

OLLSCOIL NA hÉIREANN.

THE NATIONAL UNIVERSITY OF IRELAND.

SUMMER EXAMINATIONS, 1975.

M I 2 S

MATRICULATION EXAMINATION.

MATHEMATICS.

FIRST PAPER.

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(This Paper must be answered wholly in Irish or wholly in English).

Not more than *eight* questions should be attempted.

1. (a) One cubic centimetre of water weighs one gram and mercury is 13.6 times as heavy as water. A spherical container of internal radius 10 cms. is filled with mercury. Find the weight of the mercury to the nearest kilogramme. 57

(b) If the above amount of mercury be filled into a right circular cylinder of base radius 15 cms. find the height to which the mercury will rise in the cylinder. 5.2

2. Money is borrowed at 12% per annum compound interest. Show it will take $\pounds r = \pounds 1.12$ to repay a loan of $\pounds 1$ at the end of one year. 15682

How much will be required to repay $\pounds 12,500$ at the end of two years.

A loan of $\pounds A$ is to be repaid in two equal instalments at the end of each of the first two years. Prove that

$$Ar^2 - Pr - P = 0$$

where $\pounds P$ is the amount of each instalment.

Find P if $A = 12,500$.

3. Solve for x, y, z ,

$$(1, -1, 1)$$

$$x + 2y - 3z + 4 = 0$$

$$3x - y + 4z - 8 = 0$$

$$4x + 3y + z - 2 = 0$$

4. (a) Show that the sequence

$$1, 5, 9, 13, \dots, (4n - 3)$$

is arithmetic and find the sum

$$1 + 5 + 9 + \dots + (4n - 3).$$

(b) If $a.b$ means $a \times b$ show that the sequence

$$\frac{1}{1.2}, \frac{1}{2.3}, \frac{1}{3.4}, \dots, \frac{1}{n(n+1)}$$

is not arithmetic.

Find p and q if

$$\frac{1}{n(n+1)} \equiv \frac{p}{n} - \frac{q}{n+1}$$

and find the sum of the first n numbers in the above sequence.

5. Find the complex roots z_1, z_2 , of the equation

$$3z^2 - 2z + 1 = 0.$$

Prove that

$$|z_1| + |z_2| = |z_1 + z_2| \sqrt{3}$$

2. Find in its simplest form the value of $z_1^3 + z_2^3$.

6. Use the binomial theorem to find a, b, c, d , if

$$\{x + \sqrt{(1-x^2)}\}^6 + \{x - \sqrt{(1-x^2)}\}^6 = a + bx^2 + cx^4 + dx^6.$$

7. A set of points $E(x, y)$ in the x, y , plane is defined by

$$x \geq 0, y \geq 0, x + y \leq 1, 2y - x - 1 \leq 0.$$

Illustrate on a diagram the set E and find, when $(x, y) \in E$

(1) the maximum value of $3x + 2y$.

(2) the maximum value of $x^2 + y^2$.

$$\frac{1 \pm i\sqrt{2}}{3}$$

2

2

2

3

1

8. State when a set of four real or complex numbers, $\{a, b, c, d\}$, form a group under the operation of multiplication.

Find x if $\{1, -1, x, 1/x\}$ form a group under multiplication.

OR

An experimental measurement yields the results

1.95, 2.15, 2.15, 2.17, 2.45, 2.45, 2.5

2.26

when repeated. Find the mean and standard deviation.

9. If $f(x) = x^3 + 3x^2 - 9x + 5$, $-4 \leq x \leq +4$
find $f'(x)$ from first principles.

Find the equations of the tangents to the curve $y = f(x)$
at the points where $x = -3$, $x = 0$ and $x = 1$.

10. Differentiate by rule

$$2x/(x^2+1), (3x+2)/\{(x-1)(x+2)\}.$$

$$.0 = -v - v = 0.$$

$$.0 = 1 + 2v - 1 + v = 0.$$

$$.0 = 1 + 2v - 1 + v = 0.$$

$$.0 = 1 + 2v - 1 + v = 0.$$