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LEAVING CERTIFICATE EXAMINATION, 2000

MATHEMATICS - ORDINARY LEVEL

PAPER 1 (300 marks)

THURSDAY, 8 JUNE - MORNING, 9.30 to 12.00

Attempt **SIX QUESTIONS** (50 marks each).

Marks may be lost if all necessary work is not clearly shown.

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1. (a) Express 400 grammes as a fraction of 1 kilogramme. Give your answer in its simplest form.

(b)

1 euro = IR£ 0.787564
1 euro = DM 1.95583

(i) Calculate the value of IR£100 in euro, correct to two places of decimals.

(ii) Hence, calculate the value of IR£100 in Deutschmarks (DM), correct to two places of decimals.

(c) A person has annual tax free allowances of IR£7400.

The person pays income tax at the rate of 24% on the first IR£12 400 of taxable income and at the rate of 46% on the remainder.

(i) Calculate the amount of income tax paid on the first IR£12 400 of taxable income.

(ii) Calculate the person's gross income if the total annual income tax paid is IR£5138.

2. (a) Find the value of $5x - 3y$ when $x = \frac{5}{2}$ and $y = \frac{2}{3}$.

(b) Solve for x and y

$$x - 3y = 1$$
$$x^2 - y^2 = 0.$$

(c) Write as a power of 3

(i) 243
(ii) $\sqrt{27}$.

Hence, solve for x the equation

$$\sqrt{3}(3^x) = \left(\frac{243}{\sqrt{27}}\right)^2.$$

3. **(a)** Express p in terms of t and k when

$$tp - k = 7k, \qquad t \neq 0.$$

(b) **(i)** Show that $x = 2$ is a root of $3x^3 + 8x^2 - 33x + 10 = 0$.

(ii) Find the other roots of $3x^3 + 8x^2 - 33x + 10 = 0$.

(c) **(i)** $f(x) = ax^2 + bx - 8$, where a and b are real numbers.

If $f(1) = -9$ and $f(-1) = 3$, find the value of a and the value of b.

(ii) Using your values of a and b from **(i)**, find the two values of x for which

$$ax^2 + bx = bx^2 + ax.$$

4. **(a)** Simplify

$$7(2 + i) + i(11 + 9i)$$

and express your answer in the form $x + yi$ where $x, y \in \mathbf{R}$ and $i^2 = -1$.

(b) Let $w = 3 - i$.

(i) Plot w and $w + 6i$ on an Argand diagram.

(ii) Calculate $|w + 6i|$.

(iii) Express $\dfrac{1}{w + 6i}$ in the form $u + vi$ where $u, v \in \mathbf{R}$.

(c) Let $z = 2+4i$.

(i) Express $z^2 + 28$ in the form $p + qi$ where $p, q \in \mathbf{R}$.

(ii) Solve for real k

$$k(z^2 + 28) = |z|(1 + i).$$

Express your answer in the form $\dfrac{\sqrt{a}}{b}$ where $a, b \in \mathbf{N}$ and a is a prime number.

5. (a) The nth term of a sequence is given by $T_n = n^2 + 1$.

(i) Write down the first three terms of the sequence.

(ii) Show that $T_1 + T_2 + T_3 = T_4$.

(b) The first term of a geometric series is 1 and the common ratio is $\frac{11}{10}$.

(i) Write down the second, third and fourth terms of the series.

(ii) Calculate S_4, the sum of the first four terms. Give your answer as a decimal.

(c) The first three terms of an arithmetic series are $5 + 10 + 15 + ...$

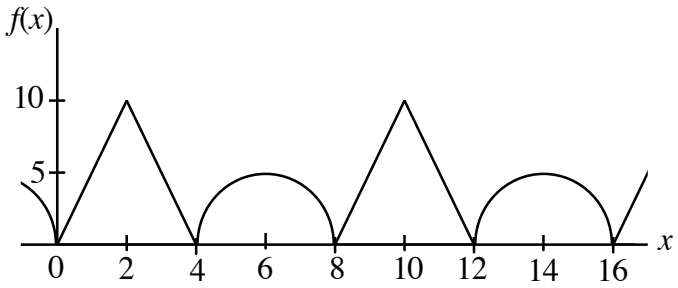
(i) Find, in terms of n, an expression for T_n, the nth term.

(ii) Find, in terms of n, an expression for S_n, the sum to n terms.

(iii) Using your expression for S_n, find the sum of the natural numbers that are both multiples of 5 and smaller than 1000.

6. (a) Differentiate $7x + 3$ from first principles with respect to x.

(b)



The graph shows portion of a periodic function $f : x \to f(x)$ which is defined for $x \in \mathbf{R}$.

(i) Write down the period and the range of $f(x)$.

(ii) Complete the following table:

x	2	8	14	20	26
$f(x)$					

(c) Let $g(x) = (2x + 3)(x^2 - 1)$ for $x \in \mathbf{R}$.

(i) For what two values of x is the slope of the tangent to the curve of $g(x)$ equal to 10?

(ii) Find the equations of the two tangents to the curve of $g(x)$ which have slope 10.

7. **(a)** Differentiate with respect to x

(i) $4x^2 + 5$

(ii) $9x - x^3$.

(b) **(i)** Find $\frac{dy}{dx}$ when $y = \frac{2x-7}{x-1}$, $x \neq 1$.

(ii) Find $\frac{dy}{dx}$ when $y = \left(x^2 + 5x - 1\right)^3$.

(c) A car, starting at $t = 0$ seconds, travels a distance of s metres in t seconds where

$$s = 30t - \frac{9}{4}t^2.$$

(i) Find the speed of the car after 2 seconds.
(ii) After how many seconds is the speed of the car equal to zero?
(iii) Find the distance travelled by the car up to the time its speed is zero.

8. **(a)** Let $p(x) = 3x - 12$.

For what values of x is $p(x) < 0$ where x is a positive whole number?

(b) **(i)** Draw the graph of

$$g(x) = \frac{1}{x} \quad \text{for } -3 \leq x \leq 3,\ x \in \mathbf{R} \text{ and } x \neq 0.$$

(ii) Using the same axes and the same scales, draw the graph of

$$h(x) = x + 1 \quad \text{for } -3 \leq x \leq 3,\ x \in \mathbf{R}.$$

(iii) Use your graphs to estimate the values of x for which

$$\frac{1}{x} = x + 1.$$

(c) Let $f(x) = x^3 - 3x^2 + ax + 1$ for all $x \in \mathbf{R}$ and for $a \in \mathbf{R}$.

$f(x)$ has a turning point (a local maximum or a local minimum) at $x = -1$.

(i) Find the value of a.

(ii) Is this turning point a local maximum or a local minimum?
Give a reason for your answer.

(iii) Find the co-ordinates of the other turning point of $f(x)$.