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LEAVING CERTIFICATE EXAMINATION, 1931.

PASS.

MATHEMATICS (I).

THURSDAY, 11*th* *JUNE*.—MORNING, 10 A.M. TO 12.30 P.M.

Seven questions may be answered. 10 (*a*) or 10 (*b*) may be answered, but not both. All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the equations :

(i) $(x+3)^2 = 13\ (x+3) + 420.$

(ii) $\dfrac{1}{\sqrt{x-2}-\sqrt{2}} + \dfrac{1}{\sqrt{x-2}+\sqrt{2},} = \sqrt{x-2}$

and verify your solutions.

2. Find the condition that the roots of the equation $ax^2+bx+c=0$ are equal.

Find the values of m for which the equation $m^2x^2+(m^2+m)x+1=0$ has equal roots, and hence find these equal roots.

3. Find (i) the sum of the first n natural numbers, (ii) the sum of the second n natural numbers.

The side AB of a triangle ABC is divided into 100 equal parts, and from the points of section parallels to BC are drawn to meet AC. From the points of section of AC parallels to AB are drawn to meet BC. Find the number of points of intersection of all these parallels, which lie inside the triangle.

4. Prove that $\log_a mn = \log_a m + \log_a n$ and that $\log_a m^n = n\log_a m$.

Show that $x=2\frac{1}{4}$ satisfies the equation $x^y=y^x$ where $y\equiv x\sqrt{x}$.

5. Find the values of x and y which satisfy the equations $2(x^2+y^2)-(x+y)=9$ and $2xy=3$.

6. Express $(l^2+m^2+n^2)\ (x^2+y^2+1)-(lx+my+n)^2$ as the sum of

three squares, and hence find values of x, y that satisfy the equation $(l^2+m^2+n^2)(x^2+y^2+1)=(lx+my+n)^2$.

7. Show that if an expression is a common factor of two given expressions, it is also a factor of their sum or difference.

Hence or otherwise find the common and other factors of x^4-6x^2-7x-6 and x^4-8x^2-5x+6.

8. Show graphically or otherwise that the value of x which makes the expression ax^2+bx+c a maximum or a minimum lies midway between the values of x which satisfy the equation $ax^2+bx+c=k$, where k has any value.

Find the maximum value of $8-3x-2x^2$.

9. A number of men agreed to share equally in the purchase of a certain number of tickets in a sweepstake at 10/- per ticket. Six of them, however, withdrew, and the others calculated that by subscribing an additional £1 each they could buy 20 tickets more than was originally intended. If only five of the men had withdrawn, an additional 10/- each from the remainder would have procured 5 tickets more than was originally intended. Find the original number of men concerned and the number of tickets they proposed to purchase.

10 (a). Write out the expansion of $(1-x)^{-1}$. Hence or otherwise find the sum to infinity of the series $1+\frac{1}{3}+\frac{1}{3^2}+\cdot\ .\ .\ .\ .$

If $y=x+x^2+x^3+.\ .\ .$ to infinity, show that $x=y-y^2+y^3-y^4+$ . . . to infinity.

Or

10 (b). Factorize $a^2(b+c)+b^2(c+a)+c^2(a+b)+2abc$ and show that if the given expression is equal to O, then

$$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^n=\frac{1}{a^n+b^n+c^n}$$

when n is any odd integer.