2024.M27K
(Deferred Exam)

2024L003G1EK

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2024

Mathematics

Paper 1

Ordinary Level

2 hours 30 minutes

300 marks

Examination number					

Centre stamp

For the Examiner only					
			Section	Question	Mark
Disallowed			A	1	
A				2	
B				3	
Total Disall.				4	
				5	
				6	
			B	7	
Cumulative Check				8	
Running Total				9	
− Total Disall.				10	
= Total		↔		Total	

Grade:

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- any **five** questions from Section A – Concepts and Skills
- any **three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

Write all answers into this booklet. There is space for extra work at the back of the booklet.
If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here: [ ]

Section A — Concepts and Skills — 150 marks

Answer **any five questions** from this section.

Question 1 **(30 marks)**

(a) Fiadh has €1500 saved.
She put this money in a savings account with a rate of 3% per annum compound interest.

(i) Find how much money will be in the account after 1 year.

(ii) Hence, or otherwise, find how much money will be in the account at the end of 2 years.

(b) The standard rates and thresholds of the Universal Social Charge (USC) in Ireland for 2024 are given in the table below.

Annual Income	Rate
First €12 012	0·5%
Next €13 748	2%
Next €44 284	4%
Balance	(Top Rate)

Source: www.revenue.ie

(i) At what annual income does a worker start paying the top rate of USC?

Mike's annual income for 2024 is €65 000.

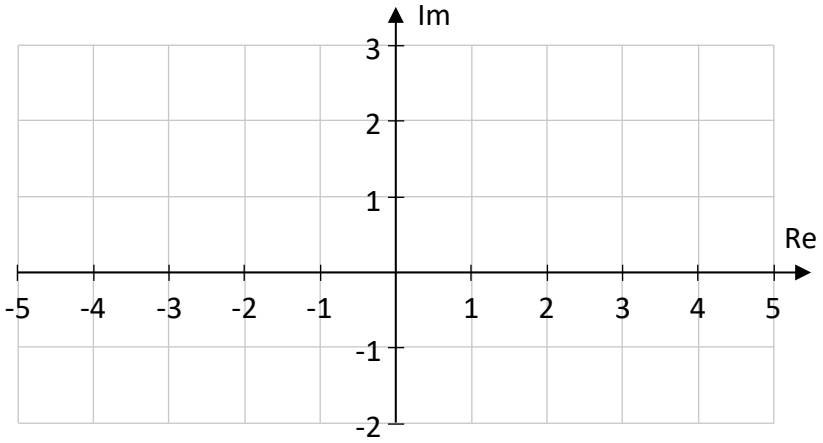
(ii) Work out how much he will pay in USC for 2024.

Question 2 **(30 marks)**

In this question, $i^2 = -1$.

(a) $z_1 = -4 + 2i$ and $z_2 = 2 - i$ are two complex numbers.

(i) **Plot** and **label** z_1 and z_2 on the Argand Diagram below.



(ii) Write down the value of $\overline{z_1}$, where $\overline{z_1}$ is the complex conjugate of z_1.

$\overline{z_1} =$

(iii) Work out the modulus of z_1. That is, find $|z_1|$.
Give your answer in the form $a\sqrt{b}$.

(b) $u = 1 + 4i$ and $v = a + bi$ are two complex numbers, where $a, b \in \mathbb{Z}$.
Given that:

$$u - 2v = -3 + 10i$$

find the value of a and the value of b.

a = _____ b = _____

Question 3 **(30 marks)**

(a) Solve the following inequality in $x \in \mathbb{N}$:

$$5(x-3)+2<11$$

List your solutions. Remember that $x \in \mathbb{N}$.

$x = \{$ $\}$

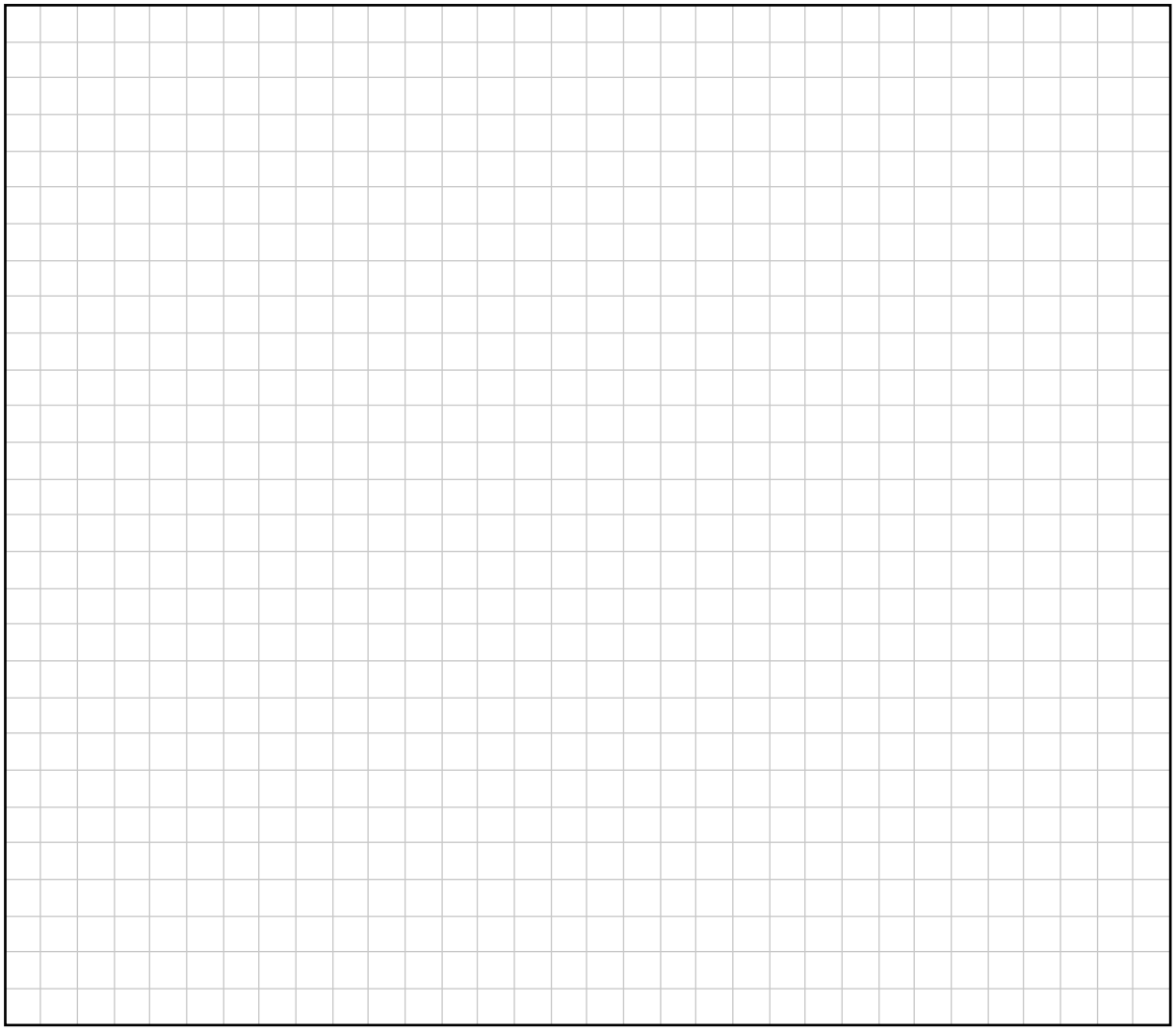
(b) Write the following expression as a single fraction, where $x \neq 0,\ x \neq -1,\ x \in \mathbb{R}$.

$$\frac{1}{x} - \frac{2}{x+1}$$

Give your answer in its simplest form.

(c) Use algebra to solve the simultaneous equations:

$$\begin{aligned} xy &= 6 \\ x - y &= 1 \end{aligned}$$



Question 4 **(30 marks)**

(a) $f(x)$ is the following function in $x \in \mathbb{R}$:

$$f(x) = (x+3)^2 + 5$$

(i) Find the value of $f(0)$.

(ii) Write $f(x)$ in the form $ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$.

(b) Solve the following equation in x:

$$3x^2 - 4x - 9 = 0$$

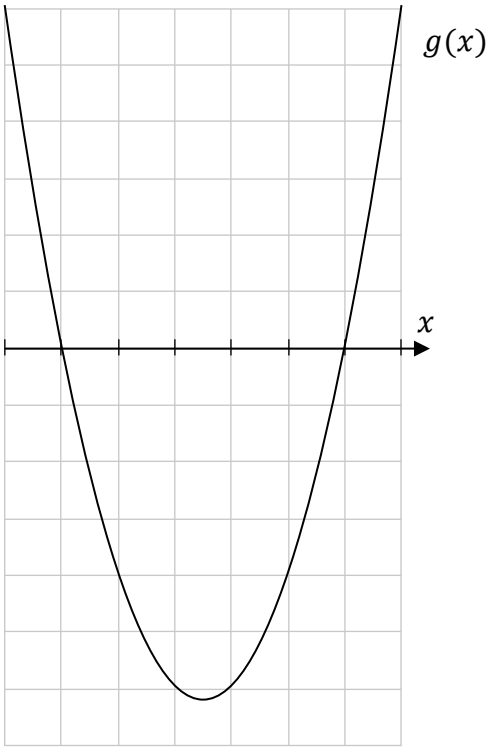
Give each answer correct to 2 decimal places.

(c) The graph of the quadratic function, $g(x)$, is shown on the diagram below (drawn to scale). The y-axis is not shown.

Using the information below, **draw** the y-axis into the diagram.

- The **roots** of $g(x)$ are $x = -2$ and $x = 3$.
- $g(0) = -6$.

Label the y-axis, and the points where $g(x)$ cuts both the x-axis and the y-axis.



Question 5 **(30 marks)**

(a) In Caoimhe's gas bill the volume of gas used is converted to kilowatt-hours.

The current reading on Caoimhe's gas meter is $12\ 518$ m^3.
The previous reading on Caoimhe's gas meter was $12\ 398$ m^3.

(i) The conversion rate is 1 m$^3 = 11{\cdot}5452$ kilowatt-hours.

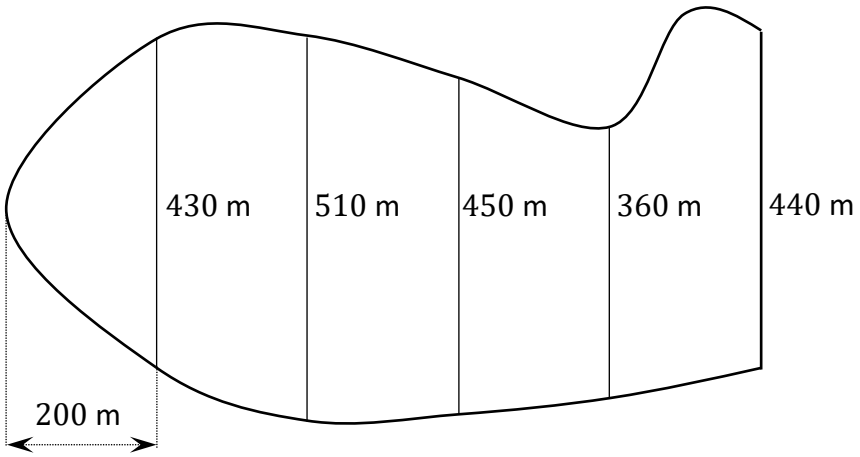
Find the volume of gas used since the previous reading, in m^3, and hence show this is equal to $1385{\cdot}424$ kilowatt-hours.

(ii) Gas costs $13{\cdot}46$ cent per kilowatt-hour.
Find the cost of the gas used by Caoimhe during this period.
Give your answer in euro, correct to 2 decimal places.

Caoimhe's total gas bill is €$227{\cdot}47$ before VAT.

(iii) Work out Caoimhe's total gas bill after VAT at 9% has been added.
Give your answer in euro, correct to 2 decimal places.

(b) Caoimhe wants to estimate the area of her farm, in m^2. She measures at intervals of 200 m, starting at the right-hand edge of the farm. These measurements are given on the diagram, correct to the nearest metre.



Use the **Trapezoidal Rule** (and the measurements in the diagram above) to work out an estimate for the area of the farm, in m^2.

Question 6 **(30 marks)**

(a) $h(x)$ is the following function in $x \in \mathbb{R}$:

$$h(x) = x^3 - 6x^2 + 7x + 6$$

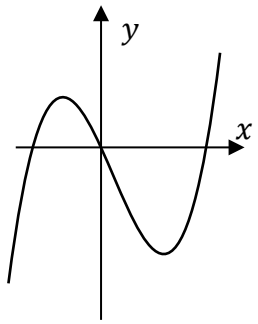
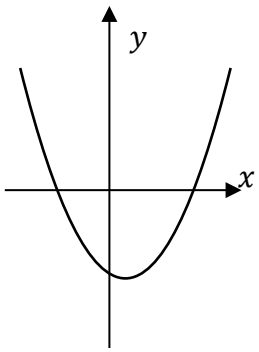
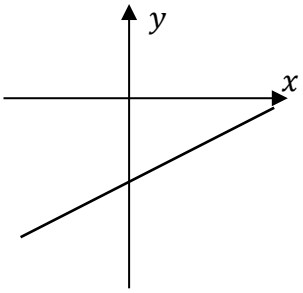
(i) Find $h'(x)$, the derivative of $h(x)$.

(ii) Hence, find the slope of the tangent when $x = 1$.

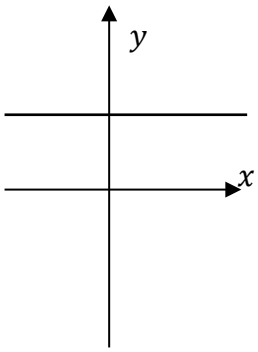
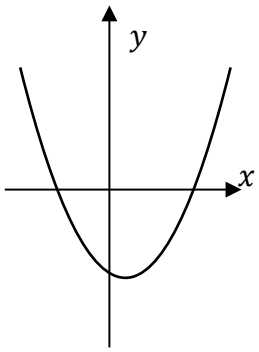
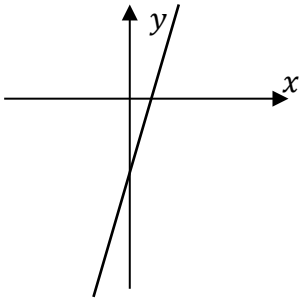
The graph of the function $h(x)$ is shown on the co-ordinate diagram below, for $-0{\cdot}5 \leq x \leq 4{\cdot}5$, $x \in \mathbb{R}$.

(iii) **Draw** the tangent to the graph of $h(x)$ at the point where $x = 1$ on the diagram above. Use the same axes and scales.

(b) The first three diagrams below show the graphs of the functions a, b and c.
These functions are cubic, quadratic, and linear, respectively.

a	b	c
		

The next three diagrams below show the graphs of the functions d, e and f.
Each one of these is the slope function (derivative) of either a, b or c (above).

d	e	f
		

Write d, e and f in the correct place in the table to match each function to its slope function.

Function	**Slope function** (d, e or f)
a	
b	
c	

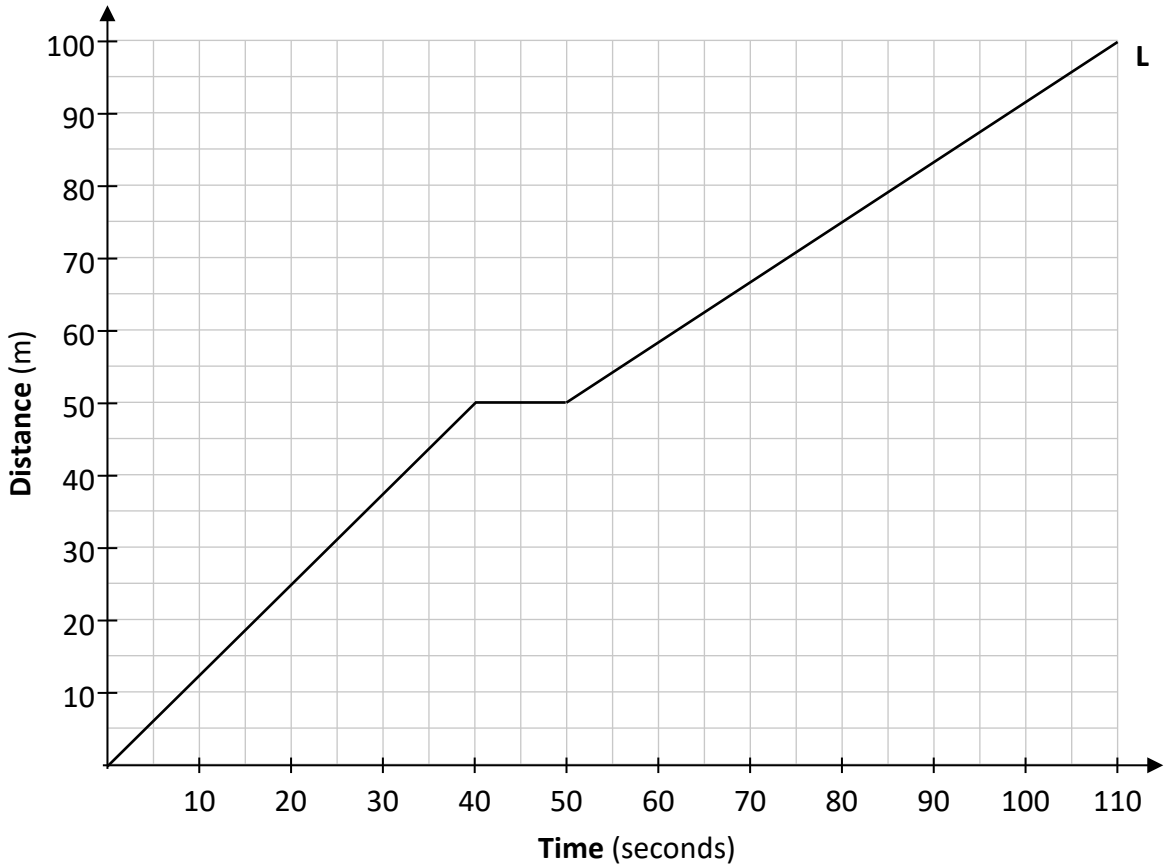
Section B — Contexts and Applications — 150 marks

Answer **any three questions** from this section.

Question 7 **(50 marks)**

Lillian and Darragh visit a 50 m swimming pool.

Lillian swims 50 m at a constant speed.
She stops for a rest after her first 50 m and then swims a second 50 m at a different constant speed.
The graph, labelled **L**, shows the distance that Lillian swims for the first 110 seconds of her swim.



(a) **(i)** How long does Lillian stop for?

Answer: _____ seconds

(ii) Find Lillian's speed for the first 50 m.
Give your answer in metres per second.

(iii) Does Lillian's **speed** increase, decrease or stay the same for the second 50 m? Give a reason for your answer.

Lillian's speed:	Increases	Decreases	Stays the same
(Tick (✓) **one** box only)	☐	☐	☐

Reason:

(b) Darragh swims at a constant speed of 10 m every 7 seconds.

(i) How many seconds does it take Darragh to swim 50 m?

(ii) Darragh starts swimming 10 seconds after Lillian.
He swims 50 m, stops for 5 seconds and then swims a second 50 m at the same constant speed as in part **(b)(i)**.

Graph the distance Darragh swims on the diagram **on the previous page** and **label** it **D**. Use the same axes and scales.

This question continues on the next page.

(c) The average speed for the world record of the women's 100 m freestyle is $1{\cdot}934$ metres per second.

Work out how long it took her to swim the 100 m, in seconds.
Give your answer correct to 2 decimal places.

(d) The formula for finding the cost of heating a swimming pool to a certain temperature is given by the equation:

$$C = 1{\cdot}163\ P\ V\ (T_2 - T_1)$$

where:

- $P =$ cost of electricity per kWh
- $V =$ volume of water in the swimming pool in m^3
- $T_1 =$ the average air temperature in degrees Celsius
- $T_2 =$ the pool temperature in degrees Celsius, when heated.

Mike has a swimming pool with a volume of 9 m^3 and wants to heat it to $25℃$.
The cost of electricity is $€0{\cdot}431$ per kWh.

(i) Work out the cost of heating the swimming pool when the average air temperature is $13℃$. Give your answer in euro, correct to 2 decimal places.

(ii) Work out the average air temperature (T_1), when the cost of heating Mike's swimming pool to $25℃$ is $€21{\cdot}20$.

Give your answer in degrees Celsius, correct to 1 decimal place.

Question 8 **(50 marks)**

A firework is set off from a harbour.

(a) The height of the firework above sea level can be modelled by the function:

$$h(t) = -4{\cdot}8t^2 + 41{\cdot}65t + 20$$

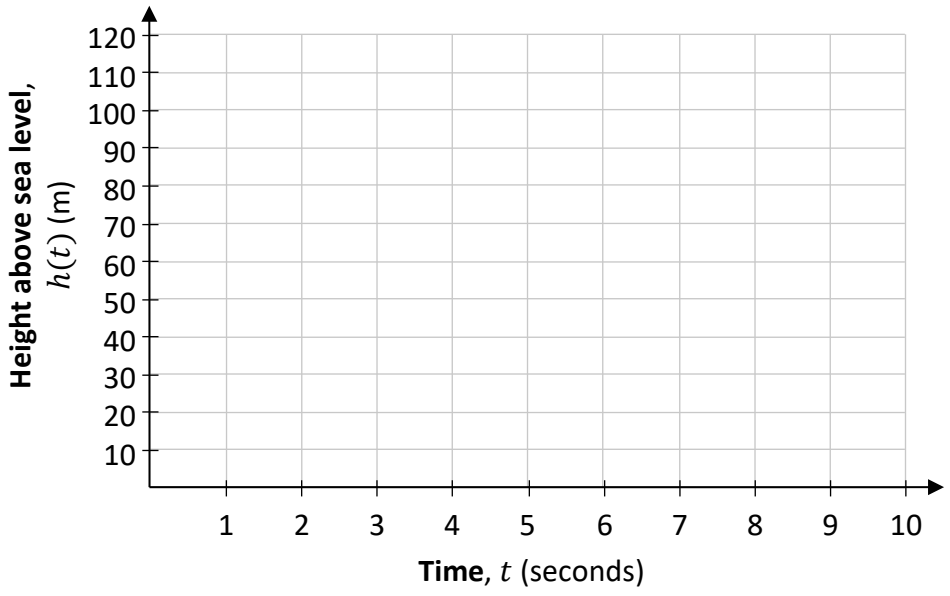
where $h(t)$ is the height of the firework above sea level, in metres, and t is the time in seconds from when the firework is set off, with $t \geq 0$, $t \in \mathbb{R}$ and $h(t) > 0$.

(i) Show that the firework is set off from a height of 20 metres above sea level.

(ii) Complete the table below to show the values of $h(t)$ for the given values of t. Give each value of $h(t)$ correct to the nearest whole number.

Time (seconds)	0	1	2	3	4	5	6	7	8
Height above sea level (m)	20			102					46

(iii) Draw the graph of $h(t)$ on the axes below, for $0 \leq t \leq 8, t \in \mathbb{R}$.



(iv) The firework fails to explode.

Continue your graph in **part (a)(iii)** on the previous page, to find how long it takes, from the time of set off, for the firework to reach sea level. That is, find the value of t for which $h(t) = 0$.

Show your work on the graph and write your answer into the grid below.

$t =$ _____ seconds

This question continues on the next page.

(b) The height above sea level of a second firework can be modelled by the function:

$$g(t) = -5t^2 + 45t + 20$$

where $g(t)$ is the height of the firework above sea level in metres, and t is the time in seconds from when the firework is set off, with $t \geq 0$, $t \in \mathbb{R}$ and $g(t) > 0$.

The firework is designed to explode at its maximum height.

(i) Use **calculus** to find the value of t when the second firework is at its maximum height.

$t =$ _____ seconds

(ii) Use your answer from **part (b)(i)** to find the maximum height of the second firework.

(c) A pyrotechnician uses a type of gunpowder when making fireworks.
The gunpowder mixture is made up of:

$\frac{3}{4}$ potassium nitrate

$\frac{3}{20}$ charcoal

$\frac{1}{10}$ sulphur

(i) Express the ratio of potassium nitrate : charcoal : sulphur in the ratio $a : b : c$, in its simplest form, where $a, b, c \in \mathbb{N}$.

(ii) There are 25 grams of charcoal in the gunpowder mixture.
Work out how many grams of potassium nitrate are in this mixture.

Question 9 **(50 marks)**

(a) A company sells memory sticks.

(i) A memory stick costs €19·99.
The cost was reduced to €16·99 in a sale.

Find the percentage discount on the memory stick.
Give your answer correct to the nearest percent.

(ii) The company buys the memory sticks from the USA.
They buy 10 000 memory sticks at $9·50 each.

Use the exchange rate €1 = $1·09 to work out the **total** cost of the memory sticks, in euro. Give your answer in euro, correct to 2 decimal places.

(b) Seán boarded a flight in Shannon travelling to New York. The flight took off at 09: 45.
New York's time zone is 5 hours behind Ireland's time zone.
The total flight time from Shannon to New York is 7 hours 35 minutes.

What is the local time when Seán's flight is due to land?

(c) The table below shows units of computer storage and the number of bytes they are equal to.

Unit of computer storage	Number of bytes
1 gigabyte	2^{30}
1 terabyte	2^{40}
1 petabyte	2^{60}

(i) Write the number of bytes in one gigabyte in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{Z}$.
Give the value of a correct to 3 significant figures.

(ii) A desktop computer has 1 terabyte of storage.
A NASA super computer has $7{\cdot}6$ petabytes of storage.

Work out how many desktop computers are needed to equal the storage of the NASA super computer. Give your answer correct to the nearest whole number.

This question continues on the next page.

(d) At the start of 2021, the total amount of data in the world was estimated to be approximately 64 zettabytes.

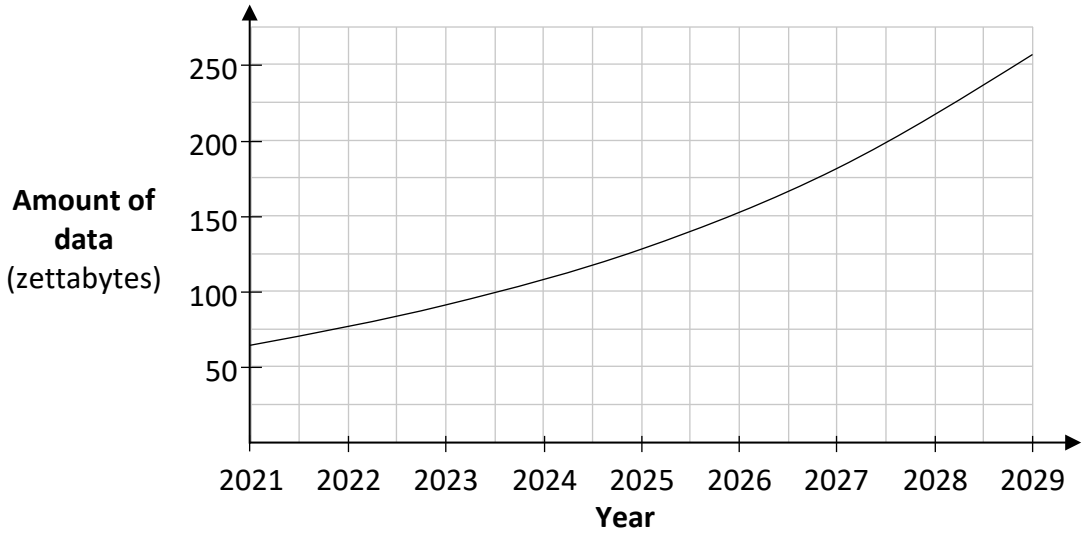
The predicted growth up to the start of 2029 is modelled by the function:

$$d(t) = 64(1{\cdot}19)^t$$

where $d(t)$ is the amount of data in the world, and t is the time in years since the **start** of 2021, with $0 \leq t \leq 8$, $t \in \mathbb{R}$.

(i) Use $d(t)$ to find the predicted amount of data, in zettabytes, in the world by the **start** of 2026. Give your answer correct to 1 decimal place.

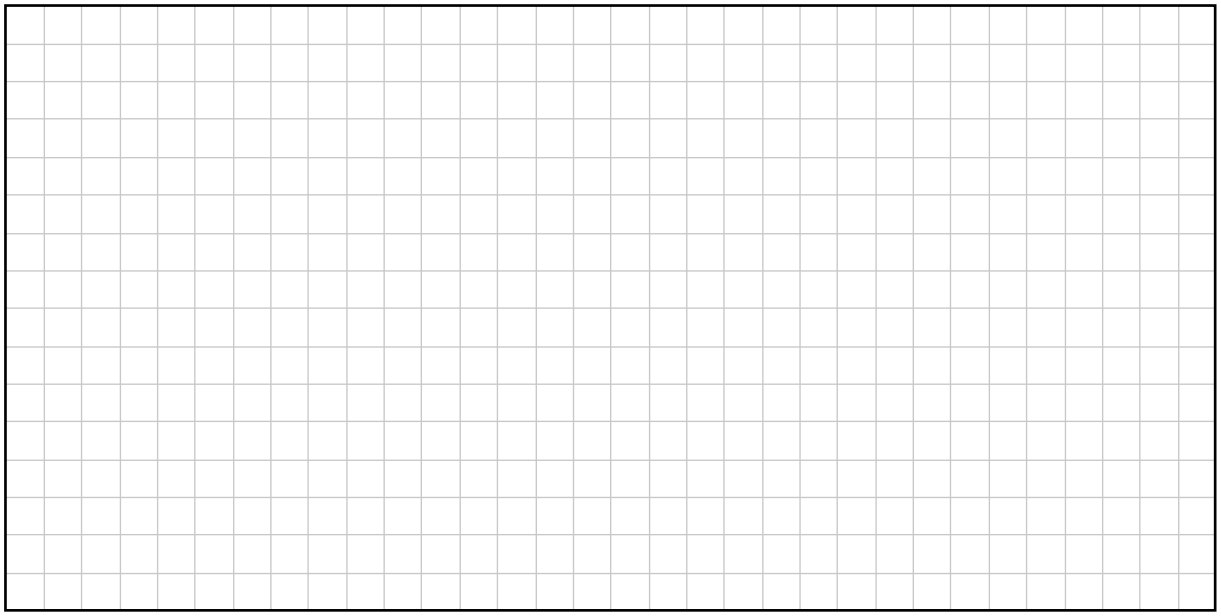
The graph below shows the predicted amount of data in the world from the start of 2021 to the start of 2029.



(ii) In which year will the predicted amount of data first reach 200 zettabytes? Show your work on the graph.

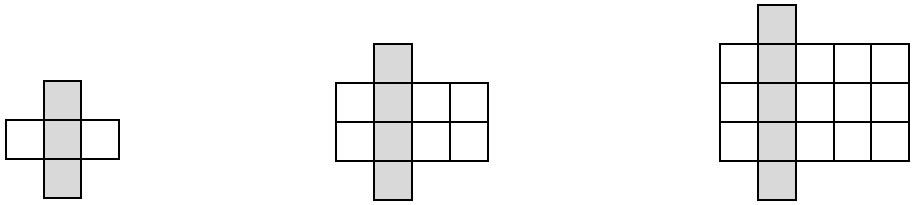
(iii) Use suitable values from the graph **on the previous page** to estimate the number of years it takes for the amount of data to double, according to $d(t)$.

Show your work on the graph and give your answer correct to the nearest whole number.



Question 10 **(50 marks)**

The first three patterns in a sequence are shown below.
Each pattern contains grey tiles and white tiles.



Pattern 1 **Pattern 2** **Pattern 3**

(a) Draw the fourth pattern in the sequence into the box below.

(b) Complete the table below to show the number of grey tiles, the number of white tiles, and the total number of tiles in the first four patterns.

Pattern	Grey tiles	White tiles	Total
1	3	2	5
2			10
3	5		
4		20	

(c) **(i)** Find a formula, in terms of n, for the number of **grey** tiles in pattern n of the sequence, where $n \in \mathbb{N}$.

(ii) Using your formula from **part (c)(i)**, or otherwise, work out the number of grey tiles in the 25th pattern.

(iii) Find the **total** number of grey tiles in the first 25 patterns of the sequence.

(iv) Find how many **white** tiles are in the 8th pattern of the sequence.

This question continues on the next page.

(d) The **total** number of tiles in pattern n of the sequence is T_n.
The value of T_n can be found using this formula, where $b, c \in \mathbb{R}$ are constants.

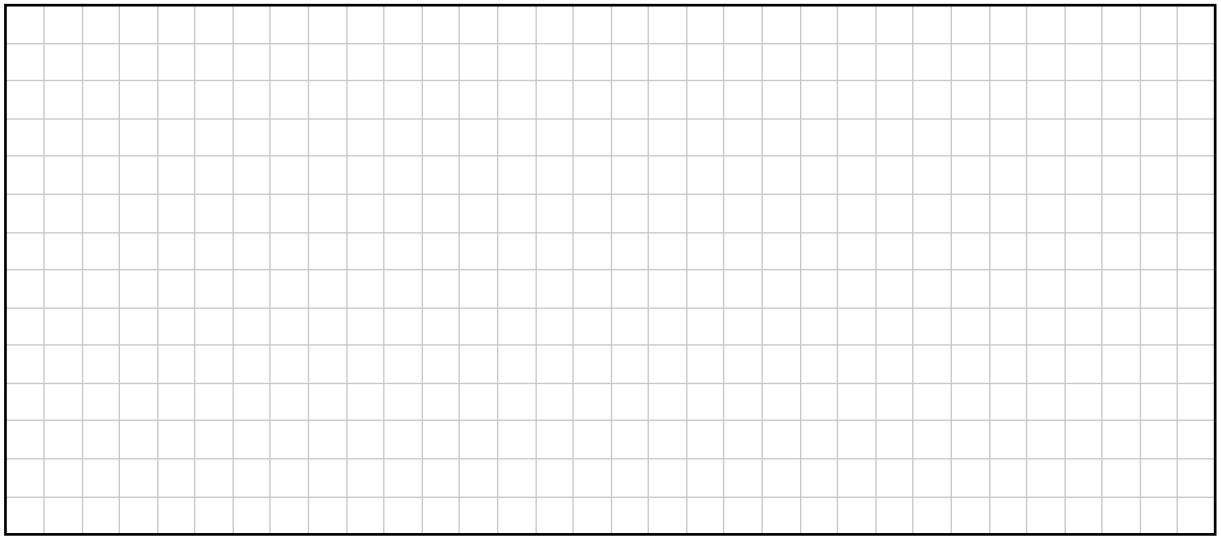
$$T_n = n^2 + bn + c$$

(i) Remember $T_2 = 10$.
Use substitution in the formula for T_n to show that:

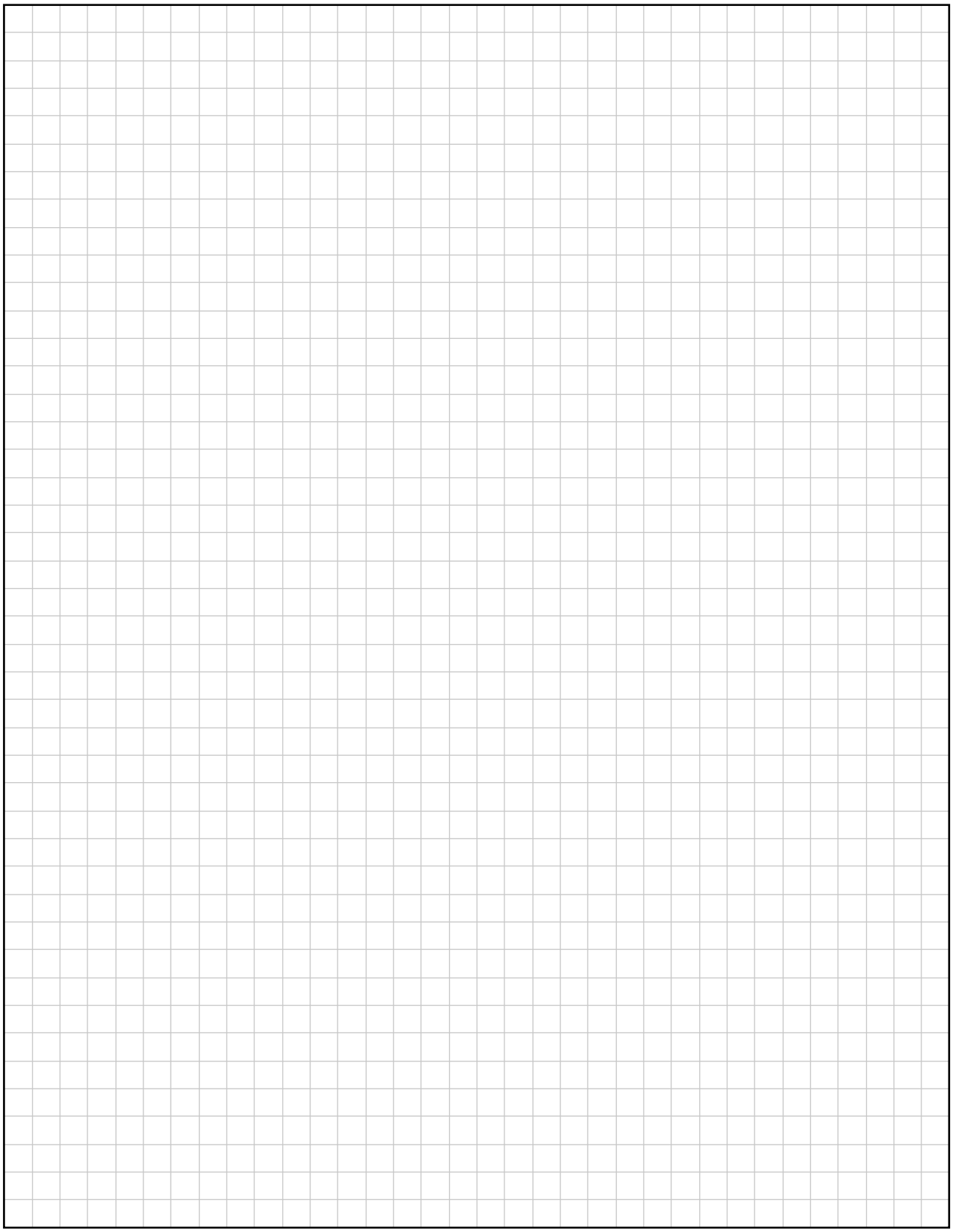
$$2b + c = 6$$

(ii) Solve the following simultaneous equations to find the value of b and the value of c:

$$b + c = 4$$
$$2b + c = 6$$



Page for extra work.
Label any extra work clearly with the question number and part



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Leaving Certificate – Ordinary Level

Mathematics Paper 1

2 hours 30 minutes