Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2025

Mathematics

Paper 2

Higher Level

2 hours 30 minutes

300 marks

Examination Number

Date of Birth ☐☐/☐☐/☐☐

For example, 3rd February 2005 is entered as 03 02 05

Centre Stamp

For the Examiner only				
		Section	Question	Mark
		A	1	
			2	
Disallowed			3	
A			4	
B			5	
Total Disall.			6	
		B	7	
Cumulative Check			8	
Running Total			9	
− Total Disall.			10	
= Total		↔	Total	

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- **any five** questions from Section A – Concepts and Skills
- **any three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

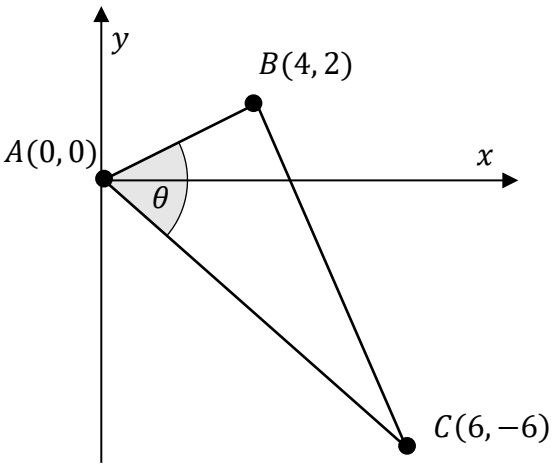
Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **any five questions** from this section.

Question 1 **(30 marks)**

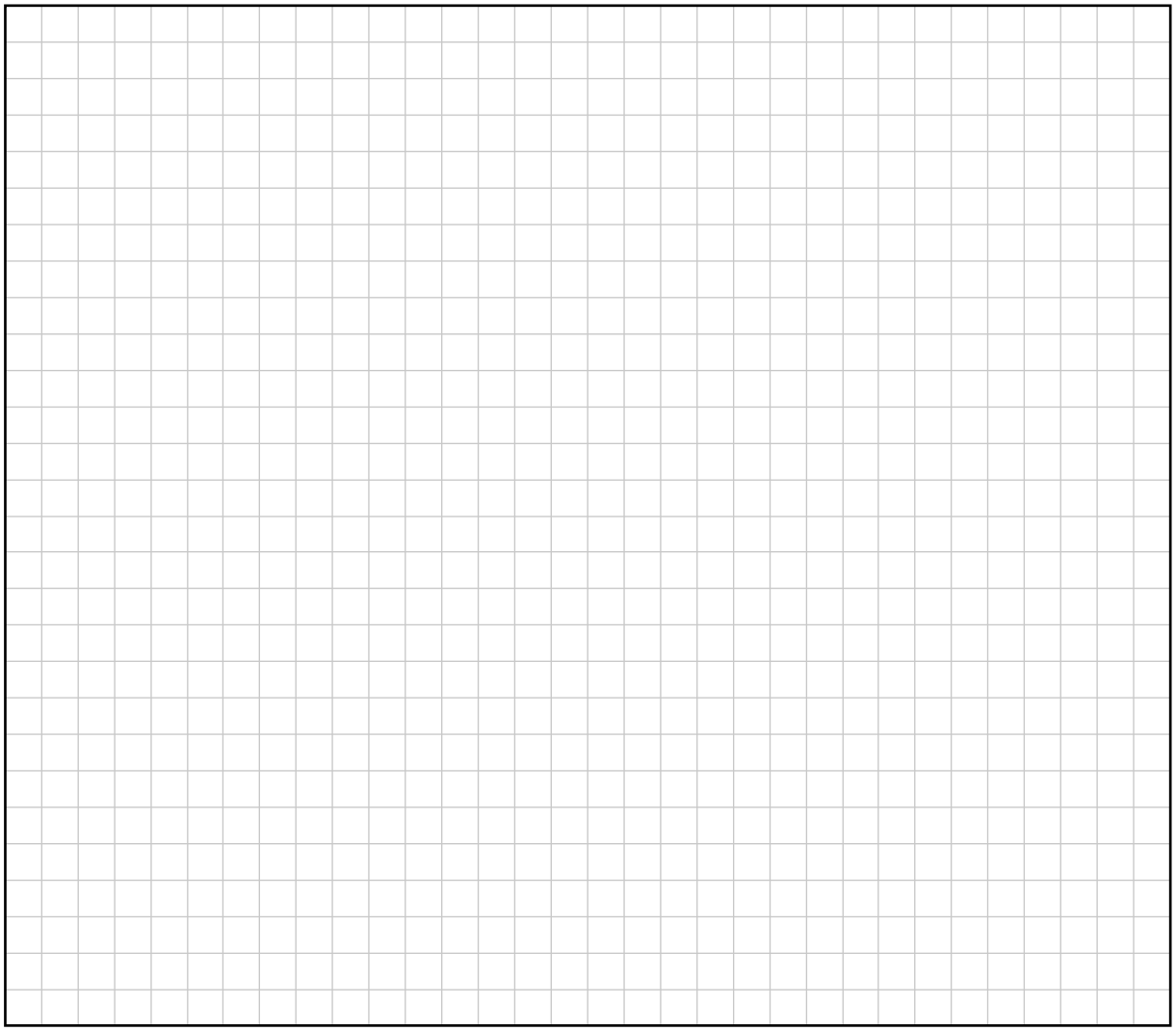
The three points $A(0,0), B(4,2)$, and $C(6,-6)$ form a triangle, as shown in the diagram below. θ is the acute angle between $[AB]$ and $[AC]$.



(a) **(i)** Find slope of the line AB **and** the slope of AC.

(ii) Hence, or otherwise, find the value of θ. Give your answer correct to 1 decimal place.

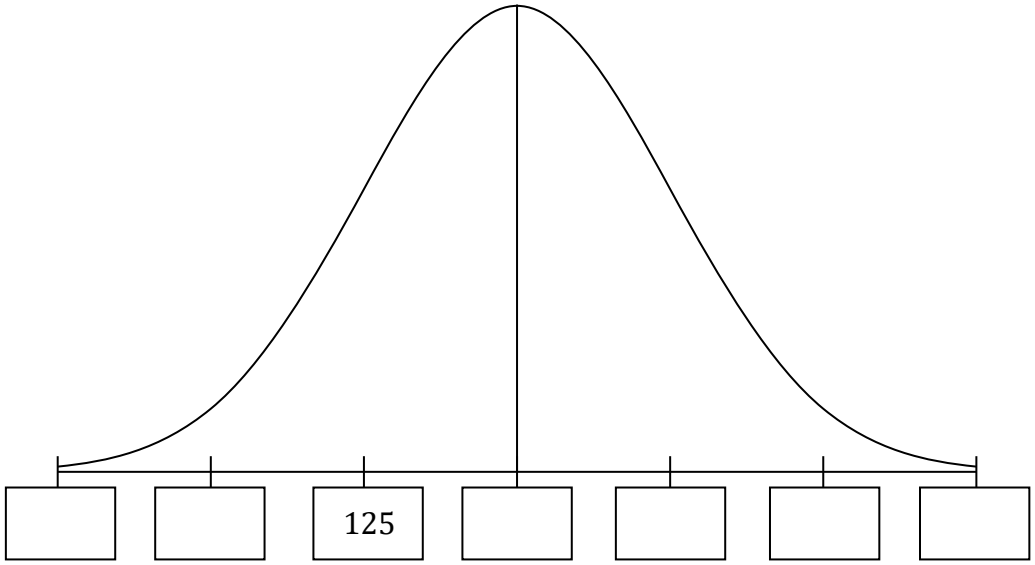
(b) Find the shortest distance from B to the line AC.
Give your answer in the form $p\sqrt{q}$, where $p, q \in \mathbb{N}$.



Question 2 **(30 marks)**

(a) In an exam, the marks awarded to $30\,000$ candidates were normally distributed, with a mean mark of 180 and a standard deviation of 55 marks.

The scaled diagram below shows the distribution of marks on the exam.



(i) Fill in the 6 missing numbers in the diagram above.

(ii) Using the empirical rule, or otherwise, find the **number** of candidates who were awarded a mark between 70 and 235.

(b) A spinner has exactly four different sectors.
In a game, the spinner is spun **once**.

The probability of landing on each sector is given in the table below.
The amount of money that you get for landing on each sector is also shown.

Sector	**A**	**B**	**C**	**D**
Probability	$\frac{3}{5}$	$\frac{1}{20}$	$\frac{1}{4}$	$\frac{1}{10}$
Amount of money	€0	€100	€10	€25

(i) Work out the **cost** to play this game, so that the expected value of the profit for someone playing the game would be $-$ €0·50.

(ii) Each spin of the spinner is independent.
The first time that it is spun, it lands on sector **C**.

The spinner is spun a second time.
Find the probability that it lands on sector **C** again, given that it landed on sector **C** the first time. Justify your answer.

Question 3 **(30 marks)**

A, B, and C are three events.

$P(A) = 0{\cdot}3,\ P(B) = 0{\cdot}15,$ and $P(A \cap B) = 0{\cdot}05.$

(a) **(i)** Find the value of $P(A \backslash B)$, that is, the probability that A happens but B does **not** happen.

(ii) Find the value of $P(A \cup B)$.

$P(C) = x$, where $x \in \mathbb{R}$, $0 \leq x \leq 1$.

(b) A and C are **mutually exclusive**.

(i) Write down the value of $P(A \cup C)$, in terms of x. Remember that $P(A) = 0{\cdot}3$.

(ii) Find the value of $P(A|C)$, the probability that A happens given that C has happened.

(c) $P(B \cap C) = 0{\cdot}08$ and $P(B|C) = \frac{1}{3}$.

Find the value of $x = P(C)$.

Question 4 **(30 marks)**

(a) $\cos A = \frac{1}{\sqrt{2}}$ and $\sin A = -\frac{1}{\sqrt{2}}$, where $0 \leq A \leq 2\pi$.

Find the value of A.

(b) $f(x) = \cos x - \sin x$, where $-2\pi \leq x \leq 2\pi$.

Find the value of $f\left(-\frac{\pi}{6}\right)$.

Give your answer correct to 4 decimal places.

(c) Solve the equation:

$$2\sin(2x) = \sqrt{3},$$

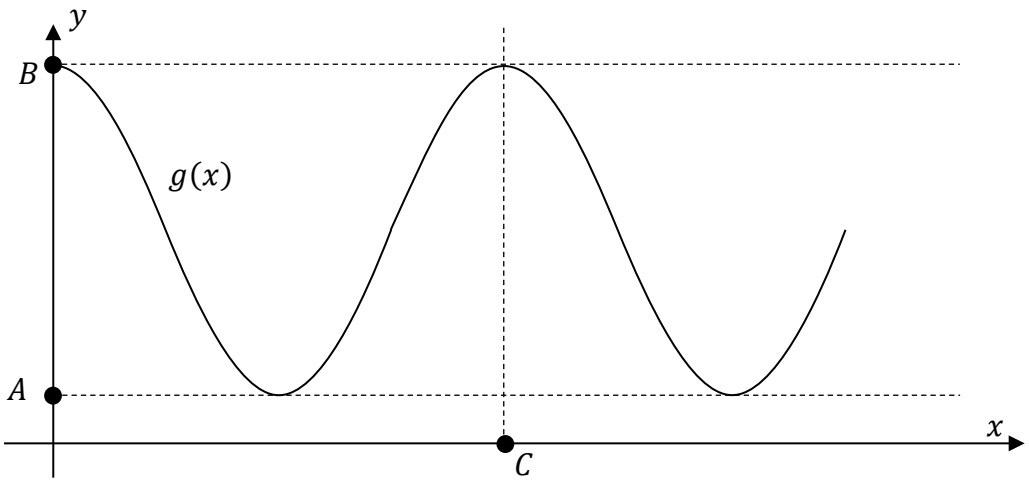
where $0 \leq x \leq 180°$.

(d) The function $g(x)$, where $x \in \mathbb{R}$, is defined as follows:

$$g(x) = 140 + 110\cos(3x)$$

Part of the graph of $y = g(x)$ is shown on the diagram below.

The horizontal lines $y = A$ and $y = B$ are shown, where $A, B \in \mathbb{N}$.
The vertical line $x = C$ is also shown, where $C \in \mathbb{R}$.



Find the value of A, the value of B, and the value of C. Write C in terms of π.

$A =$ _____ $B =$ _____ $C =$ _____

Question 5 **(30 marks)**

(a) Let ABC be a triangle. Prove that, if a line l is parallel to BC and cuts $[AB]$ in the ratio $s:t$, where $s, t \in \mathbb{N}$, then it also cuts $[AC]$ in the same ratio.

Diagram:

Given:

To Prove:

Construction:

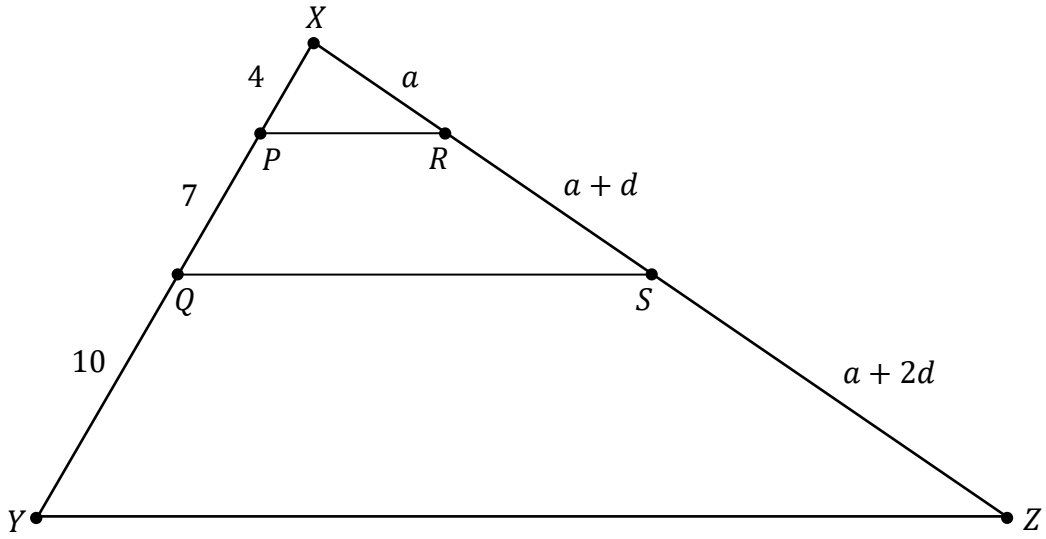
Proof:

(b) XYZ is a triangle.

The points P and Q lie on $[XY]$ and the points R and S lie on $[XZ]$.
The lines PR, QS, and YZ are all parallel.

As shown in the diagram, $|XP| = 4$, $|PQ| = 7$, and $|QY| = 10$.
$|XR| = a$, $|RS| = a + d$, and $|SZ| = a + 2d$, where a and d are positive constants.

a is **not** equal to 4.



Find one possible value of a ($a \neq 4$), **and** the corresponding value of d.

$a =$ _____ and $d =$ _____

Question 6 **(30 marks)**

(a) The equation of the circle s is $x^2 + y^2 - 4x + 2y - 20 = 0$.

(i) Write down the co-ordinates of the centre of s **and** the radius of s.

Centre: (,) Radius:

The equation of the circle t is $(x + 14)^2 + (y - 11)^2 = 225$.

(ii) Show that the circles s and t touch externally.

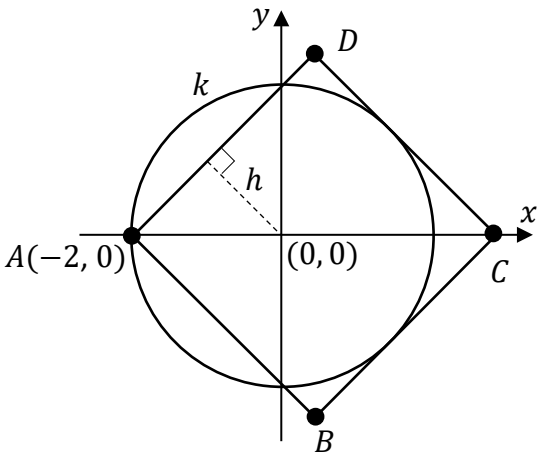
(b) The circle k has centre $(0, 0)$ and radius 2 units.

The point $A(-2, 0)$ is on the circle k.

$ABCD$ is a square.

The point C is on the x-axis, and BC and CD are tangents to k, as shown.

h is the perpendicular distance from $(0, 0)$ to AD.



(i) Find the value of h.

Give your answer in the form $\sqrt{a}$, where $a \in \mathbb{N}$.

(ii) Find the area of the square $ABCD$.

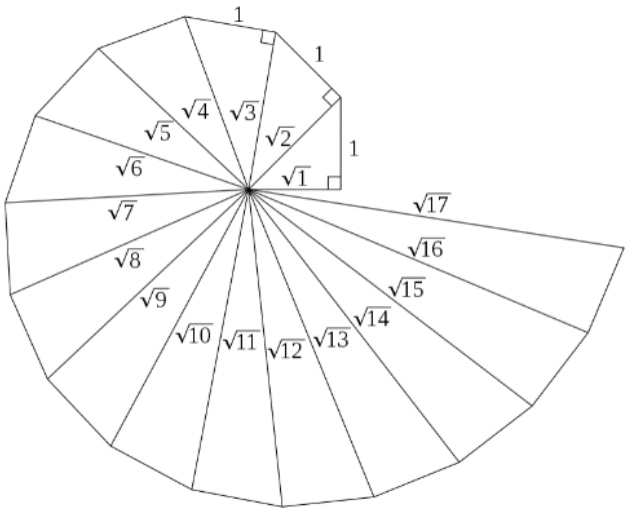
Give your answer in the form $p + q\sqrt{2}$ square units, where $p, q \in \mathbb{N}$.

Section B — Contexts and Applications — 150 marks

Answer **any three questions** from this section.

Question 7 **(50 marks)**

The first 16 triangles in a sequence of right-angled triangles are shown in the diagram on the right.



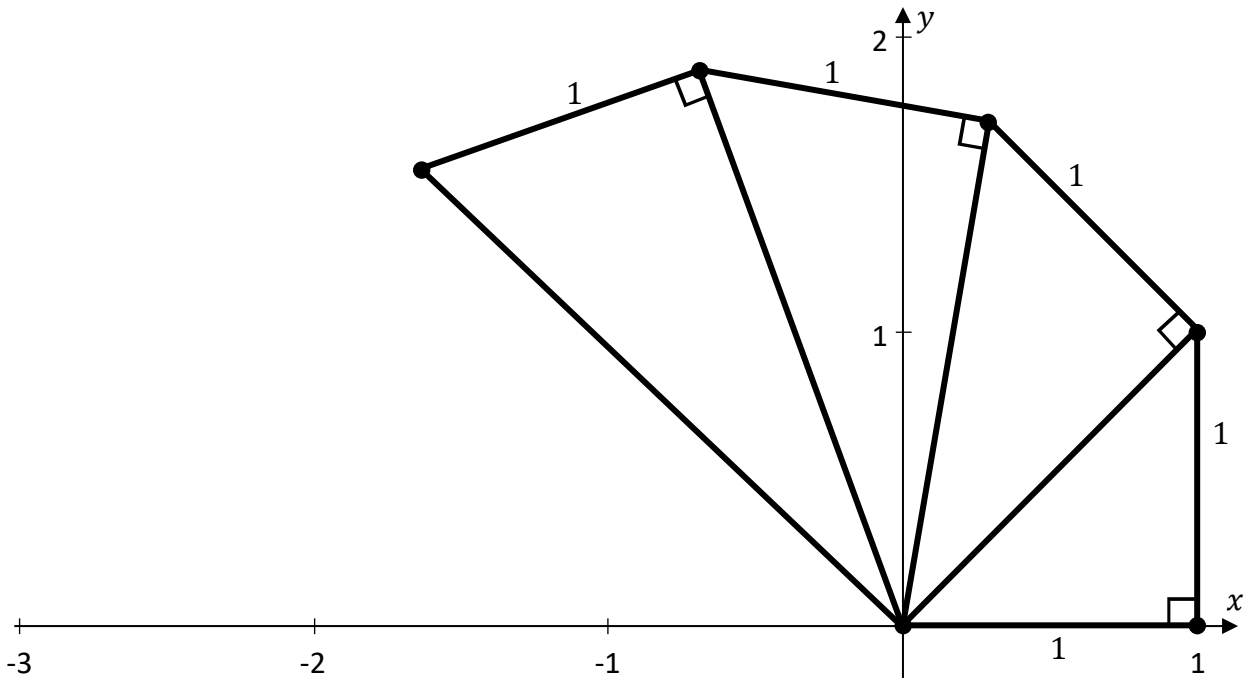
The first triangle in the sequence has sides of length 1, 1, and $\sqrt{2}$. The base line of any other triangle in the sequence is the hypotenuse of the previous triangle in the sequence.

The perpendicular height of each triangle is always 1 unit.

(a) The first four triangles in the sequence are shown on the co-ordinate axes below.

Construct the **next triangle** in the sequence on the diagram below.

Show clearly all your construction lines.
If you make any measurements, write them in an appropriate place on the diagram.



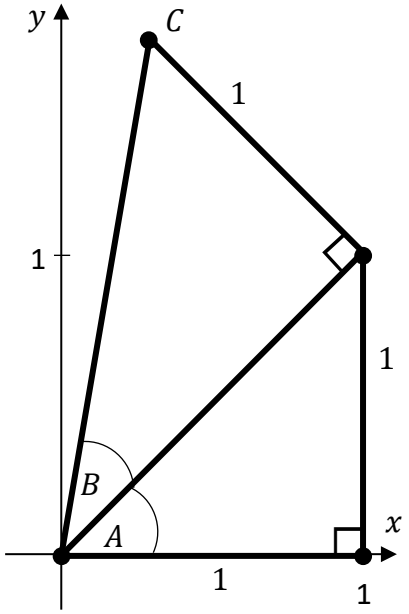
Source of top image: https://en.wikipedia.org/wiki/Spiral_of_Theodorus. Altered.

(b) **(i)** The diagram on the right shows the first two triangles in the sequence.

The angles A and B, and the point C, are marked on the diagram.

Find, in degrees, the sizes of the angles A **and** B, **and** the size of the angle $A + B$.

Give your second and third answers correct to 2 decimal places.

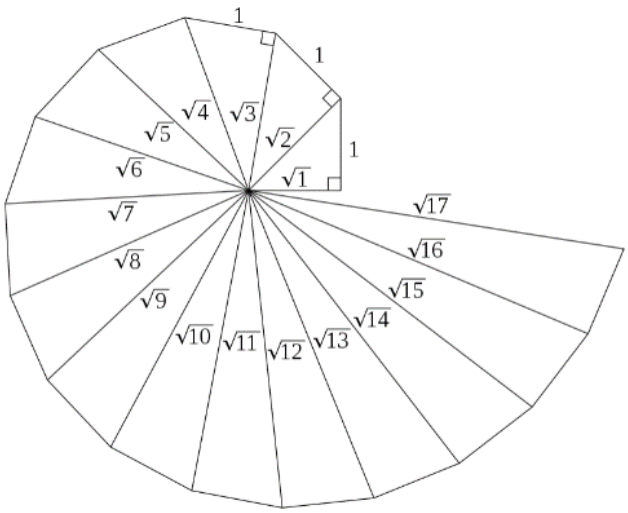


$A =$ $B =$ $A + B =$

(ii) Hence, or otherwise, find the y **co-ordinate** of the point C, correct to 2 decimal places.

This question continues on the next page.

The same diagram of the first 16 triangles in the sequence is shown on the right.



(c) Are any two triangles in the sequence **similar**?

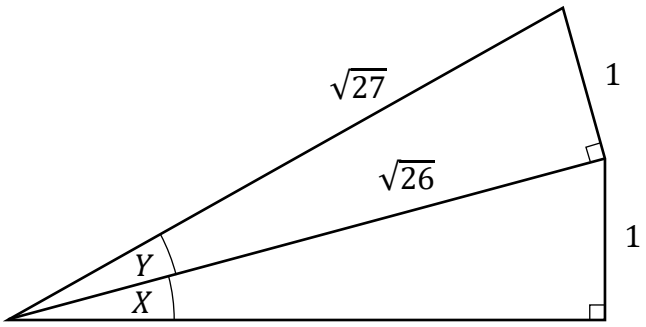
Justify your answer.

Answer (Yes or No):
Justification:

(d) The k th triangle in the sequence has a hypotenuse of length $5\sqrt{41}$, where $k \in \mathbb{N}$.

Find the value of k.

(e) Two consecutive triangles in the sequence have hypotenuses of length $\sqrt{26}$ and $\sqrt{27}$, respectively, as shown in the diagram below. Two of the angles in the diagram are marked X and Y.



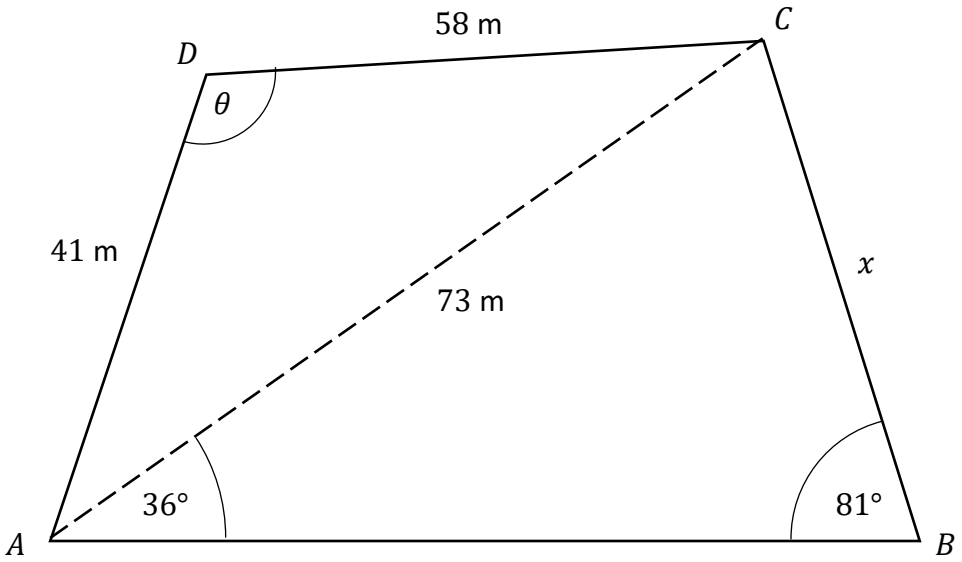
Find the value of $\sin(X + Y)$, **without** using a calculator. Show all of your working out.

Give your answer in the form $\frac{r+\sqrt{t}}{\sqrt{u}}$ where $r, t, u \in \mathbb{N}$.

Question 8 **(50 marks)**

(a) Michelle has a field in the shape of a quadrilateral $ABCD$.

$|AC| = 73$ m, $|AD| = 41$ m, $|DC| = 58$ m, $|\angle CAB| = 36°$, and $|\angle CBA| = 81°$.
$|\angle CDA| = \theta$ and $|BC| = x$.



(i) Use the triangle ABC to find the value of x, correct to 1 decimal place.

(ii) Find the value of θ, correct to 1 decimal place.

(iii) Michelle is putting topsoil on the field.
The topsoil will cover the whole field, and will have a uniform height of 20 cm.

Work out the volume of topsoil that Michelle will need, to cover the whole field.
Give your answer correct to the nearest m^3.

This question continues on the next page.

A company makes footballs of different sizes.

(b) The surface of one football is made up of 20 hexagons and 12 pentagons, each with sides of length L cm, where $L \in \mathbb{R}$.

In cm^2, the area of each hexagon is given by $\frac{3\sqrt{3}}{2}L^2$ and the area of each pentagon is given by $\frac{L^2}{4}\sqrt{5(5+2\sqrt{5})}$.

(i) Use these formulas to write the total surface area of this type of football in terms of L.

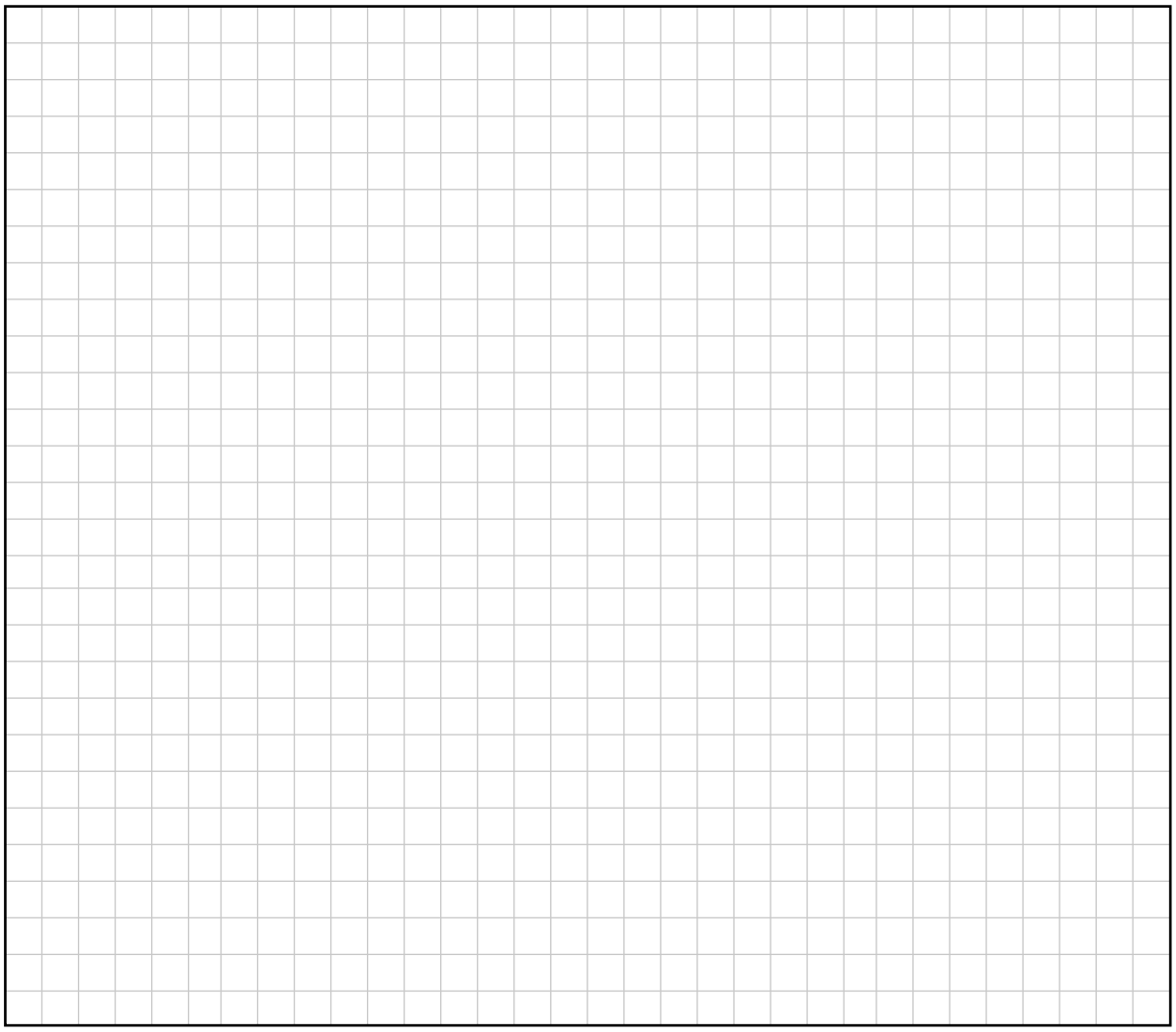
(ii) When it is inflated, this football is in the shape of a sphere with a diameter of 22 cm. Use your answer to part **(b)(i)** to find the value of L, correct to 1 decimal place.

(c) Some of the other footballs that the company makes are in packaging that is in the shape of a cube. These come in two sizes: small and large.

The surface area of a large cube is n times that of a small cube, where $n \in \mathbb{R}$.

The volume of a large cube is k times that of a small cube, where $k \in \mathbb{R}$.

Use this to write k in terms of n.



Question 9 **(50 marks)**

(a) In a large constituency with rural and urban voters, only two candidates, Alayah and David, stood for election.

After the election, a local radio phone-in show released the table below.
It was based on all of the callers who phoned in. All of these callers had voted.

For example, 10% of the callers are rural voters who voted for Alayah.

	Voted for Alayah	**Voted for David**	**Total**
Rural Voter	10%		18%
Urban Voter			
Total	61·8%		100%

(i) Complete the table.

Use the data from the table in answering parts **a(ii)**, **a(iii),** and **a(iv)**.

(ii) A caller is selected at random.
Find the probability that this caller voted for David **or** is an urban voter.

(iii) A caller is selected at random from the callers who voted for Alayah.

Find the probability that the caller is an urban voter.
Give your answer correct to 4 decimal places.

(iv) A caller is selected at random.

Let **P** be the event that the caller voted for Alayah.
Let **Q** be the event that the caller is an urban voter.

Are the events **P** and **Q** independent?
Use calculations to support your answer.

Calculations:

Answer:

This question continues on the next page.

(b) A group of 9 friends are planning a trip abroad to an Ireland hockey match.

They buy nine match tickets.
All nine tickets are in the same row and the row has exactly nine seats.

(i) How many different seating arrangements of the 9 friends are possible?

(ii) Find the number of seating arrangements in which three of the friends – Máire, Jake, and Alex – are sitting together.

(c) In the match, Ireland take 6 penalty corners.

The probability that Ireland score from a penalty corner is roughly $0{\cdot}35$.
Assume that the outcome of each penalty corner is independent.

(i) Find the probability that Ireland score from **none** of the penalty corners in the match.
Give your answer correct to 4 decimal places.

(ii) The probability that Ireland score **at least n** of their penalty corners is 12%, correct to the nearest percent.

Find the value of $n \in \mathbb{N}$.
Show all your working out.

$n =$ _____

Question 10 **(50 marks)**

Mark is analysing a large data set on driving times on the N22 from 2022.
The time taken to drive from Killarney to Cork during this period was normally distributed.
The mean was 78 minutes and the standard deviation was 6 minutes.

(a) **(i)** Find the **percentage** of drivers who completed the journey in 82 minutes or less.

(ii) Exactly 50% of the drivers took between 70 minutes and k minutes to complete the journey, where $k \in \mathbb{R}$.

Find the value of k, correct to 1 decimal place.

(b) In 2023 major road works were completed on the N22 between Killarney and Cork.
Afterwards, Mark decides to carry out a new survey to see if the driving times have changed.

He randomly selects 500 drivers to participate in the survey.
For this sample the mean driving time was 71 minutes.
Mark assumes the population standard deviation is still 6 minutes.

Carry out a hypothesis test at the 5% level of significance to see if there has been a change in driving times after the road improvements.
State your null hypothesis and your alternative hypothesis, state your conclusion, and give a reason for your conclusion.

Null Hypothesis:

Alternative Hypothesis:

Calculations:

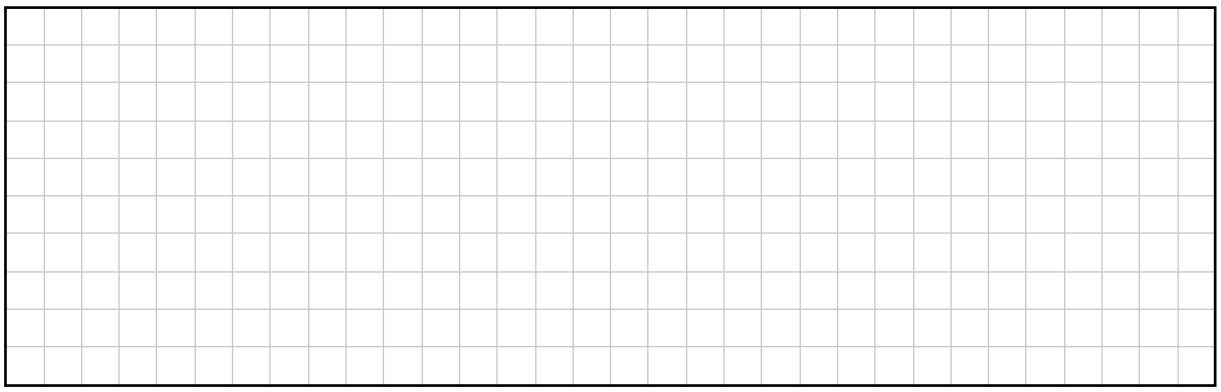
Conclusion:

Reason for your conclusion:

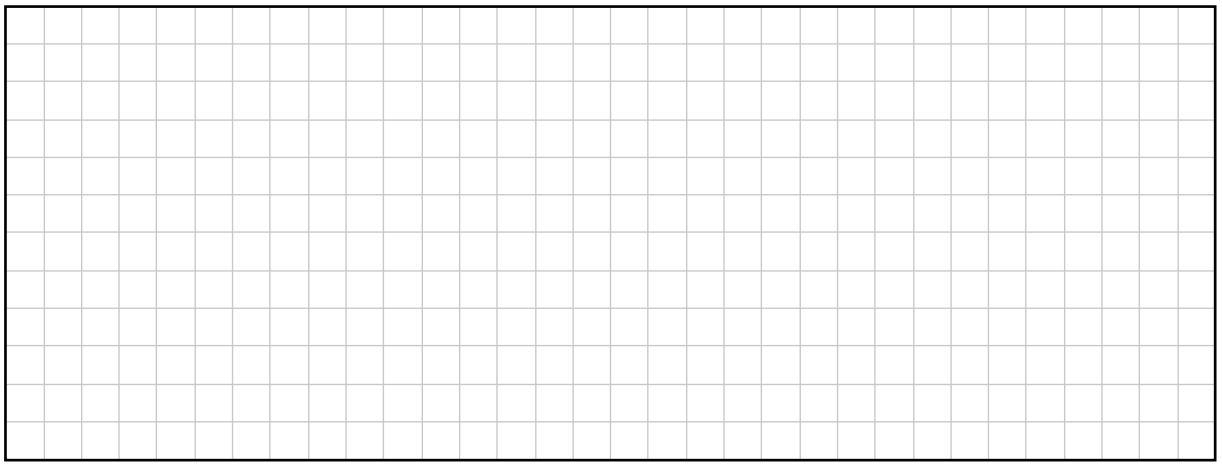
This question continues on the next page.

(c) A committee of 5 people is picked from a group of 10 people, consisting of 4 men and 6 women.

(i) Find the number of different committees which consist of 2 men and 3 women.

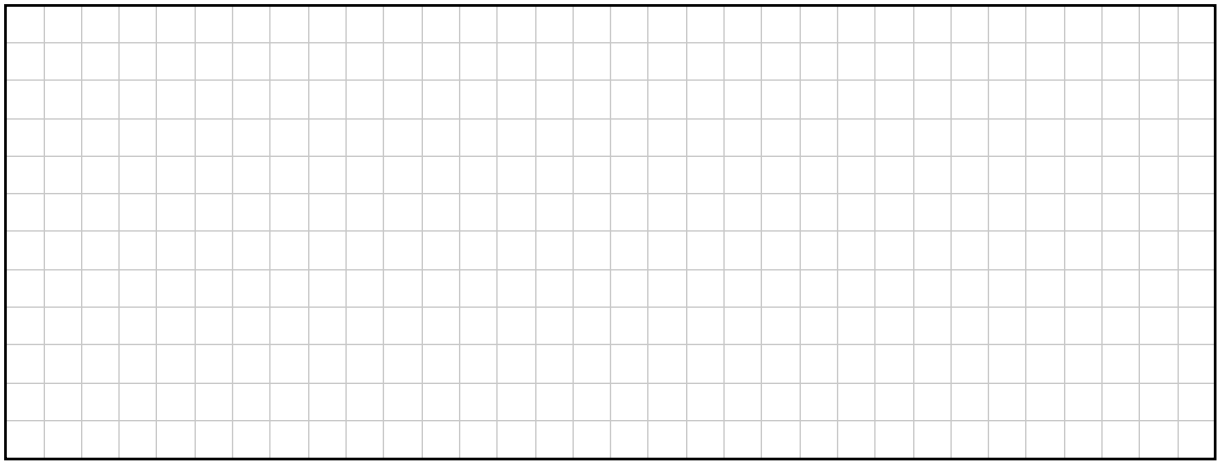


` **(ii)** Find the number of different committees that have more women than men.

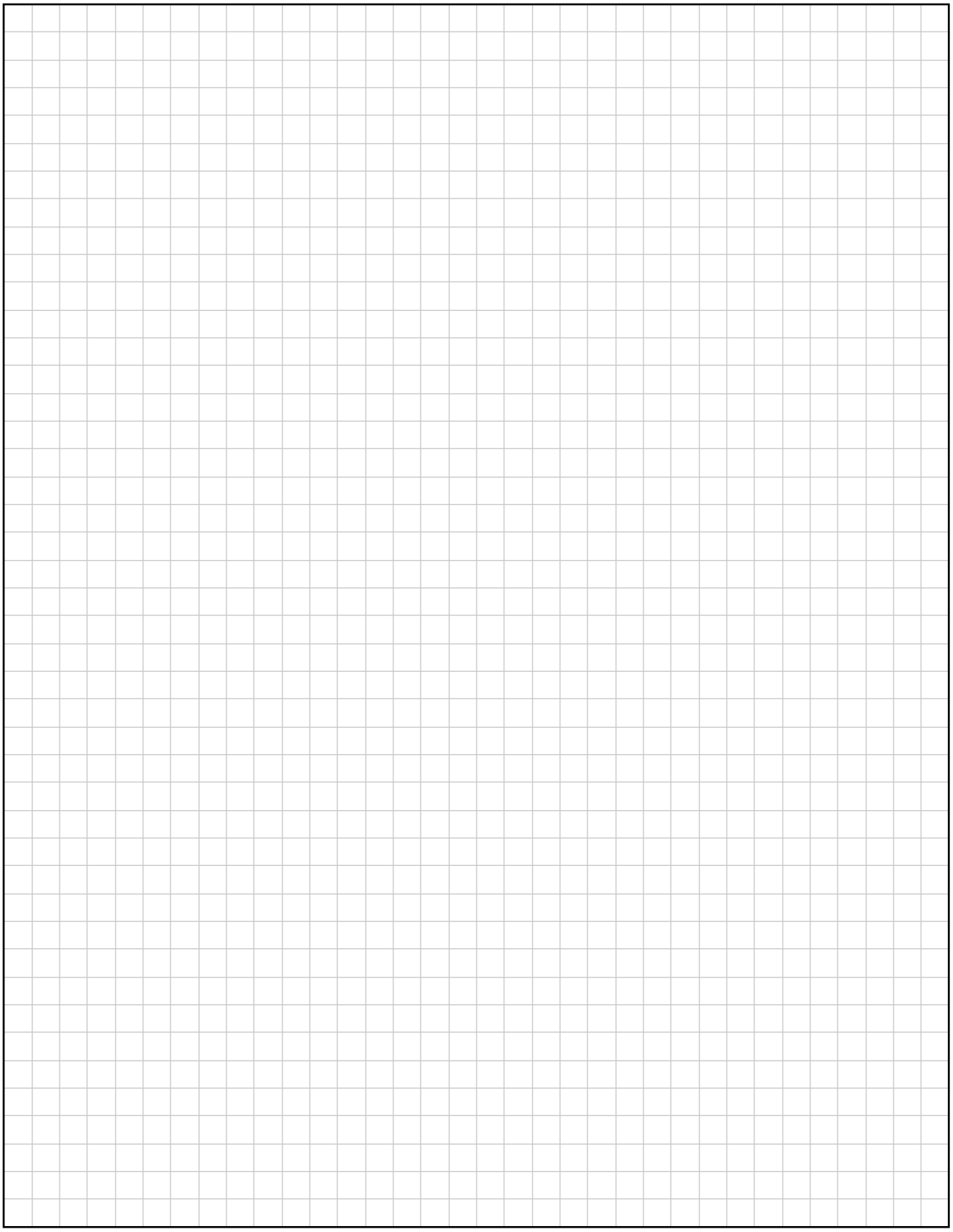


(iii) One committee is picked at random.
A **different** committee is then picked at random.

Find the probability that **none** of the 10 people are on both of these committees.



Page for extra work.
Label any extra work clearly with the question number and part.



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Leaving Certificate – Higher Level

Mathematics – Paper 2

2 hours 30 minutes