Six questions to be answered. Mathematical Tables may be obtained from the Superintendent.

1. How many circles can be drawn to touch each of three given straight lines, no two of which are parallel? Show how to construct these circles and give proof in one case. [33 marks.]

2. Show how to construct a direct common tangent to two circles and give proof. Two circles touch externally at A, and a direct common tangent touches them at P, Q respectively. Prove that the angle PAQ is a right angle. [33 marks.]

3. Show, with proof, how to divide a given straight line internally in medial section, i.e., so that the square on one part may be equal to the rectangle contained by the whole line and the other part. [33 marks.]

4. Show, with proof, how to construct a mean proportional to two given straight lines. Hence show how to construct a straight line whose length shall be \( \sqrt{3.5} \) inches. [33 marks.]

5. ABC is a triangle and D is the foot of the perpendicular from the vertex A to the base BC. Prove that the rectangle contained by AB, AC is equal to the rectangle contained by AD and the diameter of the circumcircle of the triangle ABC. Hence, or otherwise, show that the radius of the circumcircle is equal to \( \frac{abc}{4A} \), where \( a, b, c \) represent the lengths of the sides and \( A \) represents the area of the triangle. [33 marks.]

Or,

5. Prove that the areas of similar triangles are proportional to the squares on corresponding sides. A straight line is drawn parallel to the base BC of a triangle ABC cutting the sides AB, AC at P, Q respectively. If the areas of the triangles APQ, ABC are in the ratio \( 4 : 25 \), find the ratio of AP to PB. [33 marks.]
6. Prove \( \cos (A + B) = \cos A \cos B - \sin A \sin B \), given that \( A \) and \( B \) are positive and \( (A + B) < 90^\circ \).

Hence show that \( \cos 75^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}} \) [34 marks.]

7. In a triangle \( ABC \), the angle \( A = 130^\circ \), \( BC = 5 \) and \( AB = 2 \). Find the size of the angle \( B \) and the length of \( CA \). [34 marks.]