AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1963.

MATHEMATICS—GEOMETRY—PASS.

MONDAY, 10th JUNE—Morning, 10 to 12.30.

Six questions to be answered.

Mathematical Tables may be obtained from the Superintendent.

1. If a straight line is drawn parallel to one side of a triangle, prove that it cuts the other two sides proportionally.

Hence, prove that if two triangles are equiangular their corresponding sides are proportional.

(30 marks.)

2. Prove that the sum of the squares on two sides of a triangle is equal to twice the square on half the third side together with twice the square on the median which bisects the third side.

In a triangle PQR the base QR is fixed in position and magnitude and $PQ^2 + PR^2$ is constant. Show that the locus of P is a circle.

(30 marks.)

3. Show, with proof, how to construct a mean proportional between two given straight lines.

AB is a chord of a circle and Q is the foot of the perpendicular from O, the centre of the circle, to AB. The tangent at A to the circle meets OQ produced at P and OP cuts the circle at C (C is between Q and P). Prove that OC is a mean proportional between OQ and OP.

(30 marks.)

4. If the vertical angle of a triangle be bisected by a straight line which meets the base, prove that the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base together with the square on the straight line which bisects the vertical angle.

In a triangle ABC, the bisector of the angle BAC meets the base BC at D. If DB = 2", DC = 1", DA = $\sqrt{6}$", find the length of AB and the length of AC.

(35 marks.)

5. Prove that the areas of similar polygons are proportional to the squares on corresponding sides.

Given a pair of equilateral triangles show how to construct an equilateral triangle whose area is equal to the sum of the areas of the given triangles.

(35 marks.)

6. (i) Write down and prove the expansions of sin (A + B) and cos (A + B) where A, B, and (A + B) are acute.

(ii) Prove that $\sin 80^\circ = \sin 20^\circ + \cos 50^\circ$

(35 marks.)

7. In a triangle ABC the sides AB, BC, CA are 2, 5 and 4 inches, respectively, in length. Find the size of the angle BAC and the area of the triangle as accurately as possible by the use of tables.

(35 marks.)