AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1987

MATHEMATICS – ORDINARY LEVEL – PAPER I (300 marks)

THURSDAY, 11 JUNE – MORNING, 9.30 – 12.00

Attemp **Question 1** (100 marks) and **four** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used

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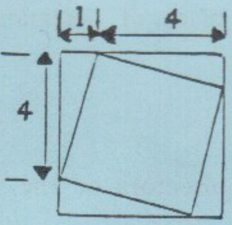
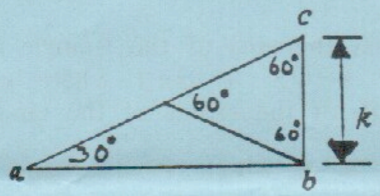


1. (i) Calculate the area of the smaller square in the diagram. The lengths 4 and 1 are shown.

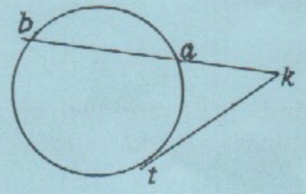
(ii) Express s in terms of p, q and r when

$$p = q\left(1 + \frac{r}{s}\right).$$

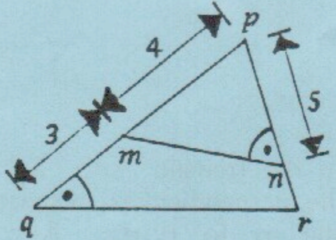
(iii) IR£1 is equivalent to 96p sterling. What is the equivalent in IR£ of £384·00 sterling?



(iv) $|bc| = k$, see diagram. Express $|ab|$ in terms of k.



(v) Find the length of the tangent $[kt]$ if $|ka| = \frac{1}{2}|kt|$ and $|ab| = 12$.



(vi) In the triangle pqr, $[mn]$ is drawn so that $|\angle pqr| = |\angle mnp|$.

Prove $|\angle pmn| = |\angle qrp|$ and hence

Calculate $|nr|$.

(vii) K is a line through the origin. Under the axial symmetry in K, $(6, -2)$ is the image of $(2, 6)$.
Find the equation of K.

(viii) $(-1, -1)$ is a point of the circle $(x - 2)^2 + (y - k)^2 = 25$, where $k > 0$. Calculate the value of k.

(ix) Find the equation of the image of the line $x - y + 2 = 0$ under the translation $(0, 0) \rightarrow (0, -2)$.

(x) If $a\vec{i} - b\vec{j} + b\vec{i} + a\vec{j} = 7\vec{i} - \vec{j}$, find the value of a and the value of b.

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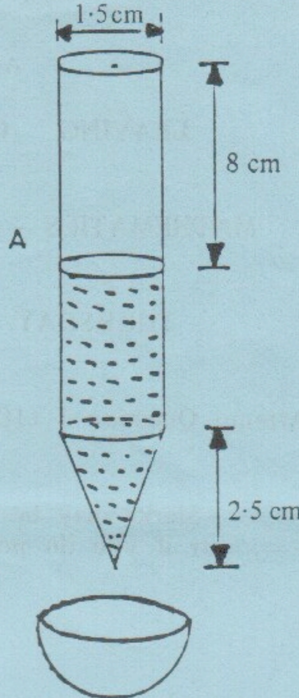
2. A container, A, see diagram is first filled to the top with liquid which can flow through the cone's vertex.

When the level of water is 8 cm lower than it was initially the bowl is full.

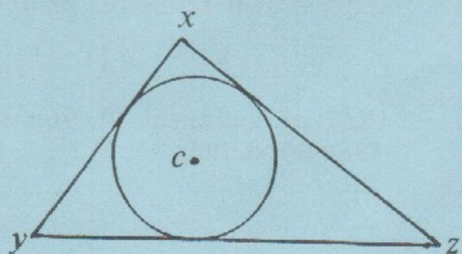
Calculate the diameter of the bowl.

A then contains 18 cm^3 of liquid, as shown. Find the height of liquid in the cylindrical part correct to one place of decimals. [Take $\pi = 3.$]



3. Prove that the bisectors of the angles of a triangle pqr are concurrent.

In the triangle xyz, the bisectors of the angles $\angle xyz$, $\angle yzx$, $\angle zxy$ meet at the incentre c, and a circle, centre c, is inscribed in the triangle.

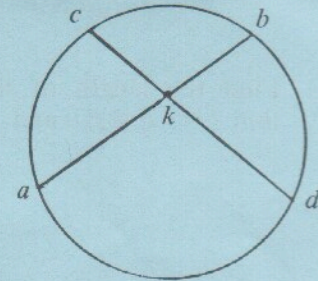


Prove that the area of the triangle xyz is $\frac{1}{2}r(|xy| + |yz| + |zx|)$ where r is the length of the radius of the circle.

If $|xy| = 9$, $|xz| = 12$ and $|\angle yxz| = 90°$, calculate the value of r.

4. (a) $[ab]$ and $[cd]$ are two chords of a circle. If the lines ab, cd intersect in k, then prove

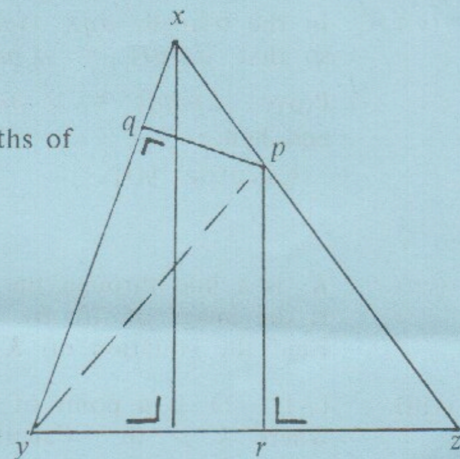
$$|ak| \,.\, |kb| = |ck| \,.\, |kd| .$$



(b) In the triangle xyz, h_1 and h_2 are the lengths of the perpendiculars, respectively from x and p to $[yz]$. Also $pq \perp xy$.

If $h_1 : h_2 = 3 : 2$ and $|xy| = |yz|$, find the value of the ratio

$$|pq| : |pr| .$$



5. M is the line $x - 2y + 6 = 0$.

Verify that the point $(4, 5)$ is in M.

Calculate the coordinates of q, the point of M which is in the X-axis.

The line K contains the point $r(2, -1)$ and $K \perp M$. Find the equation of K.

$K \cap M = \{s\}$. Find the coordinates of s without using graphs.

The area of the triangle qrs is equal to the area of the triangle formed by joining q, r and the point $(4, k)$ where $k > 0$. Calculate the value of k.

6. K_1 is the circle $x^2 + y^2 = 25$.

Write down the coordinates of the centre and the length of the radius.

Verify that the point $p(-3, 4)$ is a point of the circle K_1.

Find the equation of the tangent to K_1 at p.

K_2 is the image of K_1 under the translation $(1, 1) \rightarrow (-2, 5)$. Write the equation of K_2.

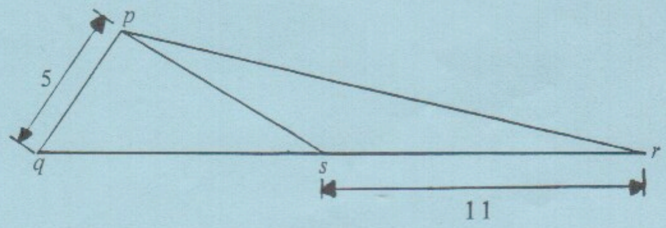
K_3 is a circle, the centre of which is midway between p and the origin and which touches K_1 at one point only.
Write an equation for K_3.

7. (a) In the triangle pqr, see diagram,

$|pq| = 5, \quad |rs| = 11,$

$|\angle pqr| = 64° 9', \quad |\angle qsp| = 30°.$

Calculate $|ps|$ and $|pr|$.



(b) Sketch the graph of

$f : x \rightarrow \cos x$

in the domain $0 \leqslant x \leqslant 2\pi, \ x \in \mathbf{R}$.

If the period of $\cos x$ is 2π, estimate from your graph the value of

(i) $\cos \frac{31\pi}{5}$

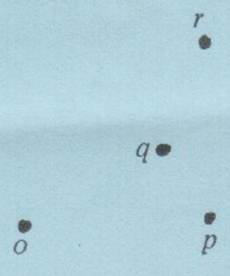
(ii) $2 \cos \left(16\pi - \frac{\pi}{6}\right)$.

8. (a) p, q, r are points and o is the origin.

Copy the diagram each time and identify on separate diagrams the points k_1, k_2 where

$(\vec{oq} + \vec{qp}) - (\vec{or} + \vec{rp}) = \vec{ok_1}$

$(\vec{oq} + \vec{qp}) + \vec{pr} = \vec{ok_2}$



(b) $oxyz$ is a parallelogram and o is the origin.

$|sy| = \frac{1}{3}|yz|$ and $|tx| = \frac{1}{3}|xy|$.

Find $|\vec{st}|$ to the nearest unit if

$\vec{x} = 4\vec{i} + 3\vec{j}$

$\vec{z} = -5\vec{i}$.