AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1965

MATHEMATICS—ALGEBRA—PASS

WEDNESDAY, 23rd JUNE - Morning, 10 to 12.30

All questions to be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the simultaneous equations

$$x^2 = 2y^2 + y - 1,$$
$$y^2 = 2x^2 - 11y + 8.$$

(25 marks)

2. (i) If the expression $x^3 - 4x^2 + px + q$ is exactly divisible by $x^2 + x - 6$, find the value of p and the value of q and factorise the expression fully.

(ii) If $c = \frac{b - x}{b + 1}$ and $b = \frac{a + x}{a - 1}$, express c in terms of $\underline{a}$ and $\underline{x}$ and hence, or otherwise, evaluate x when $a = c = 2$.

(25 marks)

3. The sum of the first 8 terms of an arithmetic progression is 120 and the 8th term is 29. Find the first term, the common difference and the sum of the first n terms.

The next N terms of the arithmetic progression after the 8th term have a sum of 741. Find N.

Find also the term of the arithmetic progression which is nearest to 200.

(30 marks)

4. Two cars, A and B, set out to cover a distance of 240 miles. The average speed of car A was less by 45 m.p.h. than the average speed of car B and the time taken by car B to cover the distance was 2 hours 40 minutes less than the time taken by car A. Find the average speed of each car.

(30 marks)

5. (i) Show that the sum to n terms of the geometric progression a, ar, ar^2, ... is $\frac{a(1 - r^n)}{1 - r}$.

The sum of the first 4 terms of a geometric progression is $\frac{5}{8}$ and the sum of the first 8 terms is $\frac{85}{128}$. If the first term of the geometric progression is 1, find the common ratio.

(ii) The three whole numbers $(10 + t)$, $(18 + t)$, $(30 + t)$ are in geometric progression. Find t.

(30 marks)

6. (i) Prove $\log_a \frac{M}{N} = \log_a M - \log_a N$.

If $x = \log_t p - \log_t (p - qy)$, show that $y = \frac{p}{q}(1 - t^{-x})$.

(ii) Solve the equation

$$4^x - 3.2^{x+2} + 32 = 0.$$

(30 marks)

7. Draw a graph of the function $\frac{1}{2}(2x - 1)(2x + 1)(3 - 2x)$ for values of x from -1 to $+2$ paying special attention to the values $-\frac{1}{2}$, 0, $\frac{1}{2}$, $1\frac{1}{2}$ of x.

Use your graph to solve as accurately as you can the equations

(i) $(2x - 1)(2x + 1)(3 - 2x) = 12$,

(ii) $2x(1 + 6x - 4x^2) = 11$.

Find also for what values of x the expression $\frac{1}{2}(2x - 1)(2x + 1)(3 - 2x)$ takes values between -1 and $+1$.

(30 marks)