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(Department of Education).

## LEAVING CERTIFICATE EXAMINATION, 1944.

## MATHEMATICS.—GEOMETRY.—PASS.

*TUESDAY*, 13*th* *JUNE*.—AFTERNOON, 3 TO 5.30.

Six questions to be attempted ; not more than four out of Section A.

All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

### SECTION A.

1. Show that the sum of a pair of opposite angles of a cyclic quadrilateral is equal to two right angles.

Prove that the internal bisectors of the angles of a quadrilateral form a cyclic quadrilateral.

2. Show how to construct a regular pentagon in each of the following cases :

(i) Given the circum-circle,

(ii) Given a diagonal in length and position. (Proof not needed in this case.)

3. What is the locus of the centres of circles of given radius which touch a given circle (i) internally, (ii) externally.

Give, without proof, the construction for drawing a circle of given radius to touch a given circle and having its centre on a given straight line. Show that there may be four, three, two, one or no solution.

4. Prove that the internal bisector of the vertical angle of a triangle divides the base internally in the ratio of the other two sides of the triangle.

If the sides of a triangle ABC be denoted by $a$, $b$, $c$ and if AD be the internal bisector of the angle A, prove that

$$BD=ab/(b+c), \quad CD=ac/(b+c).$$

5. If corresponding sides of two similar figures are of lengths $a$ and $b$, state the ratio of their areas in terms of $a$ and $b$.

Equilateral triangles are described on the sides of a right-angled triangle. Prove that the triangle on the hypotenuse is equal to the sum of the other two.

How would you describe an equilateral triangle equal in area to the sum of two given equilateral triangles ?

6. ABC is a triangle inscribed in a circle. AD is the perpendicular from A on BC and AE is the diameter passing through A. Prove that the triangles ABE, ADC are similar and hence show that

$$AB.AC=AE.AD.$$

If ABC is an equilateral triangle of side $a$, prove that $a^2=3R^2$, where R is the radius of the circum-circle.

## Section B.

7. (*a*) If $x=3\cos\theta-4\sin\theta$, $y=12\cos\theta+5\sin\theta$, find $\cos\theta$ and $\sin\theta$ in terms of $x$ and $y$, hence, or otherwise, show that

$$169x^2-32xy+25y^2=3969.$$

(*b*) If $x\cos\theta+y\sin\theta=1$, $y\cos\theta-x\sin\theta=2$, deduce a relation between $x$ and $y$ which does not contain $\theta$.

8. Define angle of elevation and angle of depression.

A man standing on a level plain observes the elevation of the top of a mountain to be 42°. He then walks 356 yards towards the mountain and finds that the elevation is 61°. Calculate the height of the mountain.

9. Prove that

$$\cos(A+B)=\cos A\cos B-\sin A\sin B,$$

in the case in which A, B and (A+B) are acute angles.

Hence show that $\cos 75°=\frac{1}{4}(\sqrt{6}-\sqrt{2})$.

Write down the corresponding expression for sin (A+B) and deduce the value of sin 75° in a form containing surds.

10. In a triangle ABC, state the value of cosA in terms of $a$, $b$, $c$.

Hence, using $\sin^2A=1-\cos^2A=(1+\cos A)(1-\cos A)$, prove that

$$\sin^2A=\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}.$$

If $2s=a+b+c$, deduce that

$$\sin A=\frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}.$$