Leaving Certificate Examination 2018

Mathematics

Paper 2
Ordinary Level

Monday, 11 June – Morning 9:30 to 12:00

300 marks

For examiner

<table>
<thead>
<tr>
<th>Question</th>
<th>Mark</th>
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Examination number

Centre stamp

Running total

Grade
Instructions

There are two sections in this examination paper.

Section A   Concepts and Skills               150 marks       6 questions
Section B   Contexts and Applications        150 marks       3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the Formulae and Tables booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:
Section A  Concepts and Skills  150 marks

Answer all six questions from this section.

Question 1  (25 marks)

(a) An experiment consists of throwing two fair, standard, six-sided dice and recording the sum of the two numbers thrown. Some of the totals are shown in the table.

(i) Complete the table.

<table>
<thead>
<tr>
<th>Die 1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>2</td>
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<td></td>
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<td>8</td>
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<tr>
<td>3</td>
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<td>9</td>
</tr>
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<td>4</td>
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<td>10</td>
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<td>11</td>
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<td>6</td>
<td></td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) Find the probability of getting a total of 7 or 11.

(iii) Find the probability of getting a total which is a prime number.

(b) A car distributor sells Ford cars and Renault cars. It has 30 cars for sale on a particular day; 18 are Ford cars and 12 are Renault cars. 7 of the Ford cars are red and 4 of the Renault cars are red. One of the 30 cars is chosen at random. What is the probability that the car chosen is a Ford car or a car which is not red?
Question 2

(25 marks)

The points $P(7, 10)$, $Q(1, 2)$ and $R(11, 4)$ are the vertices of the triangle shown. The point $U(4, 6)$ is the midpoint of $[PQ]$ and the point $V$ is the midpoint of $[PR]$.

(a) Find the co-ordinates of $V$.

(b) Show, by using slopes, that $UV$ is parallel to $QR$.

(c) Find the area of the triangle $PQR$.

(d) The point $S$ is the image of the point $Q$ under the translation $\overline{UV}$. Find the coordinates of $S$. 
Question 3  
(25 marks)

(a) (i) Find the number of different arrangements that can be made using all the letters of the word RAINBOW. Each letter is used only once.

(ii) Find the number of different 3-letter arrangements that can be made using the letters of the word RAINBOW. Each letter is used at most once.

(b) A game, called Rainbow, uses an unbiased circular spinner. The spinner has seven sectors coloured red (R), orange (O), yellow (Y), green (G), blue (B), indigo (I), and violet (V) as shown below. The table below shows the angle of each sector. It also shows the cash prize that a player wins if the spinner stops in that sector.

<table>
<thead>
<tr>
<th>Colour</th>
<th>Angle</th>
<th>Probability</th>
<th>Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>72°</td>
<td></td>
<td>€20</td>
</tr>
<tr>
<td>Orange</td>
<td>30°</td>
<td></td>
<td>€60</td>
</tr>
<tr>
<td>Yellow</td>
<td>45°</td>
<td>$\frac{1}{8}$</td>
<td>€24</td>
</tr>
<tr>
<td>Green</td>
<td>90°</td>
<td></td>
<td>€8</td>
</tr>
<tr>
<td>Blue</td>
<td>60°</td>
<td></td>
<td>€42</td>
</tr>
<tr>
<td>Indigo</td>
<td>18°</td>
<td></td>
<td>€90</td>
</tr>
<tr>
<td>Violet</td>
<td>45°</td>
<td></td>
<td>€48</td>
</tr>
</tbody>
</table>

(i) Complete the “Probability” column of the table which shows the probability of the spinner coming to rest in each sector after one spin.

(ii) Find the expected value of the prize that a player wins if they play Rainbow.
Question 4  
(25 marks)

The points $A(1, 8)$ and $B(9, 0)$ are the end-points of a diameter of the circle $w$, as shown in the diagram.

(a) Find the co-ordinates of the centre of $w$.

(b) Find the length of the radius of $w$. Give your answer in the form $p\sqrt{q}$, where $p, q \in \mathbb{N}$.

(c) Hence write down the equation of the circle $w$.

(d) Find the equation of the line that is a tangent to the circle $w$ at $A$. Give your answer in the form $ax + by + c = 0$, where $a, b,$ and $c \in \mathbb{Z}$. 
Question 5

(a) The square $ABCD$ has sides of length 7 cm. The vertices of the square $PQRS$ lie on the perimeter of $ABCD$, as shown in the diagram, with $|AQ| = 5$ cm. Find the area of the square $PQRS$.

(b) The circles $u$ and $v$ represent two wheels that are free to rotate about their centres, as shown. The radius of $u$ is 4 cm and the radius of $v$ is 6 cm.
(i) Find the length of the circumference of each circle. Give your answers in cm in terms of $\pi$.

(ii) The wheels $u$ and $v$ are in non-slip contact and therefore the rotation of one causes the other to rotate. Find the number of complete rotations wheel $u$ makes if wheel $v$ completes 100 rotations.
Question 6  

(a)  (i) Construct the triangle $ABC$, where $|AB| = 10$ cm, $\angle CAB = 60^\circ$ and $\angle ABC = 40^\circ$. Label each vertex clearly.

(ii) Measure $|BC|$, and write your answer in cm, correct to 1 decimal place.
(b) The diagram shows a parallelogram with vertices \( P, Q, R, \) and \( S \).
\( |\angle SPQ| = 115^\circ, \ |\angle QRS| = \alpha^\circ \) and \( |\angle RSP| = \beta^\circ \).

(i) Write down the value of \( \alpha \) and the value of \( \beta \).
\[ \alpha = \underline{\phantom{000}} \quad \beta = \underline{\phantom{000}} \]

(ii) Explain why the triangle \( PQR \) is congruent to triangle \( RSP \).
Give a reason for any statement you make in your explanation.
The table below shows the total rainfall, in millimetres, and the total sunshine, in hours, at Valentia, County Kerry, during the month of June from 2001 to 2010.

<table>
<thead>
<tr>
<th>Year</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rainfall (mm)</td>
<td>72</td>
<td>133</td>
<td>155</td>
<td>101</td>
<td>94</td>
<td>47</td>
<td>149</td>
<td>134</td>
<td>94</td>
<td>84</td>
</tr>
<tr>
<td>Sunshine (hours)</td>
<td>169</td>
<td>124</td>
<td>180</td>
<td>173</td>
<td>173</td>
<td>239</td>
<td>159</td>
<td>168</td>
<td>228</td>
<td>205</td>
</tr>
</tbody>
</table>

(Source: Met Éireann)

(a) Based on the data in the table above write down:
   (i) the range of the rainfall data ______________________
   (ii) the year with the highest June rainfall _____________
   (iii) the year with the least June sunshine _____________

(b) Based on the data in the table, write down the year with the best June weather and give a reason for your answer.

   Answer: ______________________

   Reason: ______________________

(c) Write the rainfall data in increasing order and hence find the median of the rainfall.

(d) (i) Find the mean number of sunshine hours for June in Valentia between 2001 and 2010.
(ii) For what years was the **sunshine data** within 5% of the mean number of sunshine hours in Valentia?

(e) Find the standard deviation of the **rainfall data**, in mm, correct to 1 decimal place.

(f) Part of a scatterplot of the data in the table is shown below. The first four data points are plotted.

(i) Complete the scatterplot.

![Scatterplot](image)

(ii) One of the numbers in the table on the right is the correlation coefficient for the data above, correct to 1 decimal place. Based on the scatterplot, select the number that you think most accurately reflects this data. Explain your choice.

<table>
<thead>
<tr>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>-0.1</td>
</tr>
<tr>
<td>-0.6</td>
</tr>
</tbody>
</table>

Explanation:
Question 8  
(50 marks)

The diagram shows a section of a garden divided into three parts. In the diagram: \( |PR| = 3.3 \text{ m} \), \( |PQ| = 6.5 \text{ m} \), \( |QT| = |QS| = 8 \text{ m} \), \( \angle QRP = 90^\circ \), \( \angle PQR = \alpha \text{°} \), and \( \angle RQS = \beta \text{°} \).

(a) Use the theorem of Pythagoras to find \( |RQ| \).

(b) Show that \( \alpha = 31^\circ \), correct to the nearest degree.

(c) Use the value of \( \alpha \) given in part (b) to find the value of \( \beta \).

\[ \beta = \]
(d) Use the Cosine Rule to find the length of $[RS]$. Give your answer correct to the nearest metre.

(e) $SQT$ is a sector of a circle whose centre is $Q$. Find the length of the arc $TS$. Give your answer in metres, correct to one decimal place.

(f) Find the area of the sector $SQT$. Give your answer in square metres, correct to one decimal place.
Question 9  
(40 marks)

(a)  
(i) Find the volume of a solid sphere of radius 0·3 cm.
Give your answer in cm³, correct to 3 decimal places.

(ii) The sphere is made of pure gold. Each cm³ of pure gold weighs 19·3 grams.
Find the number of grams of pure gold in the sphere.
Give your answer correct to 2 decimal places.

(iii) It is known that there are approximately $6·02 \times 10^{23}$ atoms in 197 grams of pure gold.
Find the number of atoms of pure gold in the sphere.
Give your answer in the form $a \times 10^n$, where $1 \leq a < 10$, $n \in \mathbb{N}$,
and where $a$ is correct to 2 significant figures.
(b) A survey was carried out on behalf of a television station to investigate the popularity of a certain show.

(i) A random sample of 1560 television viewers was surveyed. Find the margin of error of the survey. Give your answer as a percentage, correct to 1 decimal place.

(ii) In the survey, 546 of the 1560 viewers surveyed said that they liked the show. Use your answer to part (i) above to create a 95% confidence interval for the percentage of viewers who liked the show.

(iii) An executive for the television station had claimed that 40% of viewers liked the show. Use your answer to part (ii) above to conduct a hypothesis test, at the 5% level of significance, to test the executive’s claim. State your null hypothesis, your alternative hypothesis and give your conclusion in the context of the question.