PAPER II (300 marks)

1. Find, to the nearest penny, the compound interest on £540 for 3 years at 5% per annum. Find, to the nearest pound, the sum of money which would amount to £500 in 6 years at 5% per annum, compound interest.

2. (i) Given that the n^{th} term of a sequence is denoted by T_n write down T_1, T_2, and T_{n-2}, where $T_n = 2 - n$ Find the sum of the first 20 terms of the sequence.
   (ii) Nine small stones are placed in a row. Each stone is 10 metres from the next. A boy collects the stones together one by one at the middle stone in the row. Find the distance travelled by the boy in collecting the stones beginning at the middle stone, and show that this distance is less than the distance which would be travelled in collecting the stones together one by one at any other stone in the row, assuming that collection begins at that other stone.

3. (a) Find the real part and the imaginary part of the number $(2 - 3i)(-6 + i)$.
   (b) If $Z_1 = 2 - 3i$ and $Z_2 = -6 + i$ prove
   $$|Z_1, + Z_2| < |Z_1| + |Z_2|.$$
   On an Argand diagram represent Z_1, Z_2, $-Z_1$, $-Z_2$, $Z_1 + Z_2$ and $-(Z_1 + Z_2)$.

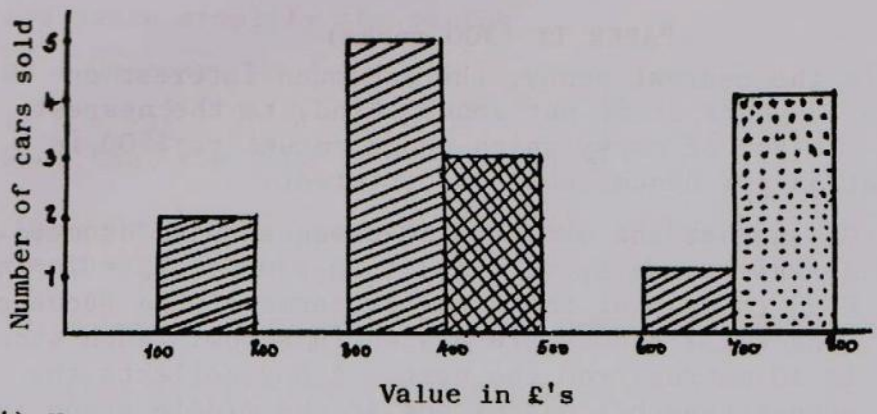
4 A.(i) State the properties of a group.
   (ii) Show that $(\{O,E\}, +)$ is a group under a suitable operation of addition, where O is the set of odd numbers and E is the set of even numbers.
   Can $\{O,E\}$, be made a group under multiplication? Give a reason.
   (You may assume that both addition and multiplication in the set of integers are associative).
   (iii) Show that the set of reflections of an equilateral triangle in its axes of symmetry is not a group under the usual operation of composition.

OR

4 B.What is the mean of the following array of numbers:

2, 3, 9, 10, 10, 13, 16.

Compute the standard deviation. (Give your answer correct to 1 place of decimals). The graph shows the number of cars, classified according to price range, sold by a garage salesman in one month:



(i) How many cars were sold?

(ii) Assuming that the value of a car in any price interval is at the midpoint of that interval estimate the total commission earned by the salesman on these sales if he is paid $2\frac{1}{2}\%$ commission on sales.

5. (a) $A = \{2,3,4\}$.
   Write down the couples of the relation R on A determined by the rule $\leqslant$, and say whether R is reflexive, symmetric, antisymmetric, transitive.

   (b) $B = \{0, 1,2,3,4\}$
   R is the relation $\{(x,y) \mid x + y = 4,\ x \in B,\ y \in B\}$
      (i) Make a graph of the relation.
      (ii) Is R a function?
      (iii) Is R^{-1} a function?
      (iv) Write down the couples of $R \circ R^{-1}$.

6. f and g are functions defined as follows
   $f : x \to x^2 = f(x)$
   $g : x \to x + 3 = g(x)$
   (i) What number is each of these: $f(4)$, $g(2)$, $f(g(0))$?
   (ii) Show that $f(g(x)) = x^2 + 6x + 9$.
   (iii) Solve $g(f(x)) = 0$.
   (iv) For what value of x is $f(g(x))$ a minimum? Compute this minimum.

7. Show that $x = i\sqrt{2}$ is a solution of the equation $x^3 - 3x^2 + 2x - 6 = 0$, and find the other solutions..
   Graph the function $f : x \to 2 + 4x - x^3 = f(x)$ in the domain $-2 \leqslant x \leqslant 3$.
   Find from the graph the values of x in the given domain for which y is negative and increasing.

8. (a) Use the binomial theorem to expand $(1 + \sqrt{2})^6$.
Which terms in the expansion are rational?
Find the value of $(0\cdot999)^6$ to 4 significant figures.

(b) Find the smallest positive integer n for which $1 - \frac{1}{n^2}$ differs from 1 by less than one millionth.

9. Differentiate from first principles with respect to x the function $f(x) = x^3$, $x \varepsilon R$.
Compute the rate of change of the function at the point $x = 0\cdot3$.
Evaluate (i) $\int_1^{1\cdot5} x\,dx$ (ii) $\int_{-1}^{+1} (3t^3 - 2t^2 - 5)\,dt$.

10. (a) Differentiate with respect to x

(i) $3x^3 - 4x^2 + 4$,

(ii) $\frac{x}{1-x}$.

(b) A trench is being dug by a group of workmen who remove x cubic feet of soil in t minutes, where $x = 12t - \frac{1}{30}t^2$. At what rate is the soil being removed from the trench at the end of the first hour?