AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1976

MATHEMATICS—ORDINARY LEVEL—PAPER I
(300 marks)

THURSDAY, 10 JUNE—MORNING, 9.30 to 12

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.

1. A solid cone is 16 cm in height and the diameter of its base is 8 cm. The cone is completely submerged in water in a cylindrical vessel of internal diameter 12 cm.
   Calculate the drop in depth of the water in the vessel when the cone is taken out.
   A solid sphere is then completely submerged in the same cylindrical vessel and the water rises to the same level as before. Find the radius of the sphere.

2. The line $y = 2x - 2$ cuts the y-axis at a and cuts the line $y = x + 1$ at b. Find
   (i) the coordinates of a and b
   (ii) the coordinates of the images of a and b under S_Y (reflection in the y-axis).
   Is the image of the line ab under S_Y perpendicular to the line $y = x + 1$? Give a reason for your answer.

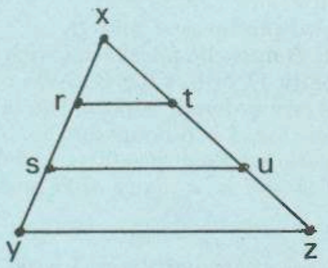
3A. K is the circle $x^2 + y^2 = 13$ and b is the point (2, 3) and o is the origin.
   (i) Verify that $b \in K$.
   (ii) Find the slope of the tangent to the circle at b and show that the tangent at b intersects the x-axis at $p(6{\cdot}5, 0)$.
   (iii) Find which is the greater: the area of the $\triangle obp$ or one quarter of the area of the circle.

OR

3B. Show, with proof, how to inscribe a regular pentagon in a given circle. (Use of the protractor is not permitted).

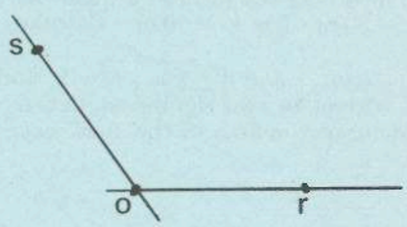
4. abc and def are two similar triangles in which $|\angle bac| = |\angle edf|$ and $|\angle abc| = |\angle def|$.
   Prove that

$$\frac{\text{area of } \triangle abc}{\text{area of } \triangle def} = \frac{|bc|^2}{|ef|^2}$$



   xyz is a triangle, as in diagram, and r, s, u, t are such that $rt \parallel su \parallel yz$ and $|xr| = |rs| = |sy|$.
   Calculate the ratios
   (i) area of $\triangle xrt$: area of $\triangle xsu$
   (ii) area of $rsut$: area of $syzu$.

5. (*a*) The diagram shows three points r, o, s. Copy the diagram into your answer book and show clearly the vectors



   (i) $\vec{or} + \vec{os}$
   (ii) $\vec{os} - 3\vec{or}$

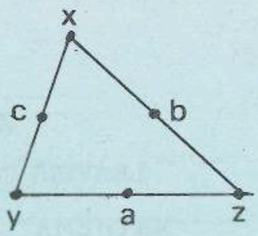
   If t is the midpoint of [rs], express $\vec{ot}$ in terms of $\vec{or}$, $\vec{os}$.

   (*b*) $\vec{i}$ and $\vec{j}$ are unit vectors along the x-axis and y-axis, respectively. When $\vec{p} = 6\vec{i} + 2\vec{j}$ and $\vec{q} = 2\vec{i} + 4\vec{j}$, find the vector $\vec{k}$ in terms of $\vec{i}$ and $\vec{j}$ such that $\vec{p} + \vec{k} = \vec{q}$.
   Indicate the vectors $\vec{p}$, $\vec{q}$, $\vec{k}$ on a diagram.
   Indicate on your diagram the vectors $u\vec{p}$ and $v\vec{q}$, where u, v are scalars if

$$u\vec{p} + v\vec{q} = 3\vec{i} + 3\vec{j}$$

[P.T.O.

6. Prove that the composition of two central symmetries is a translation. $pqrs$ is a parallelogram. Show that $S_q \circ S_p = S_r \circ S_s$.
Hence show that the composition of three central symmetries is a central symmetry.
xyz is a triangle and a, b, c are the midpoints of $[yz]$, $[zx]$ and $[xy]$, respectively.
Where is the point d such that $S_c \circ S_b \circ S_a = S_d$?
Construct the image of the triangle xyz by $S_c \circ S_b \circ S_a$.

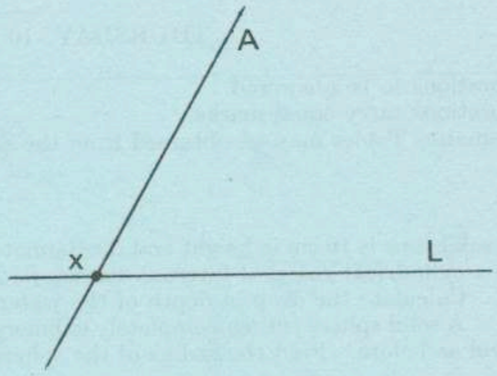


7. A and L are two lines intersecting at x. f is the projection of the plane Π on L parallel to A and g is the projection of the plane Π on A parallel to L.

(i) What is the domain and range of each of the functions f and g?

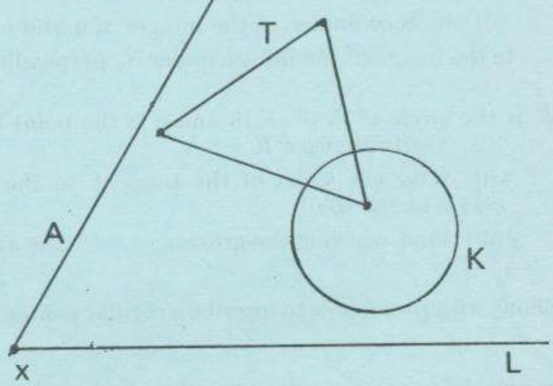
(ii) Is $g \circ f = f \circ g$? Give a reason.

(iii) Is $g \circ f$ a parallel projection? Give a reason.



T is the set of points of the three sides of the triangle; K is the circle (i.e. the set of points of the boundary), as in the diagram.

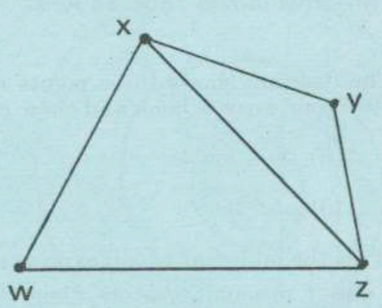
Construct $f(T)$ and $f(K)$ and say, giving a reason, whether or not $f(T \cap K) = f(T) \cap f(K)$.



8. (i) Graph the inequality $5x + 6y \leqslant 330$.
(ii) A factory makes two products A and B.
If A is bought, B must be purchased with it. B can be bought on its own.
There are already 11 orders for B on its own.
If x units of A are ordered, what is the minimum number of B that must be produced?
Production time for A is 5 hours and for B is 6 hours. In a given work period, 330 hours are available for production. Each product A yields £6 profit while each product B yields £7 profit.
How many of A and how many of B must be produced to achieve a maximum profit?

9. A level field is in the form of a quadrilateral $xyzw$. $|wx| = 50$m, $|wz| = 80$ m, $|xz| = 70$ m. Calculate $|\angle xwz|$.

If $|\angle yxz|$ and $|\angle yzx|$ are 10° and 20° respectively, calculate $|xy|$ correct to two significant figures.
Calculate the area of the field $wxyz$ correct to two significant figures.



10. State the period and range (image) of each of the functions defined for $x \in \mathbf{R}$:

(i) $x \to 2 \cos x$

(ii) $x \to -\cos x$

(iii) $x \to \sin \frac{x}{2}$.

If the periodic function $x \to a \sin bx$ has a period $\pi/2$ and a range $[-5, 5]$, find the value of a and the value of b.

(*b*) Using the same axes and the same scales, sketch the functions $x \to -\cos x$ and $x \to \sin \frac{x}{2}$ in the domain $-2\pi \leqslant x \leqslant 2\pi$.

Use these graphs to find the values of x for which

$$\sin \frac{x}{2} + \cos x = 0$$

in the given domain.