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LEAVING CERTIFICATE EXAMINATION, 1990

MATHEMATICS – ORDINARY LEVEL – PAPER II (300 marks)

FRIDAY, 8 JUNE –MORNING, 9.30 – 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used

1. (i) A person buys 30 600 Pesetas when the exchange rate is IR£1 = 170 Pesetas. A charge is made for this service. How much is this charge if the person pays IR£182·70 ?

(ii) The length of a rectangle is twice its width. The area of the rectangle is 338. Find its length.

(iii) Solve

$$4x^2 - 4(x - 2) = 9 - 7x.$$

(iv) Find k if

x^3 $+10$

$$x^3 + 2x^2 + kx + 10 = (x - 1)(x - 2)(x + 5)$$

for all $x \in \mathbf{R}$.

(v) $3, x, 75$ are numbers in geometric sequence. Find the values of x.

3 > x − 3
x >
75 > x − 75
72
36x + 3

(vi) Solve the simultaneous equations

$$3x + 1 = y - 4$$
$$30x + 10y = 5.$$

(vii) Graph the set B defined by

$$B = \{(x, y) \mid 2y - 5x \leqslant 10, \; x, y \in \mathbf{R}\}$$

and write B clearly on the set.

(viii) If $8^x = 4^3 - 2^5$, find the value of x.

(ix) The function f is defined by

$$f : \mathbf{R} \to \mathbf{R} : x \to 7 - 3x.$$

If $kf(-2) = f(24)$, find k.

(x) Differentiate $\dfrac{1 - 2x^2}{x}$ with respect to x.

2. Let $z = -2 + i$, where $i = \sqrt{-1}$.

Show that z is a solution of the equation

$$z^2 + 4z + 5 = 0.$$

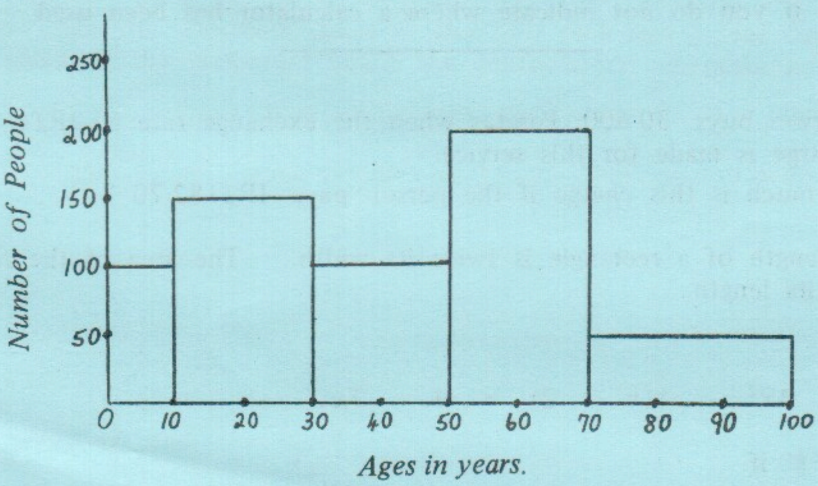
Write $3z$ and $\frac{15}{z}$ in the form $a + bi$. $15 \frac{1}{z} = -2 + i$

Plot z, $3z$ and $\frac{15}{z}$ on an Argand diagram.

w is the image of z under the central symmetry in the point $2 - i$. Express w in the form $a + bi$ and plot it on the Argand diagram.

Verify that $\left|\frac{15}{z} - w\right| > |z - w|$.

3. (a) The distribution of the ages of people living in a parish is shown in the histogram:



Complete the frequency distribution table below for this histogram:

Ages in years	0–10	10–30	30–50	50–70	70–100
Number of people		150			

How many people live in the parish ?
In which interval does the median lie ?

(b) A football match began at 3.00 p.m. A survey showed the number of people entering the grounds between 2.00 p.m. when the gates were first opened and 3.15 p.m.

Time	2.00–2.15	2.15–2.30	2.30–2.45	2.45–3.00	3.00–3.15
Number of people	100	400	550	350	100

(Note: 2.15–2.30 means 2.15 is included but 2.30 is not.)

Complete the cumulative frequency table below:-

Time	<2.15	<2.30	<2.45	<3.00	<3.15
Number of people					

Draw the cumulative frequency curve.
Use this curve to estimate the number of people who were admitted between 2.40 p.m. and 3.00 p.m.

4. If $f(x) \Rightarrow x^3 + 2x^2 - 3x$, complete the following table:

x	-3	-2	-1	0	1	2
$f(x)$					0	

Draw the graph of the function

$$f : x \rightarrow x^3 + 2x^2 - 3x$$

in the domain $-3 \leqslant x \leqslant 2,\ x \in \mathbf{R}$.

Use your graph to find, as accurately as possible, the range of values of x for which

$$0 \leqslant f(x) \leqslant 4.$$

Using the same axes and the same scales draw the graph of the function

$$g : x \rightarrow 2x^2$$

in the domain $-2 \leqslant x \leqslant 2,\ x \in \mathbf{R}$.

Use the intersection of both graphs to estimate $\sqrt{3}$.

5. (a) Solve

$$\frac{2}{x+1} = \frac{2}{x} - \frac{1}{6}$$

(b) Show that

$$\frac{11!}{3!\ 8!} = \binom{11}{3}$$

(c) Write out all the terms of the expansion of

$$(1 - 3x)^5$$

in ascending powers of x.

Find the non-zero values of x for which twice the second term equals the fourth term.

6. (a) (i) The first term, T_1, of an arithmetic sequence is 9 and the common difference is 4.
Find T_5, the fifth term and S_{10}, the sum of the first 10 terms.

(ii) In an arithmetic sequence

$$S_n = 5n - 2n^2.$$

Find T_1, T_2 and the common difference.

(b) A person, in a holiday savings scheme, saves IR£ 20 on the first day of each month for 10 consecutive months. The savings earn compound interest at the rate of 1% per month. How much are the person's savings worth at the end of the 10 months?

[Note: You may take $(1 \cdot 01)^{10}$ as $1{\cdot}1046$]

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7. Two types of cake, chocolate and coffee, are made in a small confectioner's shop. Each week, at least 250 cakes must be produced to meet orders from customers.

During each week the maximum number of chocolate cakes that can be produced is 200 and the maximum number of coffee cakes is 150.

The cost of producing a chocolate cake is IR£2 and a coffee cake is IR£1·60 and the total production cost must not exceed IR£560 in a week.

Graph the set showing the possible number of each type of cake that can be produced in a week.

If the confectioner sells the chocolate cakes at IR£2·80 each and coffee cakes at IR£2·20 each, calculate the maximum profit the confectioner can make in a week.

8. (a) Differentiate from first principles

$$5 - 4x^2$$

with respect to x.

(b) (i) Find the value of $\frac{dy}{dx}$ at $x = 1$ when

$$y = (2x^2 - 3x)(x^2 - 2x + 1).$$

(ii) Verify that $(1, 1)$ is a point of the curve

$$y = (1 - 2x)^6.$$

Find the equation of the tangent to the curve at the point $(1, 1)$.