LEAVING CERTIFICATE EXAMINATION, 1977

MATHEMATICS—ORDINARY LEVEL—PAPER II (300 marks)

TUESDAY, 14 JUNE—MORNING, 9.30 to 12

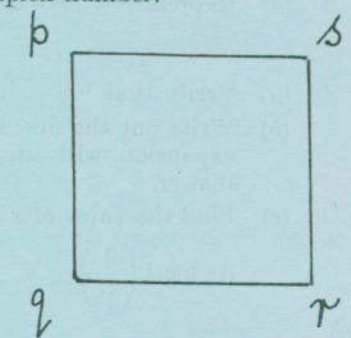
Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.

1. (*a*) Find to the nearest £ the compound interest on £5500 for 2 years at 15% per annum.
   (*b*) When invested at r% per annum, compound interest, £400 amounts to £800 after five years. Find the value of r correct to two significant figures.
   (*c*) A person borrowed £1000 for two years at 12% per annum compound interest on condition that he would repay £520 at the end of the first year and the remainder at the end of the second year. How much did the person pay at the end of the second year?

2. (*a*) Write down the formula for the sum of the first n terms of an arithmetic series.
   630 is the sum of the first 21 terms of an arithmetic series. If the 21st term is three times the 1st term, find the first term and the common difference.
   (*b*) Write down the first three terms of the geometric series which represents $0{\cdot}\dot{2}\dot{5}$. Find S_n, the sum of the first n terms of this series, and evaluate in the form $\frac{a}{b}$ $(a, b, \in \mathbf{N})$ the limit of S_n as n tends to infinity.

3. (*a*) If $z_1 = 3 - 4i$ and $z_2 = 2 + 3i$, where $i = \sqrt{-1}$, express in the form $x + iy$
   (i) $z_1 + z_2$ (ii) $2z_1 - 3z_2$ (iii) $z_1 z_2$ (iv) $\frac{z_1}{z_2}$
   Investigate if
   $$|z_1| + |z_2| > |z_1 + z_2|$$
   (*b*) If $-3 - 2i$ is a root of the equation
   $$z^2 + tz + 13 = 0 \text{ for } t \in \mathbf{R},$$
   find the value of t.
   (*c*) Express $(z^2 - 2z + 10) \div (z - 1 + 3i)$ in the form $z + u$ where u is a complex number.

4A. (*a*) Let A, B, C, D be four lines such that the square $pqrs$ maps onto itself under the axial symmetry in each line (i.e. S_A, S_B, S_C, S_D).
   Investigate if the set $\{S_A, S_B, S_C, S_D\}$ is closed under composition.



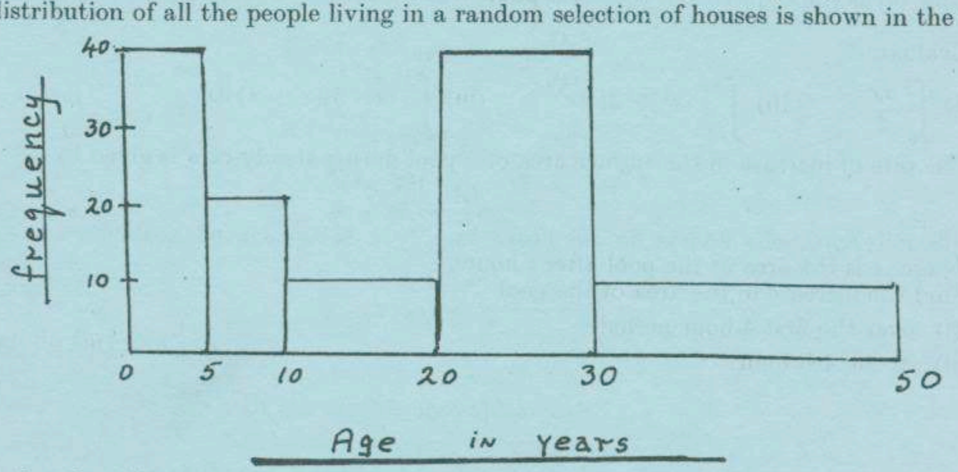
   (*b*) Write down the four subsets of the set $\{x, y\}$.
   Let P be this set of subsets. Verify that P is closed under the operation of $\cup$ and name the identity element. Is $P, \cup$ a group? Give a reason for your answer.

OR

4B. (*a*) Verify that 2 is the mean of the data

x	0	2	3	4
frequency	4	3	2	3

   and calculate the standard deviation to one place of decimals. (see Tables p. 34).
   (*b*) The age distribution of all the people living in a random selection of houses is shown in the histogram.



   (0–5 means less than 5 years, 5–10 means 5 years and over but less than 10 years etc.)
   If there are 40 people in the 20–30 age group, find
   (i) the number of people in the 5–10 age group
   (ii) the total number of people in the houses
   (iii) the mean age of the people (assuming that the age of every person in a class interval is at the mid-point of that interval i.e. take 25 as the age of every person in the 20–30 class interval.)

5. (a) Illustrate the following by a diagram in each case:
   (i) function
   (ii) injection that is not a surjection
   (iii) surjection that is not an injection.
   Give a reason why the function

$$f : R \to R : x \to 3x + 4$$

is (i) injective (ii) surjective.
Write down its inverse function, f^{-1}, in the form

$$f^{-1} : R \to R : x \to \ldots$$

Investigate if $f^{-1}(u + v) = f^{-1}(u) + f^{-1}(v)$.

(b) If $f(x) = x^3 - 6x^2 + 15x - 14$, verify that $f(2) = 0$ and find the three roots of the equation $f(x) = 0$.

6. (a) Let S be the relation defined by the set of four couples $\{(a, b), (a, c), (c, c), (b, c)\}$. Say, giving a reason, whether or not S is transitive.

(b) Let two functions f and g be defined by

$$f : R_0 \to R : x \to \frac{x-1}{x} \text{ where } R_0 = R \setminus \{0\}$$

$$g : R \to R : x \to x^2 + 1.$$

Write down the values of
(i) $f(1)$ (ii) $g(1)$ (iii) $f[g(1)]$ (iv) $g[f(1)]$
Express the composite function fg (i.e. $f \circ g$,) in the form $x \to \ldots.$ and verify that $fg \neq gf$.

(c) If $x \in \mathbf{R}$, find the set of values of x for which

$$15 + x - 6x^2 \geqslant 0.$$

7. Draw the graph of the function

$$f : x \to x^3 - 2x^2 - 5x + 6$$

in the domain $-2 \leqslant x \leqslant 3$ for $x \in \mathbf{R}$.

(a) Use your graph to find the set of x for which
   (i) $-2 \leqslant f(x) \leqslant 2$
   (ii) $f(x)$ is negative and decreasing as x increases.

(b) Express $x^3 - 2x^2 - 4x + 6$ in the form $f(x) + g(x)$ and hence use your graph to solve

$$x^3 - 2x^2 - 4x + 6 = 0.$$

8. (a) Verify that ${}^7C_3 = {}^7C_4$ and write down the six couples (x, y) for which ${}^5C_x = {}^5C_y$.

(b) Write out the first four terms of $(2x - y)^7$ in order of increasing powers of y and letting $x = \frac{1}{2}$ use the expansion, with an appropriate value for y, to calculate $(0{\cdot}99)^7$, retaining four significant figures in the answer.

(c) Find the value of x in each of the following:
   (i) $\log_{\frac{1}{3}}\left(\frac{1}{27}\right) = x$ (ii) $\log_x 2 = \frac{1}{3}$ (iii) $\log_4 \frac{x-1}{x+1} = \frac{1}{2}$.

9. (a) Differentiate from first principles

$$3x^2 + 2$$

with respect to x.

(b) If $x \to u(x)$ and $x \to v(x)$ are two functions, prove the product rule for the derivative of $u(x) \,.\, v(x)$.
Find the derivative of $(3x^2 + 2)(1 - x + x^2)$.

(c) An object moves so that its distance s, in metres, from a fixed point after t seconds is given by

$$s = 4t(8 - t).$$

How far is the object from the fixed point when it stops moving?

10. (a) Evaluate
   (i) $\int_0^2 \frac{dx}{2}$ (ii) $\int_{-1}^{1} (x^2 + 3)\, dx$ (iii) $\int_0^1 (x - 3)(x + 1)\, dx$.

(b) The rate of increase in the surface area of a pool during steady rain is given by

$$\frac{dA}{dt} = 5t + 4,$$

where A is the area of the pool after t hours.
Find the increase in the area of the pool
   (i) over the first 4-hour period
   (ii) in the 4th hour.