Six questions to be attempted, of which not more than four may be selected from Section A.
All questions are of equal value.
Mathematical Tables may be obtained from the Superintendent.

Section A.

1. From a point $P$, outside a circle, a tangent $PT$ and a secant $PBA$ are drawn. Prove that the angle $PTB$ is equal to the angle $PAT$. A straight line is drawn parallel to $PT$ meeting $TA$ and $TB$ at $X$, $Y$ respectively; prove that $XYBA$ is a cyclic quadrilateral.

2. Prove that equiangular triangles are similar.
   $ABC$ is a triangle inscribed in a circle. The bisector of the angle $A$ meets the base in $D$ and meets the circumference of the circle in $E$. Show that the triangles $ADB$, $AEC$ are similar and hence prove that:
   \[ AB \cdot AC = BD \cdot DC + AD^2. \]

3. $ABC$, $DEF$ are two similar triangles, having $BC$, $EF$ as corresponding sides. Prove that:
   \[ \triangle ABC : \triangle DEF :: BC^2 : EF^2. \]
   Deduce the corresponding theorem in the case of two similar quadrilaterals.

4. If a straight line is drawn parallel to one side of a triangle, prove that it divides the other two sides proportionally.
   Prove that three parallel straight lines cut any two intersecting straight lines proportionally.

5. Show how to construct a circle touching one side of a triangle and touching the other two sides produced.
   If the triangle is equilateral and the length of each side is $a$, prove that $4r^2 = 3a^2$, where $r$ is the radius of the above circle.

6. $ABC$ is a triangle having the base $BC$ fixed in magnitude and position. If the angle $A$ is fixed in magnitude, what is the locus of the point $A$?
   If the bisector of the angle $A$ meets the circumference of the circle circumscribed about $ABC$ at the point $X$, show that $IX = XC$, where $I$ is the centre of the circle inscribed in $ABC$. Hence deduce the locus of $I$ as $A$ varies in position.
7. From the top of a tower, 155 ft. high, the angles of depression of two points which are in the same vertical plane as the observer are 28° 16' and 44° 19'. If the two points are in the horizontal plane passing through the foot of the tower, calculate their distance apart.

8. Using the formulae for \( \sin(A+B) \) and \( \cos(A+B) \), show that
   (i) \( \sin 2A = 2 \sin A \cos A \),
   (ii) \( \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 \).

Deduce that
   \( \sin 3A = 3 \sin A - 4 \sin^3 A \),
   \( \cos 3A = 4 \cos^3 A - 3 \cos A \).

9. With the usual notation, show that the area of a triangle \( ABC \)
   is equal to \( \frac{1}{2}ab \sin C \).

   A regular pentagon is inscribed in a circle of radius 5 cm. Using the Tables, calculate (i) the area, (ii) the perimeter, of the pentagon.

10. Make out a table of values of \( 3 \sin x + 4 \cos x \) when \( x \) has the values
    \( 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 135^\circ, 150^\circ, 180^\circ \).

    Hence draw the graph of
    \( y = 3 \sin x + 4 \cos x \).

    From your graph,
    (i) find the maximum value of \( 3 \sin x + 4 \cos x \),
    (ii) solve the equation \( 3 \sin x + 4 \cos x = 2 \).