1. (i) On paying a bill a customer was allowed a discount of 10%. He paid \( \text{Ir£}180 \). How much would he have paid if he had been allowed a discount of 5%?

(ii) Write the ratio \( \frac{1}{2} : \frac{1}{8} \) in the form \( p : q \) where \( p, q \in \mathbb{Z} \).

(iii) Solve the equation \( x - \frac{6}{x - 1} = 2 \).

(iv) Factorise \( 8x^3 + 27y^3 \).

(v) If \( 3^{2x + 1} = 81 \), find \( x \) without the use of the Tables.

(vi) Find the sum of the first 100 terms of the arithmetic series \( 13 + 10 + 7 + \ldots \ldots \ldots \ldots \).

(vii) Calculate \( \frac{10!}{6!4!} \).

(viii) \( f = \{(a, b), (b, c), (c, k)\} \)

\( g = \{(a, b), (b, d), (d, k)\} \)

Write out the couples, if any, of \( f \circ g \).

(ix) Graph the set \( A \) defined by

\[ A = \{(x, y) \mid 3x + 4y \geq 24, x, y \in \mathbb{R}\} \]

and write \( A \) clearly on the set.

(x) Find the value of \( \frac{dy}{dx} \) at \( x = -1 \) when

\[ y = (1 - x)(1 - x - x^2). \]
2. Verify that the complex number \( z_1 = 3 - 2i \) is a root of the equation
\[ z^2 - 6z + 13 = 0 \]
and find \( z_2 \), the other root of the equation.

On an Argand diagram plot the complex numbers \( z_1 \) and \( z_2 \).

\( z_3 \) is the image of \( z_1 \) under the central symmetry in \( z_2 \). Express \( z_3 \) in the form \( a + ib \) and plot it on the Argand diagram.

Investigate if \( |z_1 - z_2| = |z_1| - |z_2| \).

3. (a) The distribution of the ages of people attending a meeting is shown in the histogram

![Histogram](image)

If there were 30 people in the 25 – 35 year age group, how many people were at the meeting?

(b) 100 pupils were given a problem to solve. The following grouped frequency distribution table gives the numbers of pupils who solved the problem in the given time interval:

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>0 - 10</th>
<th>10 - 30</th>
<th>30 - 50</th>
<th>50 - 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td>21</td>
<td>47</td>
<td>22</td>
</tr>
</tbody>
</table>

(Note: 0 - 10 means 0 is included but 10 is not etc.)

Verify, using the mid-interval time values, that the average time taken per pupil to solve the problem is 40 minutes.

Assuming that the standard deviation, \( \sigma \), is 22 minutes, use a cumulative frequency curve to estimate the percentage of pupils who solved the problem in the time interval \([40 - \sigma, 40 + \sigma]\) minutes.

4. The function
\[ f: x \to x^3 - 4x^2 + 4 \]
is defined for \(-2 \leq x \leq 4, x \in \mathbb{R}\).

Draw the graph of \( f \).

Find from your graph, as accurately as you can, the values of \( x \) for which
\[ (i) \quad f(x) = 2 \]
\[ (ii) \quad x^3 - 4x^2 - x + 4 = 0. \]

Find, also, the range of values of \( h \) for which
\[ f(x) - h = 0. \]
On an Argand diagram plot the complex numbers \( z_1 \) and \( z_2 \).

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\[ f(x) - h = 0 \]

has one root only.
5. (a) The \( n \)th term of a sequence is given by
\[
T_n = 2n + 1.
\]
Write down an expression for the \((n - 1)\)th term and hence deduce that the sequence is arithmetic.

Show that the sum of the first \( n \) terms is given by
\[
S_n = n^2 + 2n.
\]
Calculate the average (mean) of the first 51 terms of the sequence.

(b) Two people, \( A \) and \( B \), invested IRL5000 each for one year. \( A \) invested at 2% per month, compound interest, while \( B \) invested at 24% per annum.

Calculate by how much is the income of \( A \) greater than the income of \( B \).

[Note: You may take \((1.02)^n\) as \(1 + 0.02n\)]

6. (a) Solve the simultaneous equations
\[
\begin{align*}
2x - y &= 1 \\
x^2y &= 6.
\end{align*}
\]

(b) Write out the first three terms of the expansion of
\[
(1 - 3x)^6
\]
in ascending powers of \( x \).

Use your result to evaluate
\[
(0.997)^6
\]
correct to four places of decimals.
7. A manufacturer produces two products $P$ and $Q$. The time in hours required for the Cutting and the Finishing of each unit produced is shown in the table:

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting hours per unit</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Finishing hours per unit</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

There are at most 180 hours available for Cutting and at most 150 hours for Finishing.

Assuming that a profit of £50 is made on the sale of each unit of $P$ and a profit of £10 on the sale of each unit of $Q$ and that there is a ready sale for both products:

(i) Graph the set of all possible sales of $P$ and $Q$.
(ii) Graph the set of all possible sales of $P$ and $Q$ that yield a profit of £700 and say what is the maximum sale for $P$ and the maximum sale for $Q$ that would yield this profit.
(iii) Find the sales of $P$ and $Q$ that yield a maximum profit.

8. (a) Differentiate from first principles

$$1 - x^3$$

with respect to $x$.

(b) Find the value of $\frac{dy}{dx}$ at $x = 2$ when

$$y = \frac{x^3 - 4x}{x^4 - 1}$$

(c) Show that the $x$-axis is a tangent to the graph of

$$y = (x^3 - 4x)^5$$

at $x = 2$.

(d) The boundary of a rectangular field is 100 m in length. If one side of the field is of length $x$ metres, show that the area, $y$, of the field is given by

$$y = x(50 - x)$$

Hence show that the maximum area of a rectangular field having a boundary of length 100 m is 625 m².