AN ROINN OIDEACHAIS **M.47**



**LEAVING CERTIFICATE EXAMINATION, 1989**

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MATHEMATICS – ORDINARY LEVEL – PAPER I (300 marks)

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THURSDAY, 8 JUNE – MORNING, 9.30 – 12.00

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Attempt **Question 1** (100 marks) and **four** other questions (50 marks each)

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**Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used.**

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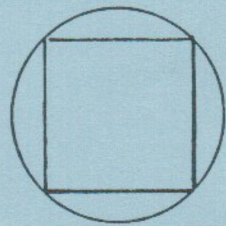
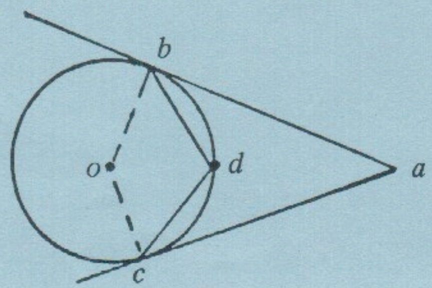
1. (i) A square is inscribed in a circle.
The diameter of the circle is 4 cm in length.
Find the area of the square.

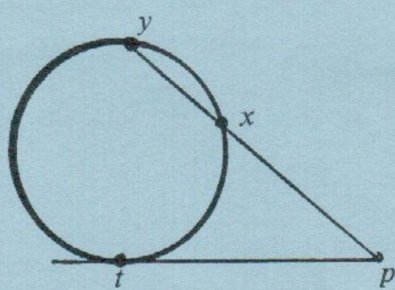


(ii) If $y = \dfrac{x + a}{x - 2a}$, express $x$ in terms of $a$ and $y$.

(iii) $ab$ and $ac$ are tangents to the circle, centre $o$.
If $|\angle bac| = 40°$,
find $|\angle bdc|$ .




(iv) If $|px| = 5$ cm, $|xy| = 2{\cdot}2$ cm
and $pt$ is a tangent, calculate $|pt|$.



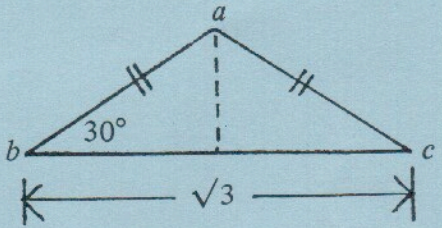

(v) Calculate $|ab|$ when $a$ is $(-1, -1)$ and $b$ is $(3, 2)$.

(vi) $a(-1, 4)$ and $b(11, 0)$ are points. $c$ is a point on $[ab]$ such that $\frac{|ac|}{|cb|} = \frac{3}{1}$

Calculate the coordinates of $c$.

(vii) The points $(0, 0)$ $(0, -6)$ and $(8, 0)$ are on a circle. Find the centre of the circle.

(viii) Find $|ab|$ in the isosceles triangle $abc$, if $|bc| = \sqrt{3}$ and $|\angle abc| = 30°$.




(ix) Assuming the period of $\sin x$ to be $2\pi$, or otherwise, write two values of $x$, other than $\frac{\pi}{6}$, such that $\sin x = \frac{1}{2}$.

(x) If $\vec{a} = 3\vec{i} + 3\vec{j}$ and $b = -\vec{i} - 5\vec{j}$, where $o$ is the origin, find $\overrightarrow{ox}$ in terms of $\vec{i}$ and $\vec{j}$ such that

$$\overrightarrow{ab} + \overrightarrow{ox} = \vec{o}.$$

OVER →

2. A rectangular box, complete with lid has the following internal dimensions

length 63 cm
breadth 42 cm
height 21 cm

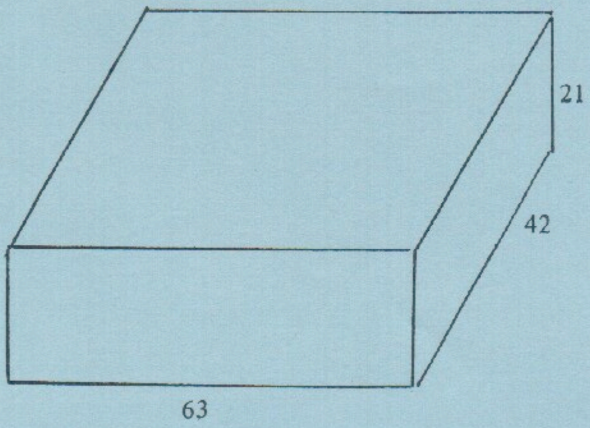
Find its capacity (internal volume) in $cm^3$.

Find the volume in $cm^3$, taking $\pi = \frac{22}{7}$, of the largest

(i) sphere

(ii) cylinder

that the box will hold, the height of the cylinder being perpendicular to a side.



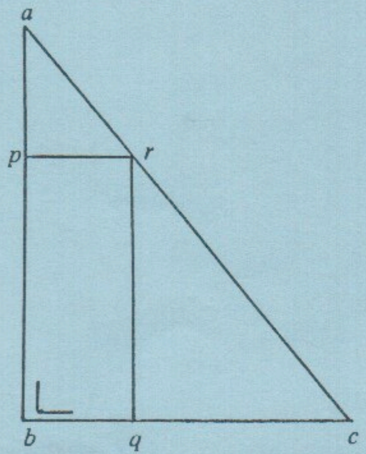

3. Prove that the areas of two triangles of equal height are proportional to the lengths of their bases.

$bqrp$ is a rectangle, see diagram.
Prove $|ap| : |pb| = |bq| : |qc|$.

If $|ap| : |pb| = 1 : 2$, prove

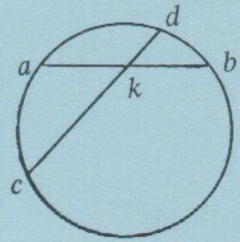
$$\frac{\text{area of the triangle } apr}{\text{area of the rectangle } bqrp} = \frac{1}{4}.$$

If the area of the triangle $abc$ is 36, find the area of the rectangle $pbqr$.



4. (i) $[ab]$ and $[cd]$ are two chords of a circle. If the lines $ab$ and $cd$ intersect internally in $k$, prove

$$|ak| \cdot |kb| = |ck| \cdot |kd| .$$



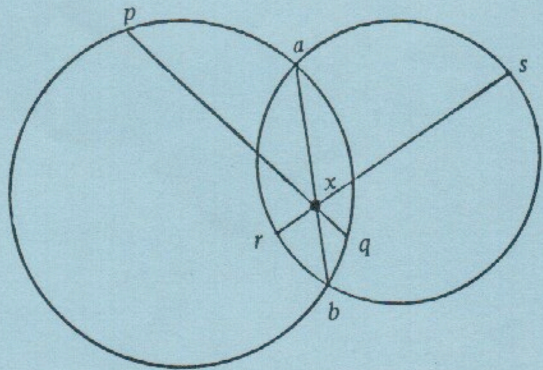

(ii) In the diagram, chords $[pq]$, $[rs]$, $[ab]$ intersect at the point $x$.

Prove:

$$|px| \cdot |xq| = |sx| \cdot |xr|.$$

Prove:

$$|\angle qpr| = |\angle qsr| .$$




5. The line $L$ contains the points $p$ $(3, -1)$ and $q$ $(1, 0)$.

$M$ is the line $x + 2y = 11$.

(i) Calculate the slope of $L$.

(ii) Verify $L \parallel M$.

(iii) Find the equation of the line $K$ which is perpendicular to $L$ and contains $p$.

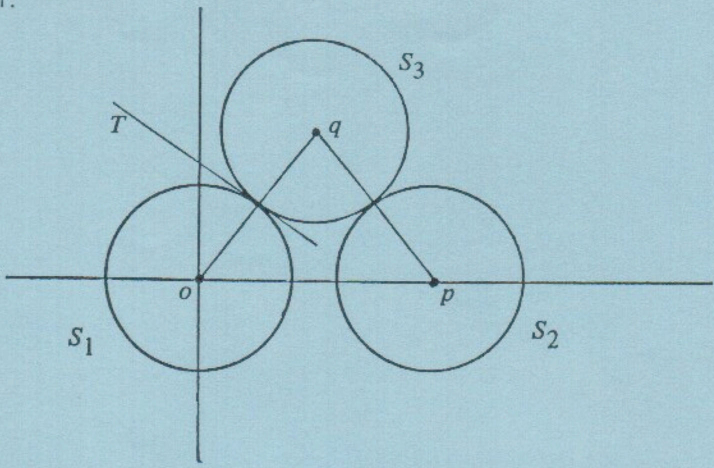
(iv) Calculate the coordinates of the point $K \cap M$.

(v) $H$ is the line through $q$ parallel to $K$. Calculate the area of the rectangle formed by $H$, $K$, $L$ and $M$.

6. Points $o$ (0, 0), $p$ (12, 0) and $q$ (6, 8) form a triangle, in which $|qp| = |qo|$.

Circles $S_1$, $S_2$ and $S_3$, each of radius length 5, have $o$, $p$ and $q$, respectively, as centres

$S_1$, $S_3$ and $S_2$, $S_3$ touch as shown.




Write the equation of (i) $S_1$

(ii) $S_2$

(iii) $S_3$ .

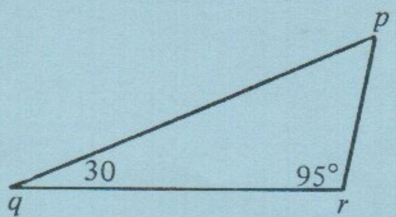
Find the equation of $T$, the tangent common to $S_1$ and $S_3$.

If the points $o$, $p$ and $q$ are on another circle, centre $k$, find $k$.

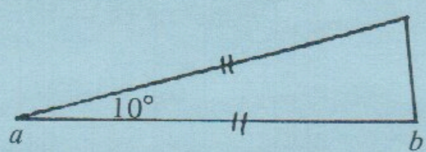
7. (a) In the triangle $pqr$

$$|\angle pqr| = 30^\circ \quad \text{and}$$
$$|\angle qrp| = 95^\circ$$
$$|qr| = 6000 \text{ m},$$

calculate $|pr|$ to the nearest metre.



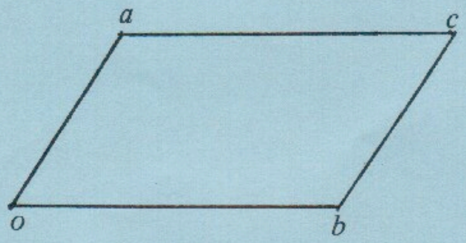

(b) It was intended to sail a boat due East from $a$ at 30 km per hour.
In fact, it sailed East 10° North at 30 km per hour.
How far, to the nearest km, was the boat from $b$ (see diagram) after one hour.




(c) Using the same axes and scales, sketch the graph of $\sin x$ and the graph of $\cos x$ in the domain $0 \leqslant x \leqslant 2\pi$.

If $\tan x = \dfrac{\sin x}{\cos x}$, where $x \neq \dfrac{\pi}{2}, \dfrac{3\pi}{2}$, show and estimate the value of $x$ other than $\dfrac{\pi}{4}$, for which $\tan x = 1$.

8. (a) $obca$ is a parallelogram, where $o$ is the origin.




(i) Copy the diagram and plot $k$ such that $\vec{k} = \vec{a} + \vec{c}$.

(ii) Express $\vec{k}$ in terms of $\vec{a}$ and $\vec{b}$.

(iii) Find the value of the scalar $r$ such that $\vec{a} + \vec{b} + r\vec{c} = \vec{0}$.

(b) $\vec{d} = \vec{i} + 5\vec{j}$ and $\vec{e} = -3\vec{i} + \vec{j}$.

(i) Express $\vec{h}$, the image of $\vec{e}$ under the central symmetry in the origin, in terms of $\vec{i}$ and $\vec{j}$.

(ii) If $y$ is the mid-point of $[dh]$, verify that $|\overrightarrow{ed}| = 2|\vec{y}|$.