutes*

*Noughts and Crosses* is a two-person game played on a $3 \times 3$ grid, made up of 9 small squares.

We call each of the 3 rows, 3 columns, and 2 diagonals a *line*.

An example of one type of line is shaded in each of the $3 \times 3$ grids below.

![Row](image1)

![Column](image2)

![Diagonal](image3)

**(a)** In the $3 \times 3$ grid below, write in each small square the number of different lines to which it belongs. Two small squares are already filled in for you – one belongs to 3 different lines, and the other belongs to 2 different lines.

![Grid](image4)

*Noughts and Crosses* can also be played on a $6 \times 6$ grid, made up of 36 small squares. A *line* of this grid is one of its rows, one of its columns, or one of its 2 diagonals.

**(b)** How many different rows are in this grid? How many different columns are in this grid?
(c) Explain why each small square of the 6×6 grid must belong to at least 2 lines.

(d) Find the maximum number of lines to which a small square in the 6×6 grid may belong. Show all of your work.
Ordinary Level Question  
(Suggested maximum time: 5 minutes)

(a) Find the maximum and the mean of the following data:

3, 5, 7, 8

Maximum = [ ]  
Mean = [ ]

(b) Gerry starts to give an example of a list of 4 numbers where the maximum is the same as the mean. Fill in the 2 missing numbers in Gerry’s list, and write down the mean of his list.

Gerry’s list = 6 , 6 , [ ] , [ ]

Mean = [ ]
Ordinary Level Question  

(Suggested maximum time: 10 minutes)

(a) Marcel has a number trick where he asks the audience to “think of a number.” He calls this number \( x \).

Marcel gives the audience the instructions in the table below and then “magically” tells them what answer they got.

Fill in the table to show the effect of each of the instructions.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Result of each instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think of a number.</td>
<td>( x )</td>
</tr>
<tr>
<td>Multiply the number by 3.</td>
<td>( 3x )</td>
</tr>
<tr>
<td>Add 2 to your answer.</td>
<td></td>
</tr>
<tr>
<td>Subtract your original number (i.e. ( x )).</td>
<td></td>
</tr>
<tr>
<td>Divide your answer by 2.</td>
<td>( x + 1 )</td>
</tr>
<tr>
<td>Subtract your original number.</td>
<td></td>
</tr>
</tbody>
</table>

(b) Marcel has a second number trick.

It is shown in the table below, but some of the instructions are missing.

Complete the table to show the missing instructions. Each instruction should be to add a number, subtract a number, multiply by a number, or divide by a number.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Result of each instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think of a number.</td>
<td>( x )</td>
</tr>
<tr>
<td></td>
<td>( 2x )</td>
</tr>
<tr>
<td></td>
<td>( 2x + 6 )</td>
</tr>
<tr>
<td>Add your original number (i.e. ( x )).</td>
<td>( 3x + 6 )</td>
</tr>
<tr>
<td></td>
<td>( x + 2 )</td>
</tr>
<tr>
<td></td>
<td>( 2 )</td>
</tr>
</tbody>
</table>
Foundation Level Question

The photograph on the right shows an American Football stadium.

The pitch is outlined in white in the centre of the photograph.

(a) What shape is the pitch?

Shape =

The width of the pitch is marked W.

(b) Measure the width of the pitch and the length of the pitch in the photograph.

Give each answer in centimetres, correct to one decimal place.

Width = cm

Length = cm

The actual width of the pitch is 160 feet.

(c) Find the actual length of the pitch.

Give your answer in feet, correct to the nearest whole number.
Foundation Level Question (Suggested maximum time: 10 minutes)

*Noughts and Crosses* is a two-person game played on a 3×3 grid, made up of 9 small squares. We call each of the 3 rows, 3 columns, and 2 diagonals a line.

An example of one type of line is shaded in each of the 3×3 grids below. The square marked A in each diagram belongs to all 3 of the lines.

(a) Each diagram below shows a 3×3 grid with one square marked B. Square B is in two different lines.

On each diagram, **shade in** one line that square B is in. Shade in a different line on each diagram.

(b) In the 3×3 grid below, write in each small square the number of different lines that it is in. Two small squares are already filled in for you – one is in 3 different lines, and the other is in 2 different lines.
Foundation Level Question  
(Suggested maximum time: 10 minutes)

Marcel has a number trick where he asks the audience to “think of a number.”
Marcel gives the audience the instructions in the table below and then “magically” tells them what answer they got.

(a) Fill in the table to show the effect of each of the instructions. The starting number in the table is 12.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Working Out</th>
<th>Result of each instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think of a number.</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Multiply the number by 3.</td>
<td>$12 \times 3 = 36$</td>
<td>36</td>
</tr>
<tr>
<td>Add 2 to your answer.</td>
<td>$36 + 2 = 38$</td>
<td></td>
</tr>
<tr>
<td>Subtract your original number (i.e. 12).</td>
<td>$38 - 12 = 26$</td>
<td></td>
</tr>
<tr>
<td>Divide your answer by 2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtract your original number.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) You can do the same thing using $n$ as your starting number. Fill in the table below to show the effect of each of the instructions in this case.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Working Out</th>
<th>Result of each instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think of a number.</td>
<td>$n$</td>
<td></td>
</tr>
<tr>
<td>Multiply the number by 3.</td>
<td>$3 \times n = 3n$</td>
<td>3n</td>
</tr>
<tr>
<td>Add 2 to your answer.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtract your original number (i.e. $n$).</td>
<td>$3n + 2 - n =$</td>
<td></td>
</tr>
<tr>
<td>Divide your answer by 2.</td>
<td></td>
<td>$n + 1$</td>
</tr>
<tr>
<td>Subtract your original number (i.e. $n$).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Foundation Level Question  
(Suggested maximum time: 5 minutes)

In a shop, the VAT rate for each item is 10%.

The shopkeeper knows the selling price of an item, **before** VAT is added.

Give an example to show how she could calculate what the selling price of this item should be, **after** VAT is added.

<table>
<thead>
<tr>
<th>Your Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price <strong>before</strong> VAT is added =</td>
</tr>
<tr>
<td>VAT @ 10% =</td>
</tr>
<tr>
<td>Selling price <strong>after</strong> VAT is added =</td>
</tr>
</tbody>
</table>
Note to readers of this document:

These sample questions are intended to help candidates and teachers prepare for the June 2015 and subsequent Junior Certificate examinations in Mathematics.

They are intended to supplement the 2015 Sample Papers issued at Higher, Ordinary, and Foundation Levels.