



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2025

Mathematics

Paper 1

Higher Level

2 hours 30 minutes

300 marks

Examination Number

<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
----------------------	----------------------	----------------------	----------------------	----------------------	----------------------

Date of Birth

<input type="text"/>	<input type="text"/>	/	<input type="text"/>	<input type="text"/>	/	<input type="text"/>	<input type="text"/>
----------------------	----------------------	---	----------------------	----------------------	---	----------------------	----------------------

For example, 3rd February 2005 is
entered as 03 02 05

Centre Stamp

<i>For the Examiner only</i>				
		Section	Question	Mark
		A	1	
			2	
<i>Disallowed</i>			3	
A			4	
B			5	
Total Disall.			6	
		B	7	
<i>Cumulative Check</i>			8	
Running Total			9	
– Total Disall.			10	
= Total		↔	Total	

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- **Any five** questions from Section A – Concepts and Skills
- **Any three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Diagrams are generally not drawn to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **any five** questions from this section.

Question 1**(30 marks)**

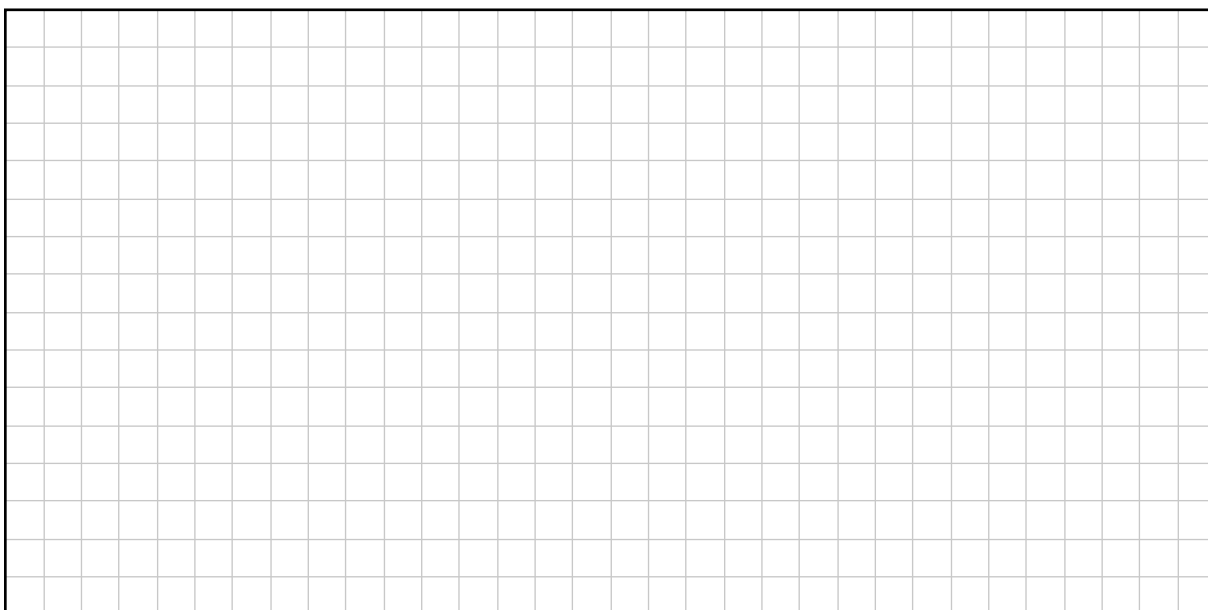
(a) Solve the following equation in x . Give each answer correct to 2 decimal places.

$$2x^2 - 8x - 11 = 0$$



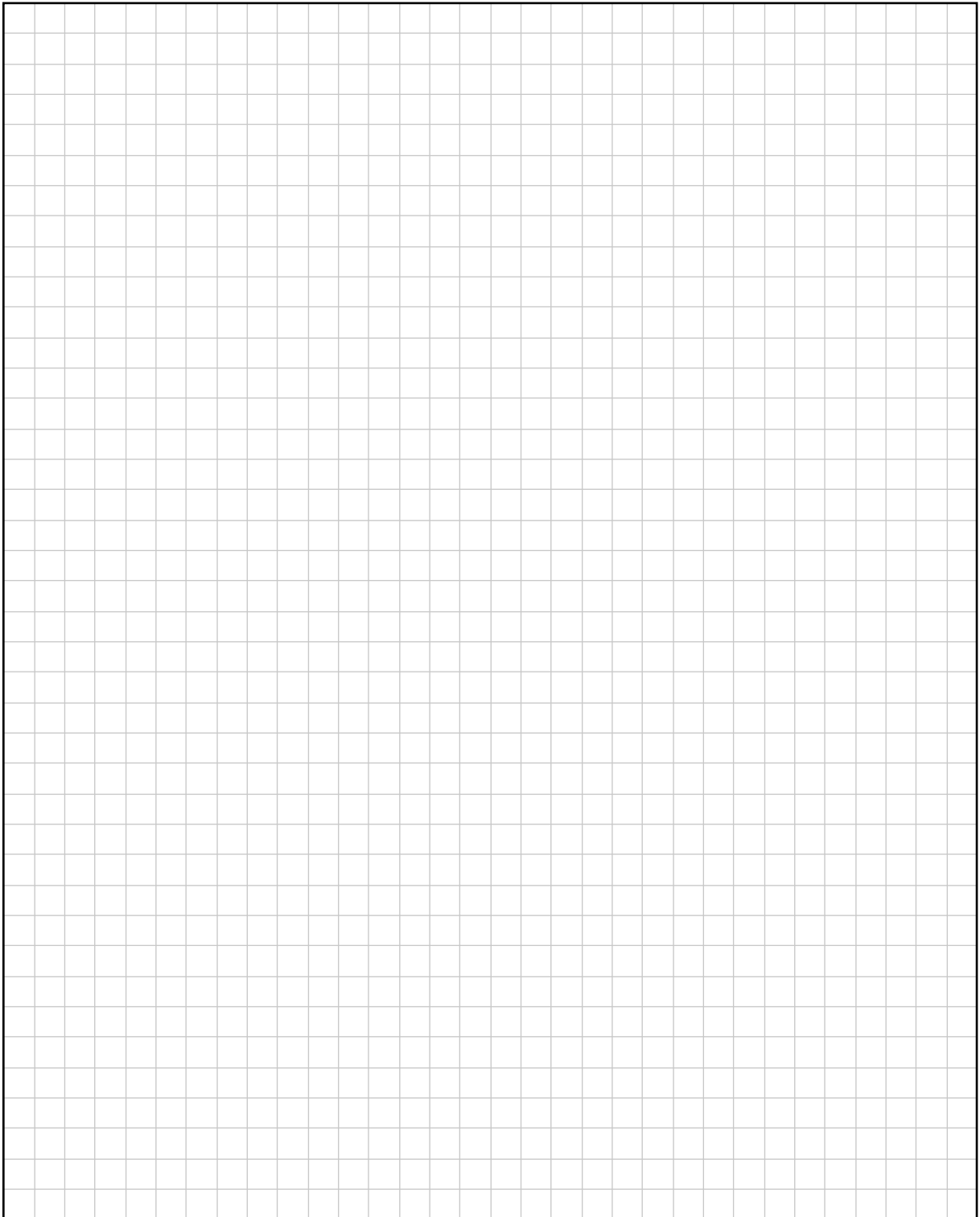
(b) Find the range of values of $p \in \mathbb{R}$ for which:

$$|2p + 7| \geq 3$$



(c) Solve the following equation for $n \in \mathbb{R}$, where $n > 0$:

$$\log_2(8 - 4n) - 2 \log_2 n = 2$$

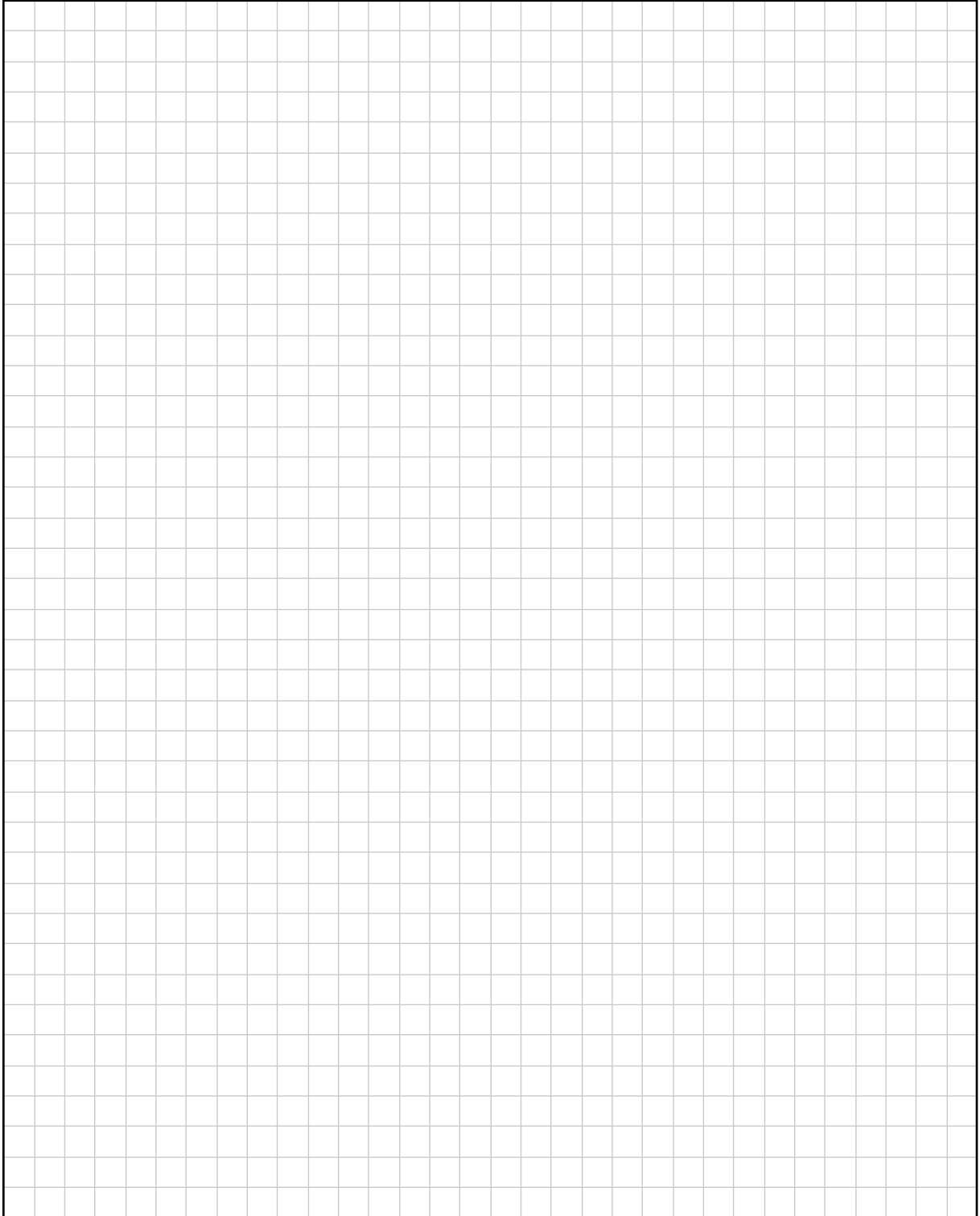


Question 2

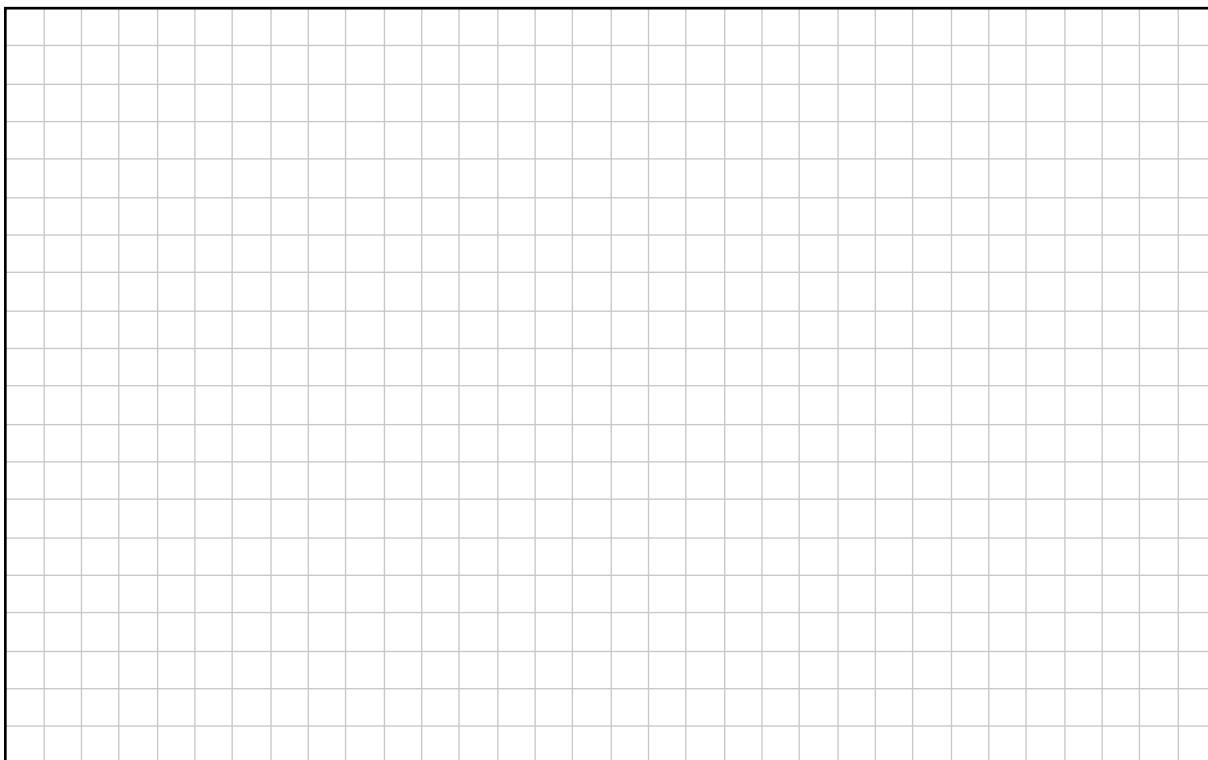
(30 marks)

(a) Prove by induction that the following identity is true for all $n \in \mathbb{N}$:

$$1 + 2 + 3 + \dots + n = \frac{n(n + 1)}{2}$$

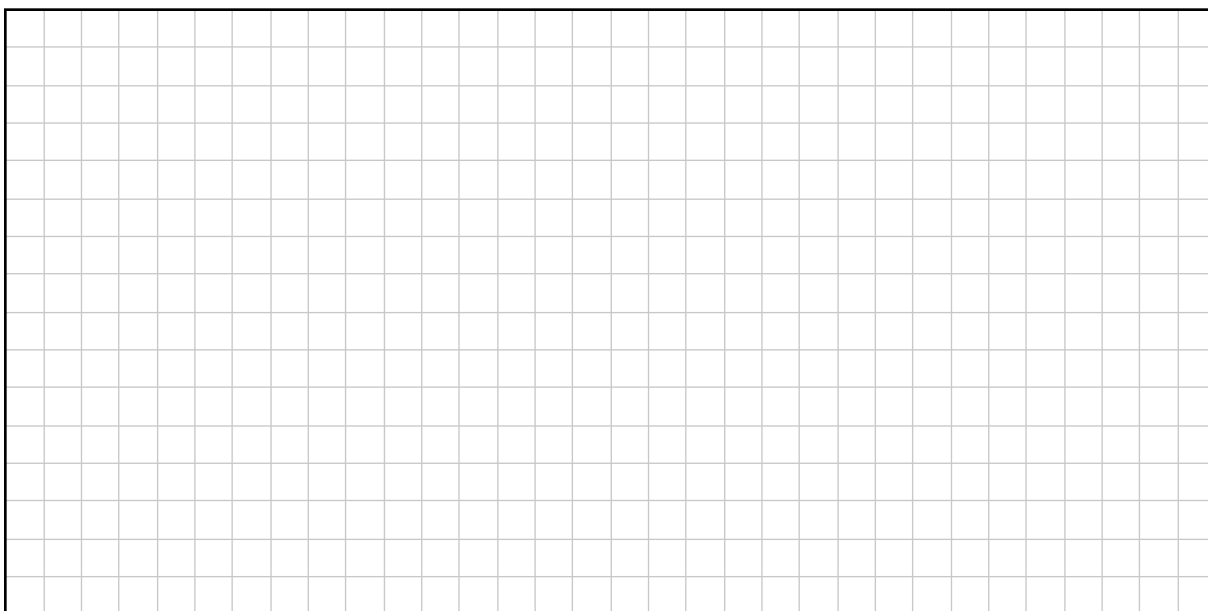


- (b) (i) Solve the equation $\frac{1}{y} + 2y = 3$, where $y \in \mathbb{R}, y \neq 0$.



- (ii) Hence, or otherwise, find the values of $x \in \mathbb{R}$ for which:

$$4^{-x} + 2(4^x) = 3$$

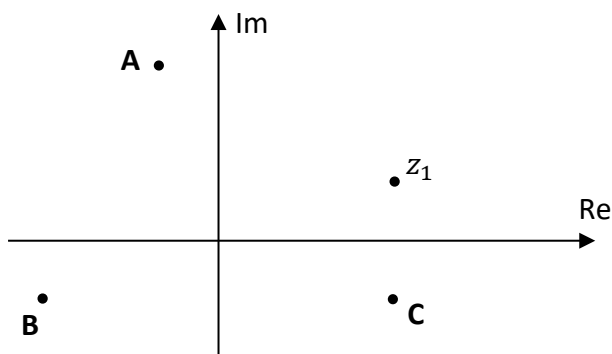


Question 3

(30 marks)

Note: In this question, $i^2 = -1$.

- (a) The complex number z_1 is shown in the Argand diagram below. Three other complex numbers are plotted, and are labelled **A**, **B**, and **C**, respectively. Write **A**, **B**, and **C** in the appropriate spaces in the table below, to show which point represents each complex number. Note that $\overline{z_1}$ is the complex conjugate of z_1 .



Complex Number	Point on the Argand Diagram (A, B, or C)
$-z_1$	
$\overline{z_1}$	
$i z_1$	

--	--

- (b) Use **De Moivre's** theorem to solve the equation $w^2 = 4i$. Give each answer in the form $a + bi$, where $a, b \in \mathbb{R}$.

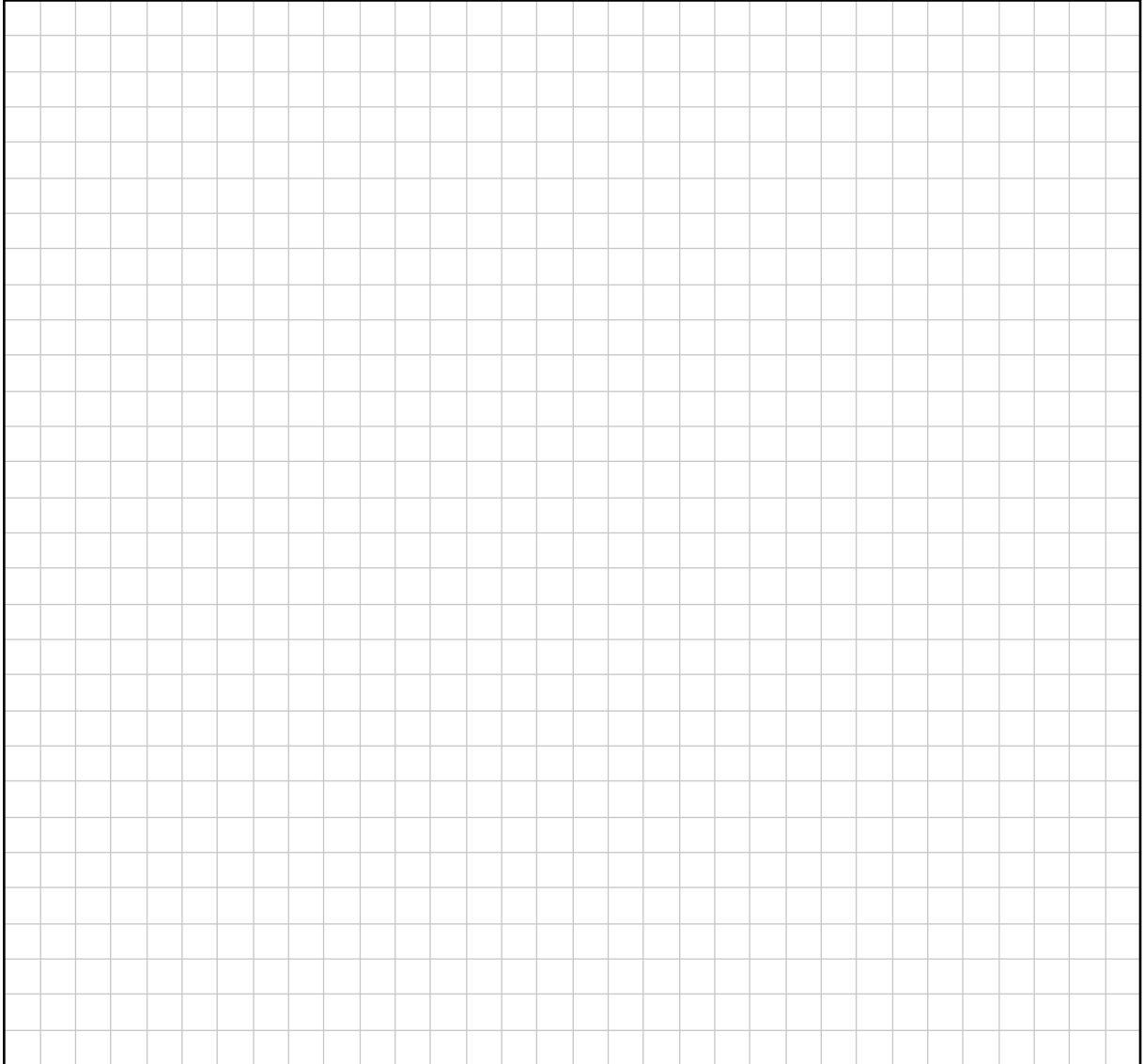
--	--

(c) $z = (3 - i)$ is a root of the equation:

$$z^3 - 2z^2 - 14z + 40 = 0$$

Find the other two roots of this equation.

One should be complex and the other should be real.



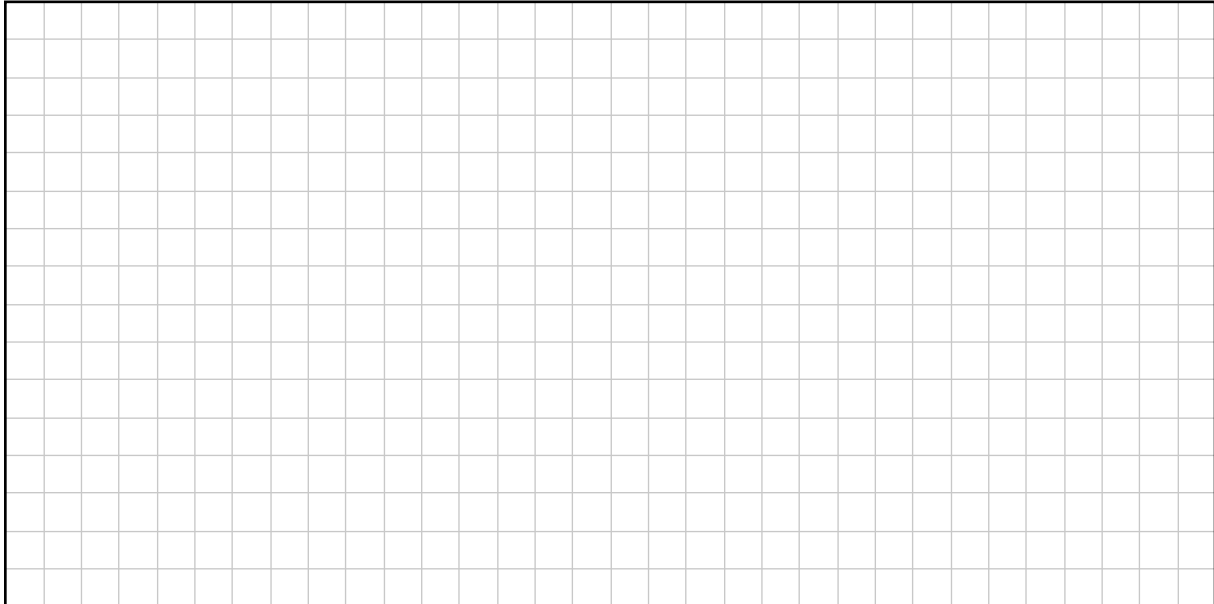
Question 4

(30 marks)

(a) Solve the following pair of simultaneous equations in $x, y \in \mathbb{R}$:

$$y = 3x^2 - 32x + 54$$

$$y = 2x + 14$$



(b) $f: \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function.

The graph of $y = f(x)$ has one local maximum point, which is at $(7, 12)$.

Is $y = f(x)$ **injective** (one-to-one) on the interval $x \in [6, 8]$?

Justify your answer.

Hint: draw a sketch of the graph of $y = f(x)$ near the point $(7, 12)$.

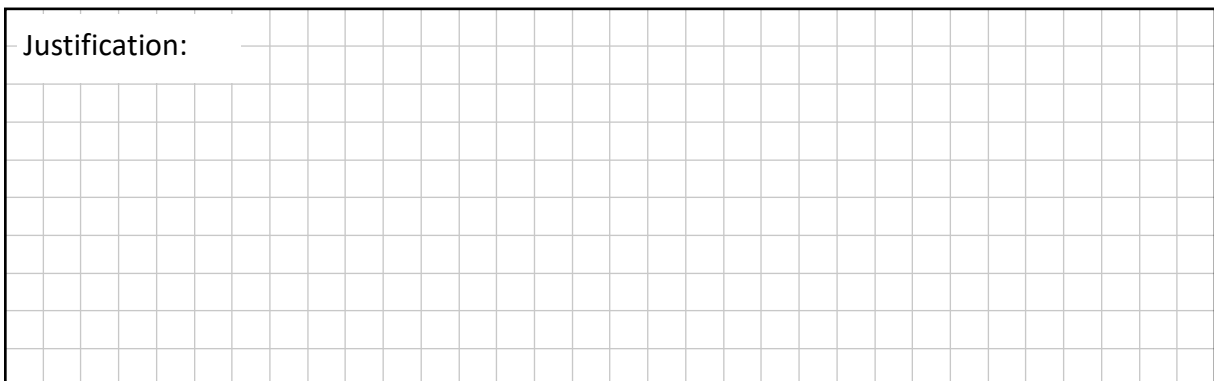
Is $y = f(x)$ injective on $[6, 8]$?

Yes

No

We don't have enough information to be sure

Tick (\checkmark) **one** box only



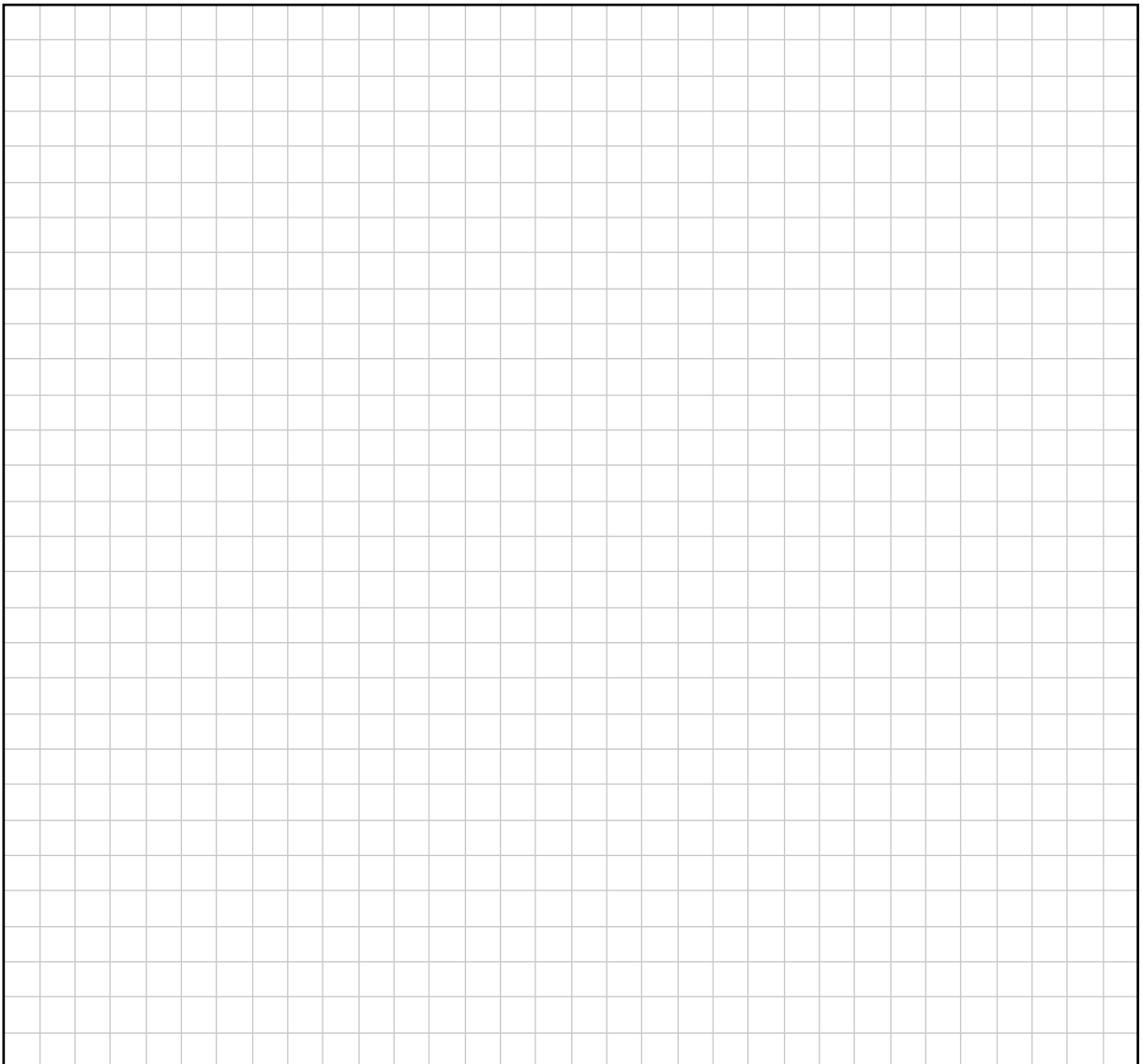
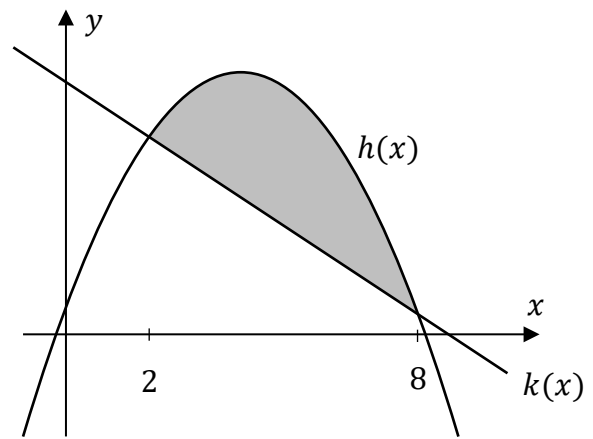
- (c) The diagram shows the graphs of the functions $h(x)$ and $k(x)$, where:

$$h(x) = -\frac{1}{2}x^2 + 4x + 1$$

$$k(x) = 9 - x$$

Note that $h(2) = k(2)$ and $h(8) = k(8)$.

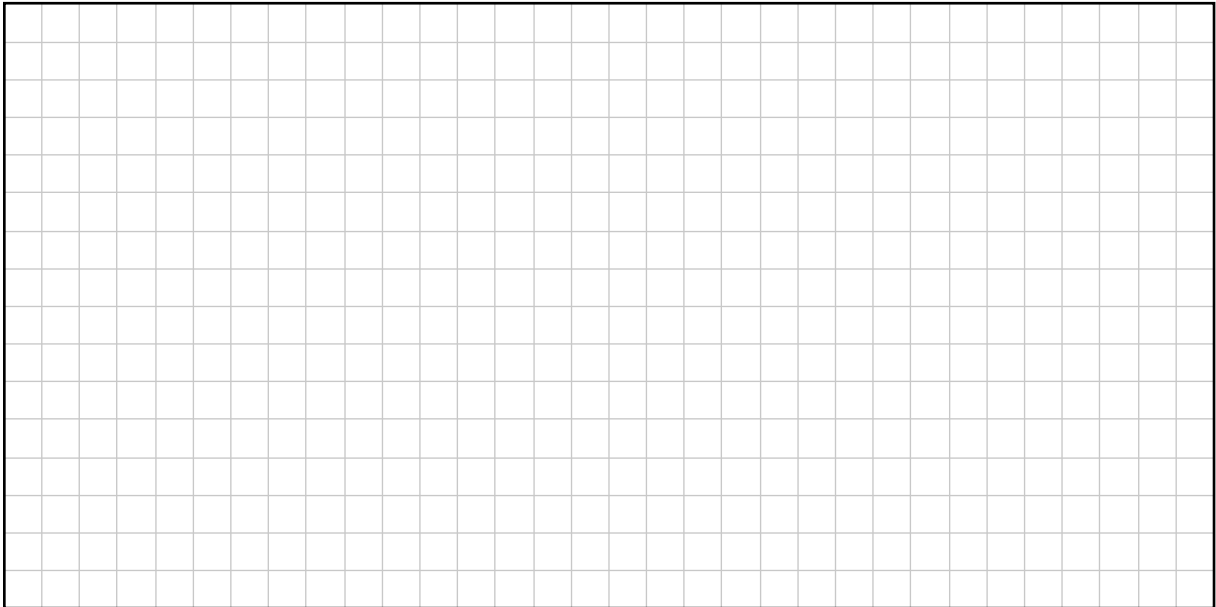
Use integration to find the **area** enclosed between the graphs of $h(x)$ and $k(x)$, from $x = 2$ to $x = 8$.



Question 5

(30 marks)

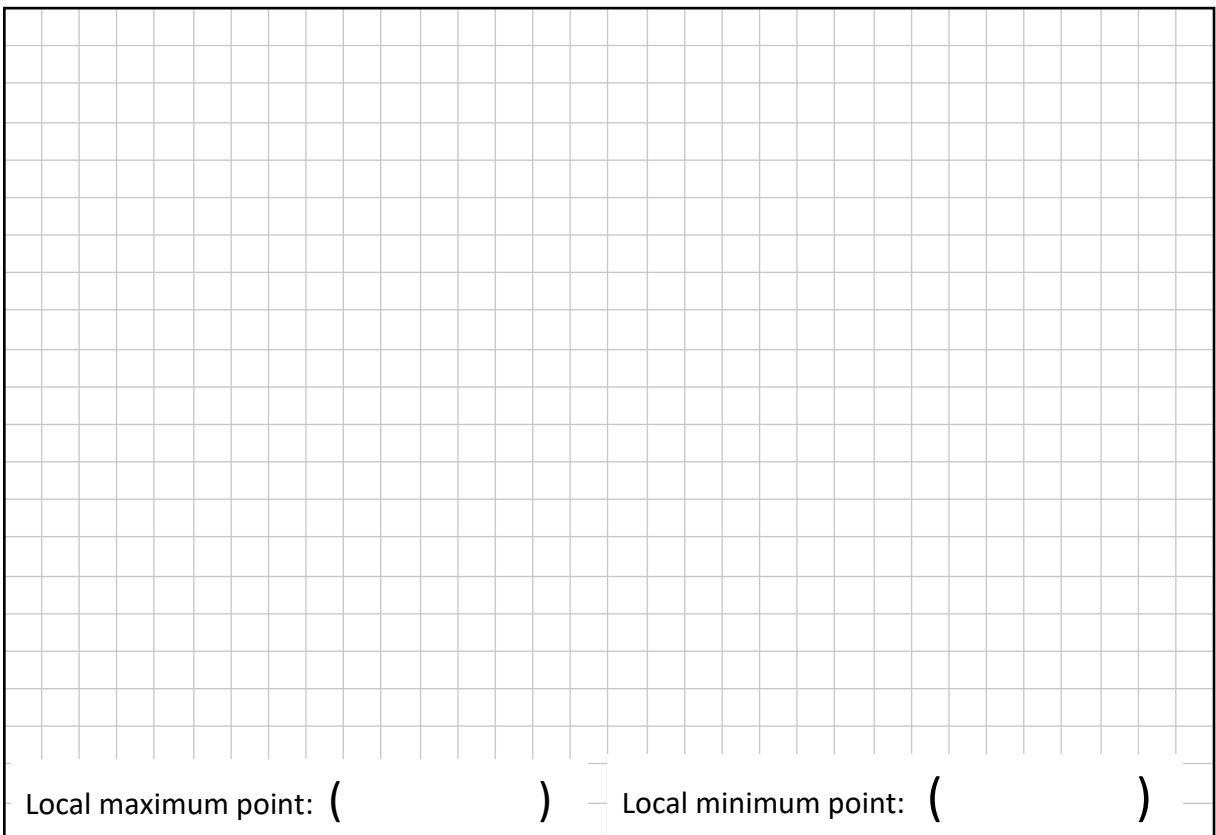
- (a)** Multiply out and simplify $(x - 1)^3$.



- (b)** A cubic function $h(x)$ has the following equation in $x \in \mathbb{R}$:

$$h(x) = x^3 - 6x^2 - 15x + 10$$

Find the co-ordinates of the local maximum point and the local minimum point of $h(x)$.

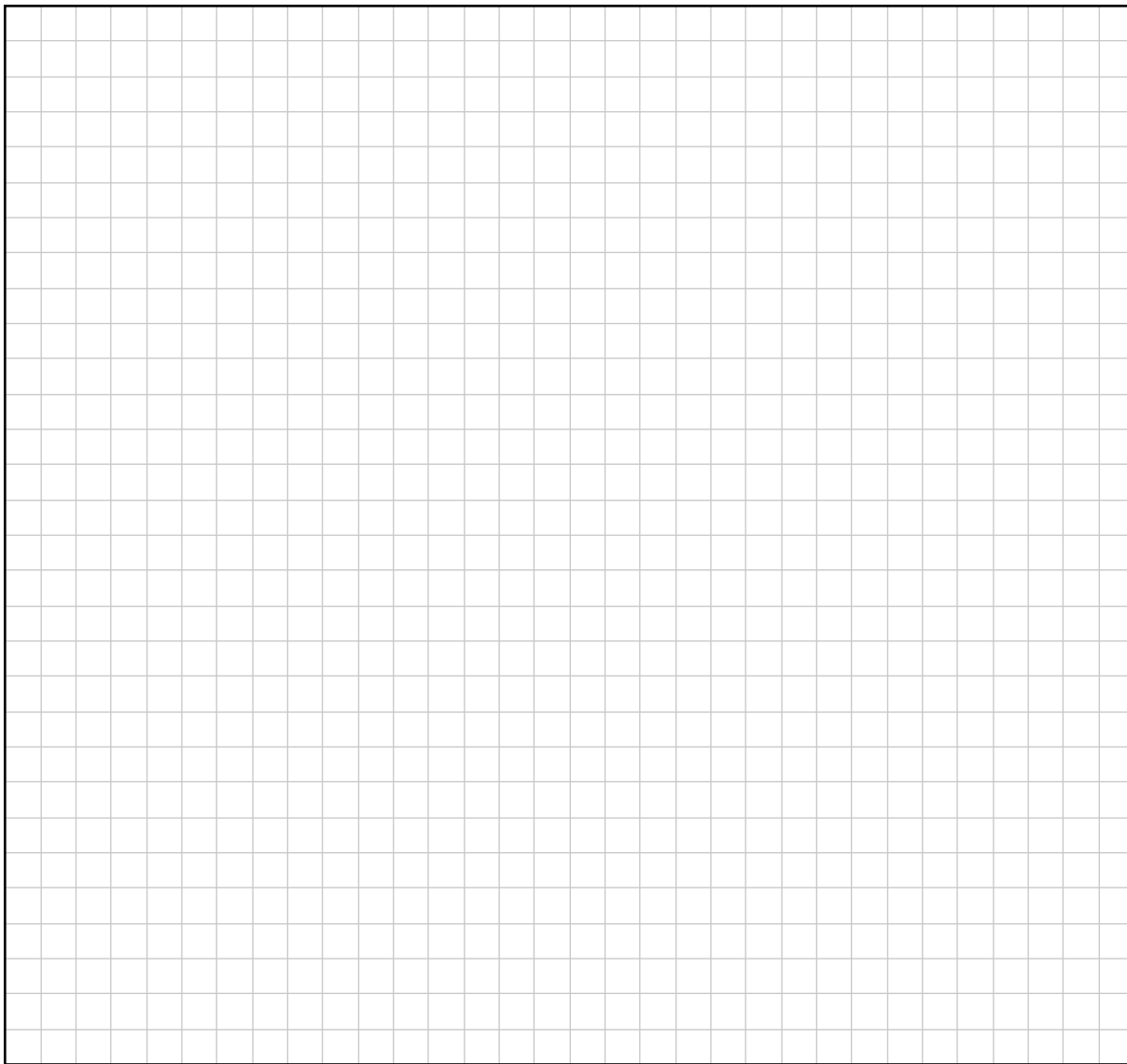


Local maximum point: () Local minimum point: ()

(c) f is the following function, where $x \in \mathbb{R}, x \neq \frac{1}{2}$:

$$f(x) = 2x + 5 + \frac{1}{2x - 1}$$

Find the values of x for which $f'(x) = 0$.



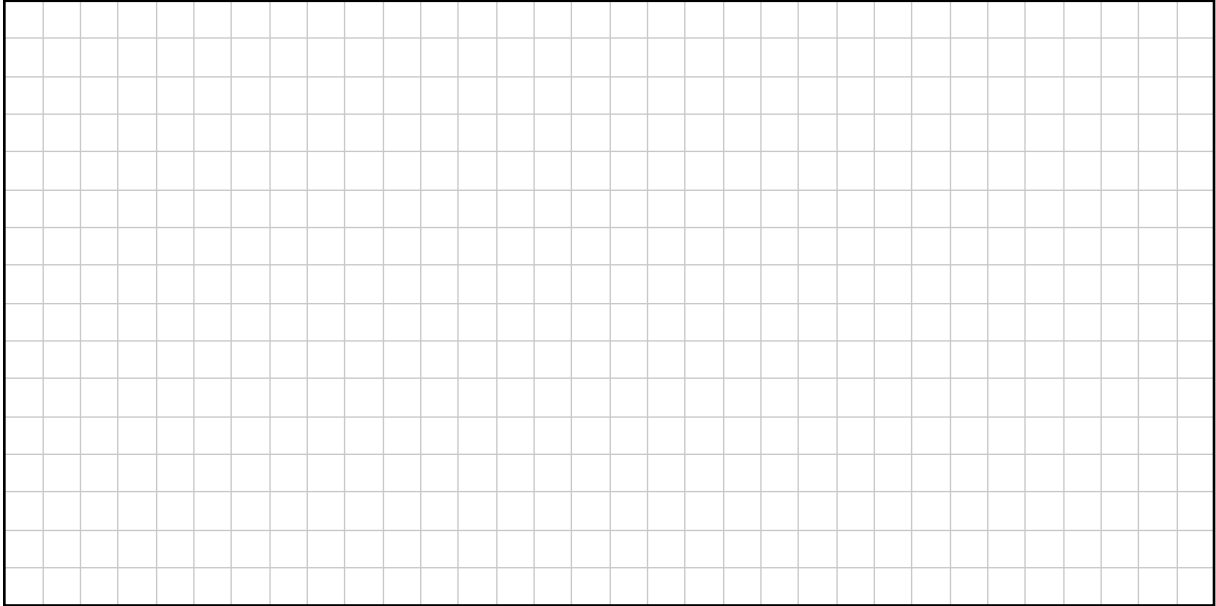
Question 6

(30 marks)

- (a)** The population of a region is now 6.7 million people.
This population is growing at a rate of 10% per year.

At this growth rate, the population will be greater than 20 million for the first time n years from now, where $n \in \mathbb{N}$.

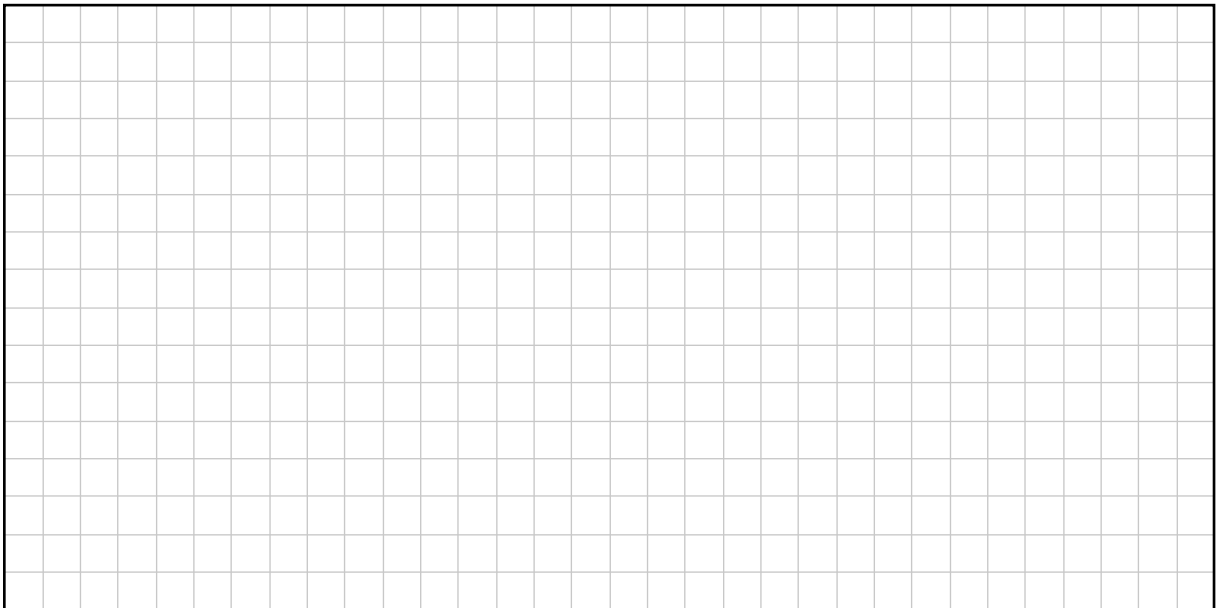
Find the value of n .



- (b)** f is the following function, where $x \in \mathbb{R}$ is in radians:

$$f(x) = \sin 3x \cos x$$

Use the product rule to find the value of $f' \left(\frac{\pi}{4} \right)$.



Answer **any three questions** from this section.

Question 7**(50 marks)**

Air pressure is the pressure caused by the weight of air pressing down on the earth.

Air pressure decreases as the height above sea level increases.

Air pressure, P , can be estimated using the formula:

$$P(h) = A e^{k h}$$

where $k = \frac{\ln 0.88}{1000}$, A is a constant, and h is the height in metres above sea level.

Use the formula $P(h) = A e^{k h}$ to answer parts **(a)** to **(d)** of this question.

- (a)** The air pressure at sea level ($h = 0$) is 101.3 units.

Show that $A = 101.3$.

- (b)** Find the air pressure at the top of Mount Everest, which is 8849 m above sea level.
Give your answer correct to 2 decimal places.

- (c) Altitude sickness is caused by low air pressure.
 Some people start to feel sick when the air pressure is at 80 units.
 At what height above sea level would these people start to feel sick?
 Give your answer correct to the nearest metre.

- (d) A ball, **B**, is at sea level, so $h = 0$.
 A second ball, **C**, is at a height above sea level of H metres, where $H > 0$.
 Each ball is raised up 40 m, so that the air pressure on each decreases.
 Which ball, if any, will have a **greater** decrease in air pressure?
 Justify your answer.

Ball with the greater decrease:

B

C

B and C will have the same decrease

Tick (✓) **one** box only

Justification:

This question continues on the next page.

- (e) John is researching the connection between air pressure and height above sea level. On a website, he finds a different formula to estimate air pressure, P :

$$P(h) = 101.325(1 - 2.2557 \times 10^{-5} \times h)^{5.2558}$$

where $h \geq 0$ is the height in metres above sea level.

Use this formula to answer parts (e)(i) and (e)(ii).

- (i) Find the air pressure at the top of Mount Everest, which is 8849 m above sea level. Give your answer correct to 2 decimal places.

- (ii) Work out the value of h for which the air pressure is 80 units, according to this formula. Give your answer correct to the nearest metre.

Question 8

(50 marks)

Jon has decided to put money into a pension fund for the next 25 years.

He puts €285 into his pension fund at the start of every month for 300 months, at an interest rate of 0.35% per month.

- (a) (i) Write the total amount of money in his pension fund at the end of the 25 years as the sum of a series. Include the first two and last two terms of the series.

- (ii) Find the total amount of money in the pension fund at the end of the 25 years. Give your answer correct to the nearest cent.

This question continues on the next page.

- (b) At the end of the 25 years, Jon also puts an additional lump sum into his pension fund. His total pension fund is now €165 646.20.

At this point, Jon retires, and stops paying money into his pension fund. Instead, he is now going to take out € A per month at the end of each month, for the next 20 years.

Assuming that the interest rate remains at 0.35% per month, work out the maximum value that A can be. (That is, the value of A that would mean there is no money left in the pension fund after 20 years.)

Before he retired, Jon worked for a company making two types of speed camera, **X** and **Y**.

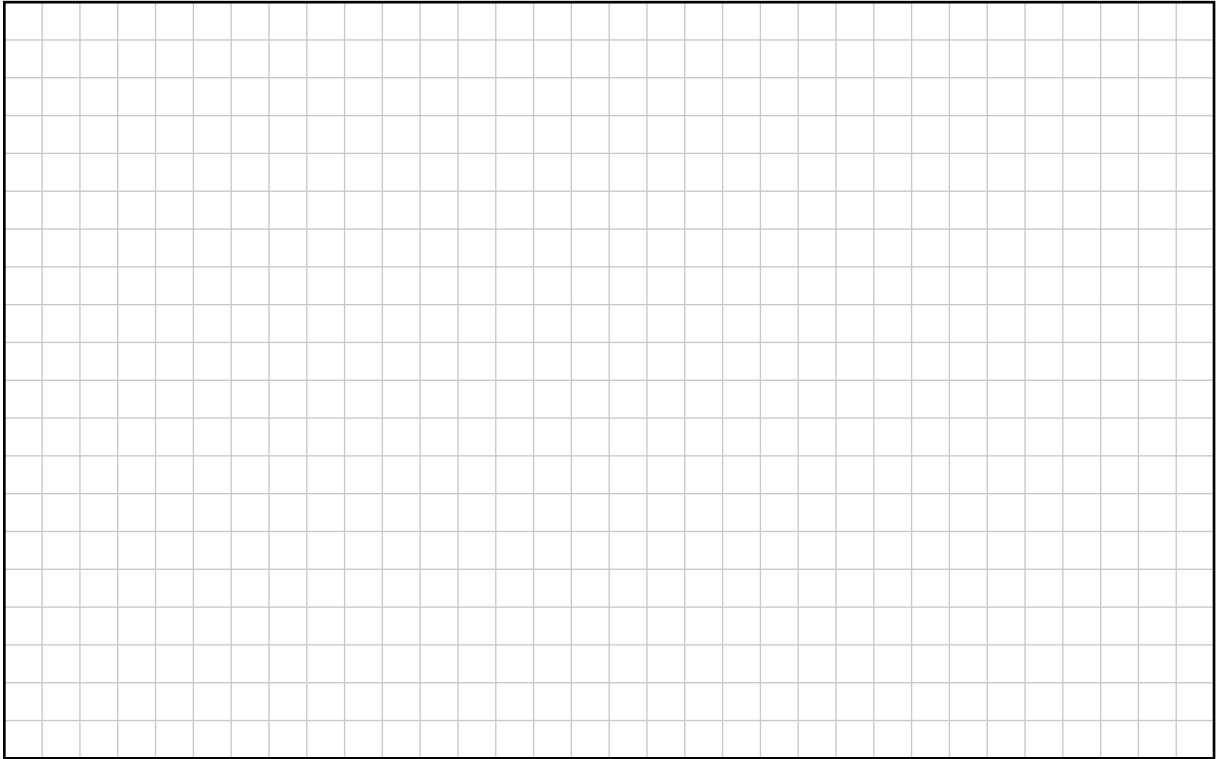
- (c) In 2020, the company made x cameras of Type **X**, and y cameras of Type **Y**.
In 2020, 40% of the cameras the company made were Type **X**.

- (i) Use this to write y in terms of x .

In 2021, the company increased the number of Type **X** cameras it made by 1500, and increased the number of Type **Y** cameras by 900.

As a result, in 2021, 50% of the cameras the company made were Type **X**.

(ii) Work out the **total** number of cameras the company made in **2021**.

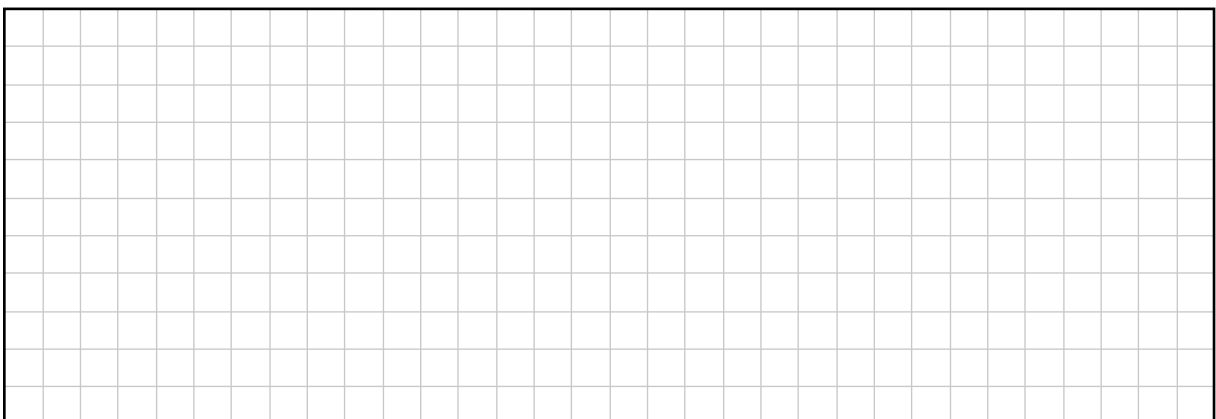


(d) According to a Type **X** speed camera, a car is travelling at 112 km/hour.

The company claims that their Type **X** speed camera can measure speed with a maximum error of 3.5%.

Use this to work out the **maximum actual** speed of the car.

Give your answer in km/hour, correct to 1 decimal place.

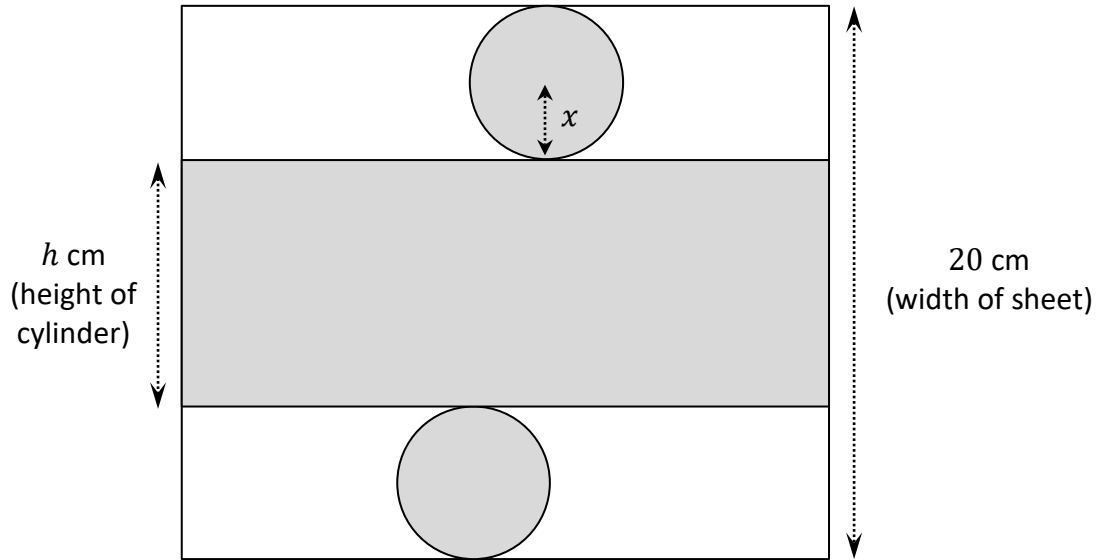


Question 9

(50 marks)

- (a) The net of a closed cylinder with a radius x cm and a height h cm is cut out from a rectangular sheet of tin. The width of the sheet is 20 cm.

The diagram below shows the sheet of tin, as well as the net of the cylinder cut from this sheet (the shaded region). The net is made up of a rectangle with a width of h cm (which is the height of the cylinder), and two circles, each of radius x cm. Each circle has two tangents in the diagram.



- (i) Use this information to write h in terms of x .

--

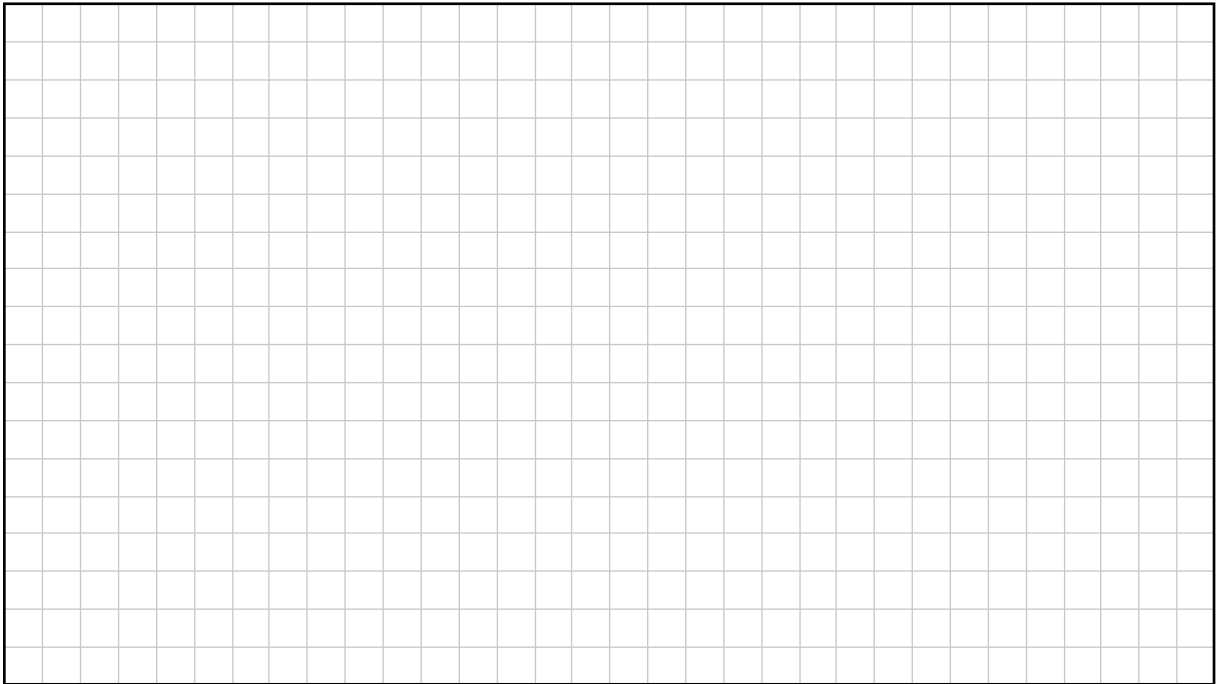
- (ii) Show that the volume of the cylinder can be written as:

$$V(x) = 20\pi x^2 - 4\pi x^3$$

--

- (iii) Use differentiation to find the **maximum volume** that such a cylinder can have. Give your answer correct to the nearest cm^3 .

Remember that $V(x) = 20\pi x^2 - 4\pi x^3$.

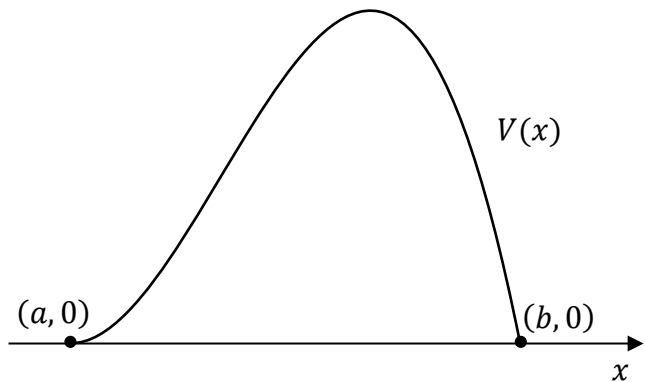


The graph on the right shows $V(x)$ for a range of values of $x \in \mathbb{R}$.

The x -axis is shown, but the y -axis is not.

The graph of $V(x)$ cuts the x -axis at the points $(a, 0)$ and $(b, 0)$, with $a < b$, as shown.

- (iv) By referring to the diagram on the previous page, or otherwise, find the value of a and the value of b .



$a =$ _____ $b =$ _____

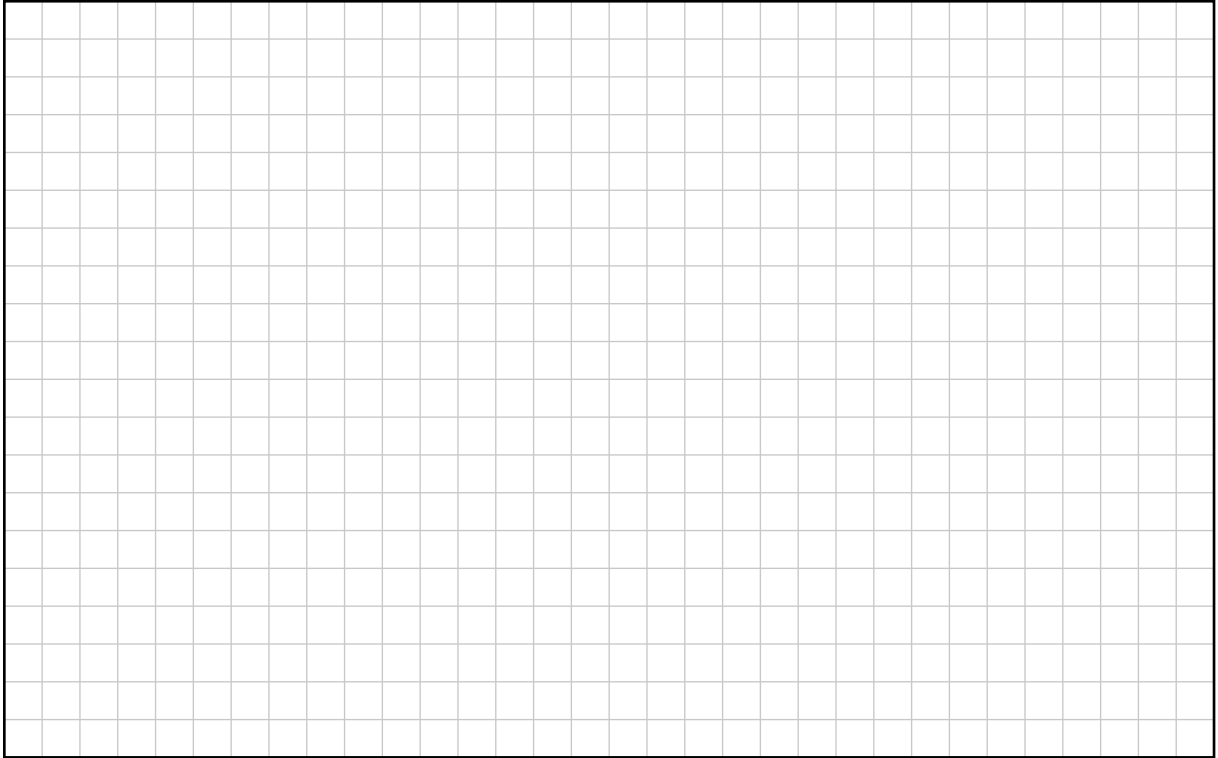
This question continues on the next page.

(b) The volume, V , of liquid in a different container can be calculated using the formula:

$$V(h) = (5h^2 + 4)^{\frac{3}{2}} - 6$$

where h is the height of the liquid in the container, in m, and V is measured in m^3 .

(i) Find the height of the liquid in the container when the volume of liquid is 203.285 m^3 .
Give your answer in metres, correct to 1 decimal place.



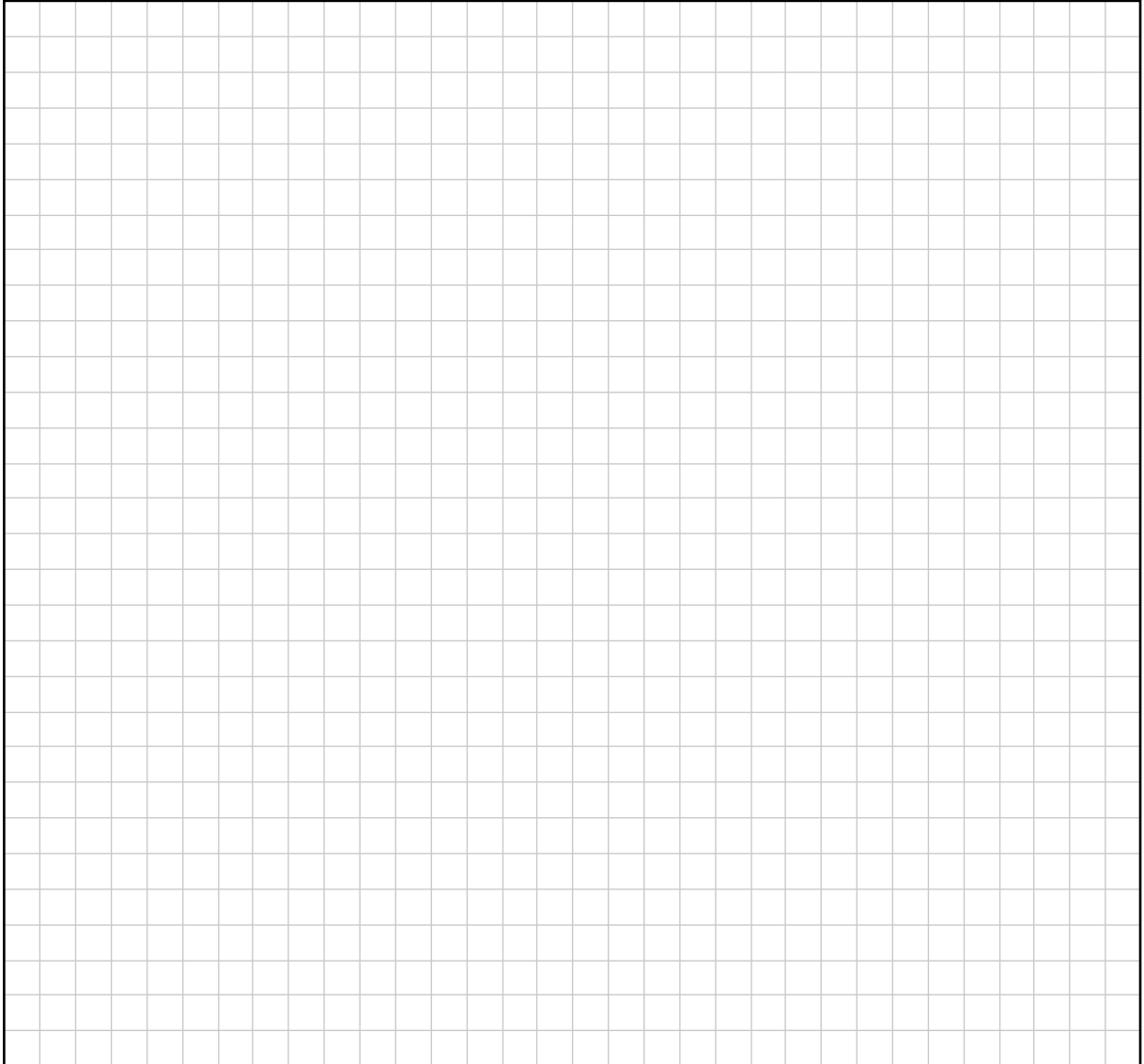
(ii) Liquid is leaking from this container.

At a certain time, the height of the liquid in the container is 1 m.

At this time, the height of the liquid is decreasing at a rate of 0.03 m/hour.

Find the rate at which the volume of liquid in the container is decreasing at this time.

Give your answer in m^3/hour .



Question 10

(50 marks)

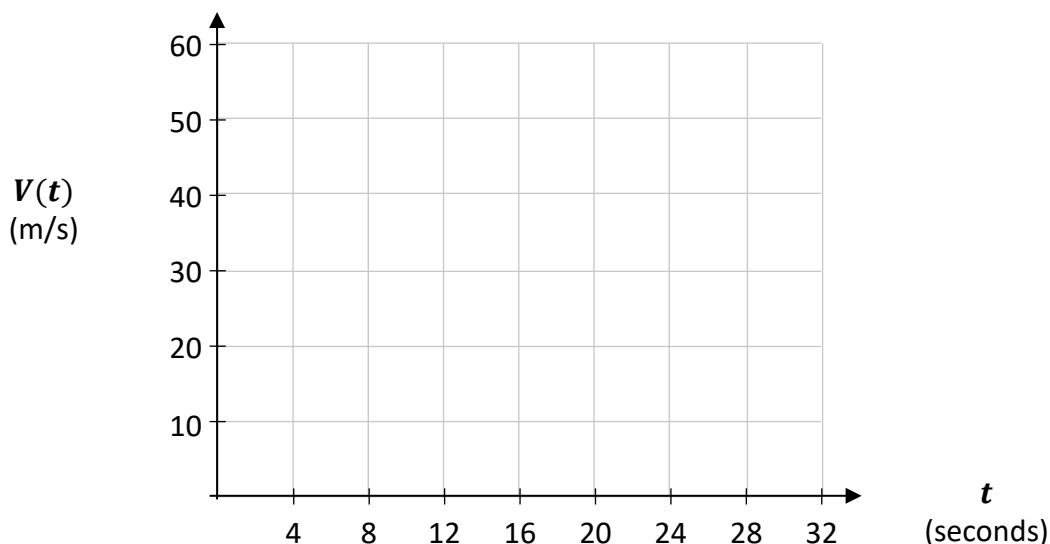
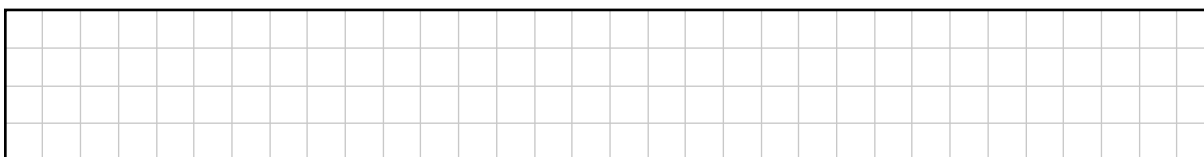
- (a) The speed, in metres per second (m/s), of a skydiver who jumps from an airplane is given by:

$$V(t) = 54(1 - e^{-0.2t})$$

where t is the time, in seconds, that has passed since the jump.

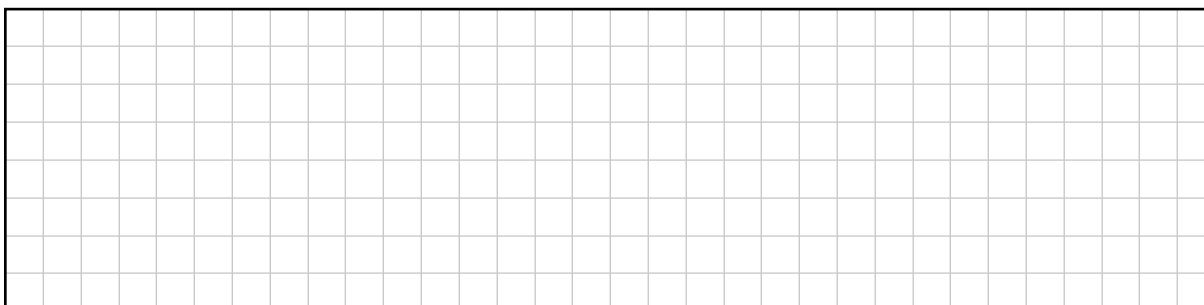
- (i) Complete the table below **and** hence draw the graph of $V(t)$ in the domain $0 \leq t \leq 32, t \in \mathbb{R}$, on the grid below. Give your values correct to 1 decimal place.

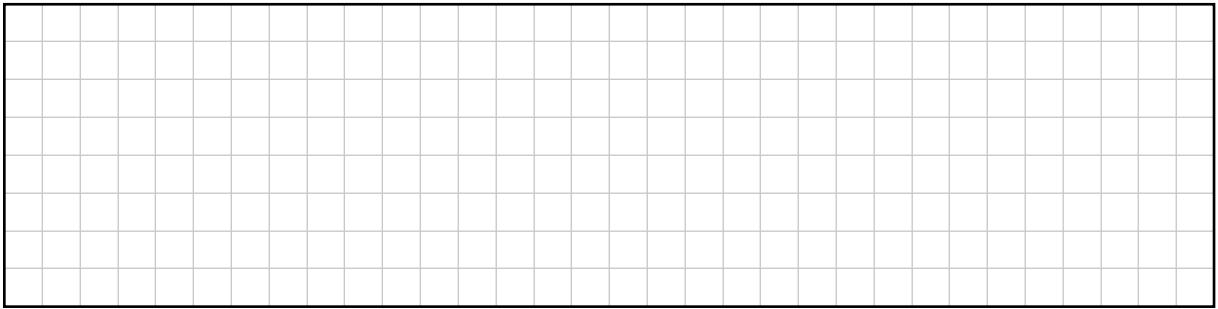
t	0	4	8	12	16	20	24	28	32
$V(t)$		29.7					53.6		



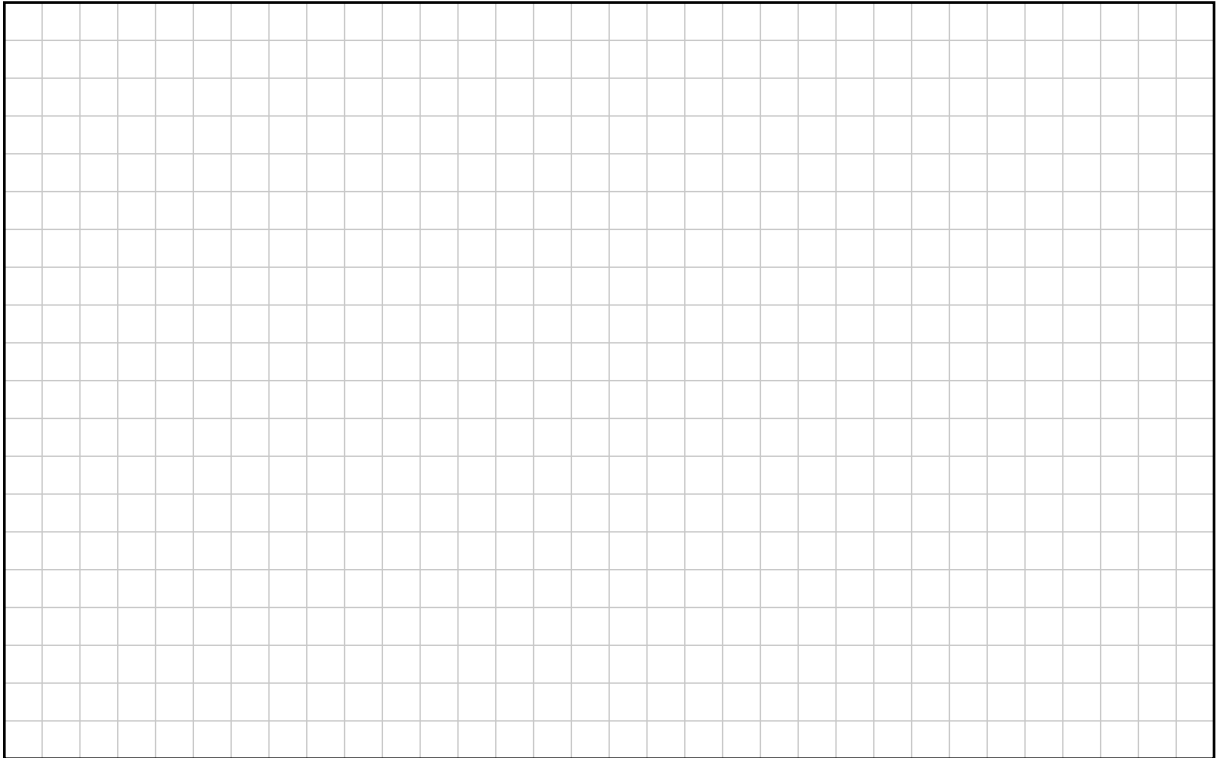
- (ii) Find the acceleration (rate of change of speed) of the skydiver one second after they jump from the airplane. Give your answer, in m/s^2 , correct to 1 decimal place.

There is more space for work on the next page.



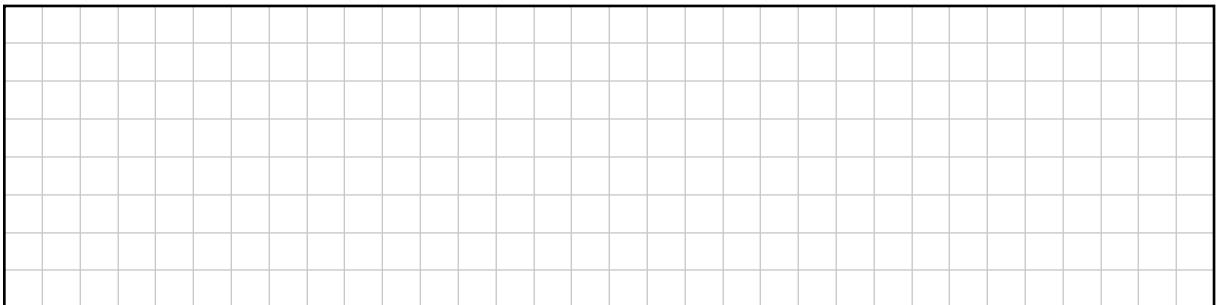


- (iii)** Use integration to find the average speed of the skydiver over the first 10 seconds after the jump. Give your answer correct to the nearest m/s.



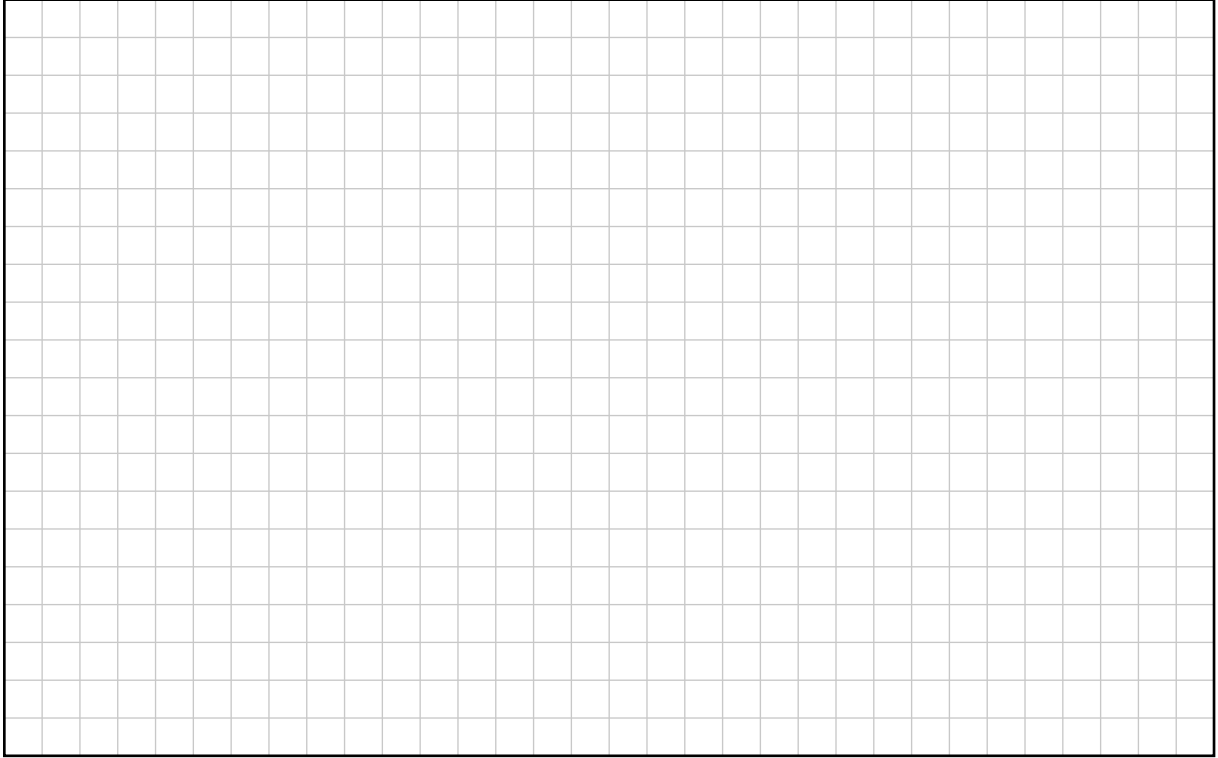
- (iv)** Find the terminal velocity of the skydiver, that is, find the value of:

$$\lim_{t \rightarrow \infty} V(t)$$



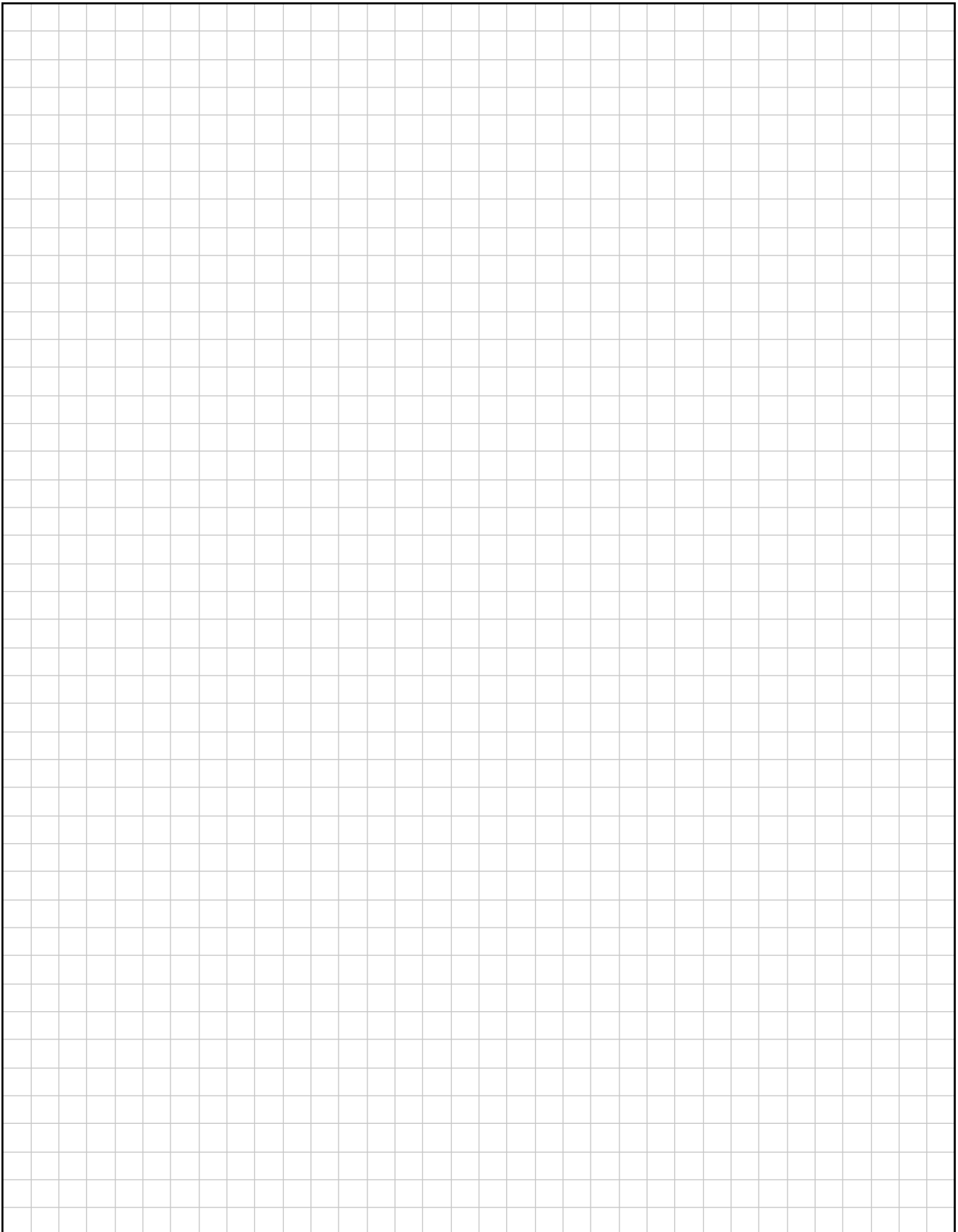
This question continues on the next page.

- (b) The airplane flies d km out to a jumpsite, at an average speed of x km/hour.
It then flies d km back from the jumpsite.
Throughout the return journey, its speed is 10% less than x km/hour.
Find the **percentage increase** in the time it will take for the return journey.
Give your answer as a percentage, correct to 1 decimal place.



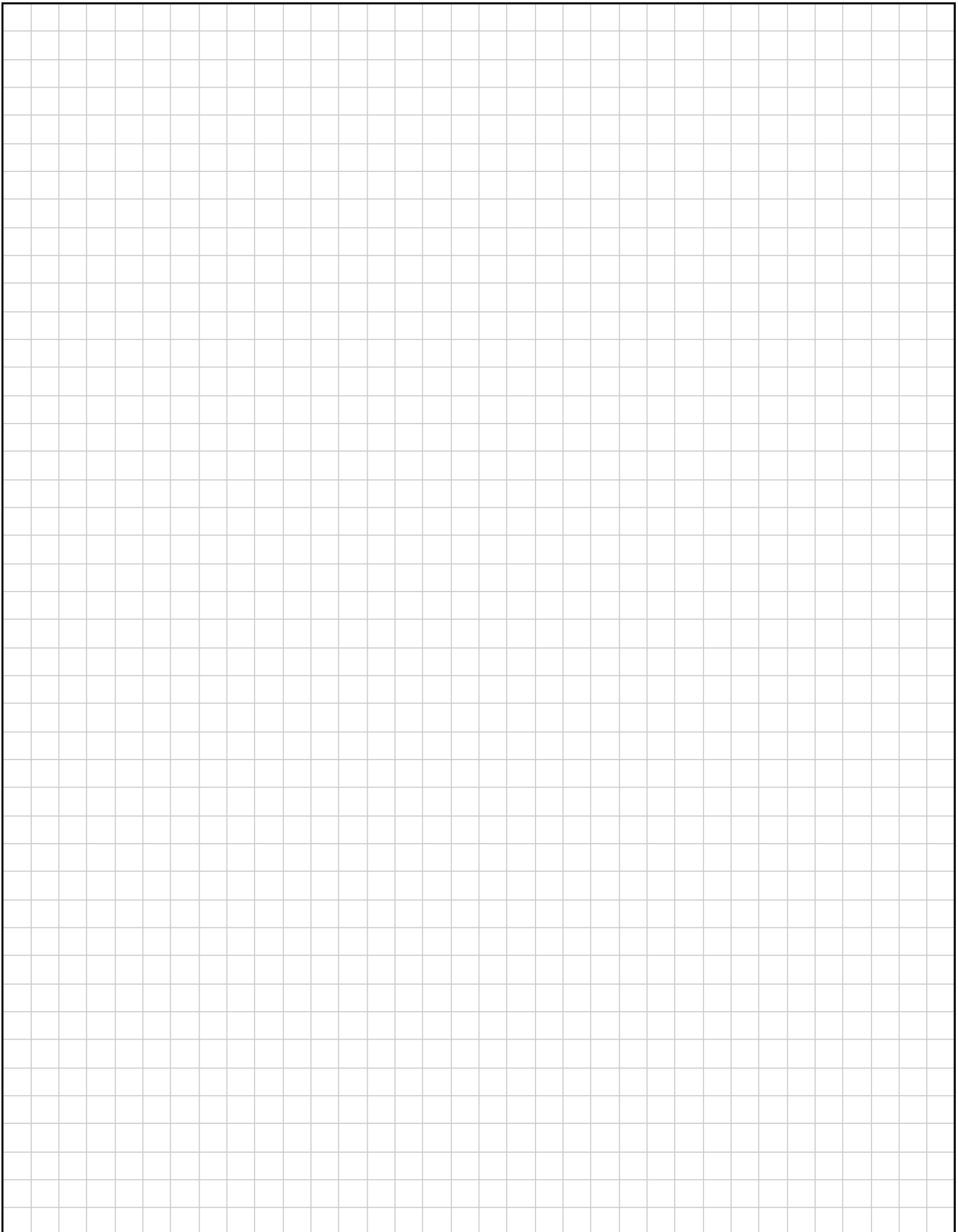
Page for extra work.

Label any extra work clearly with the question number and part.



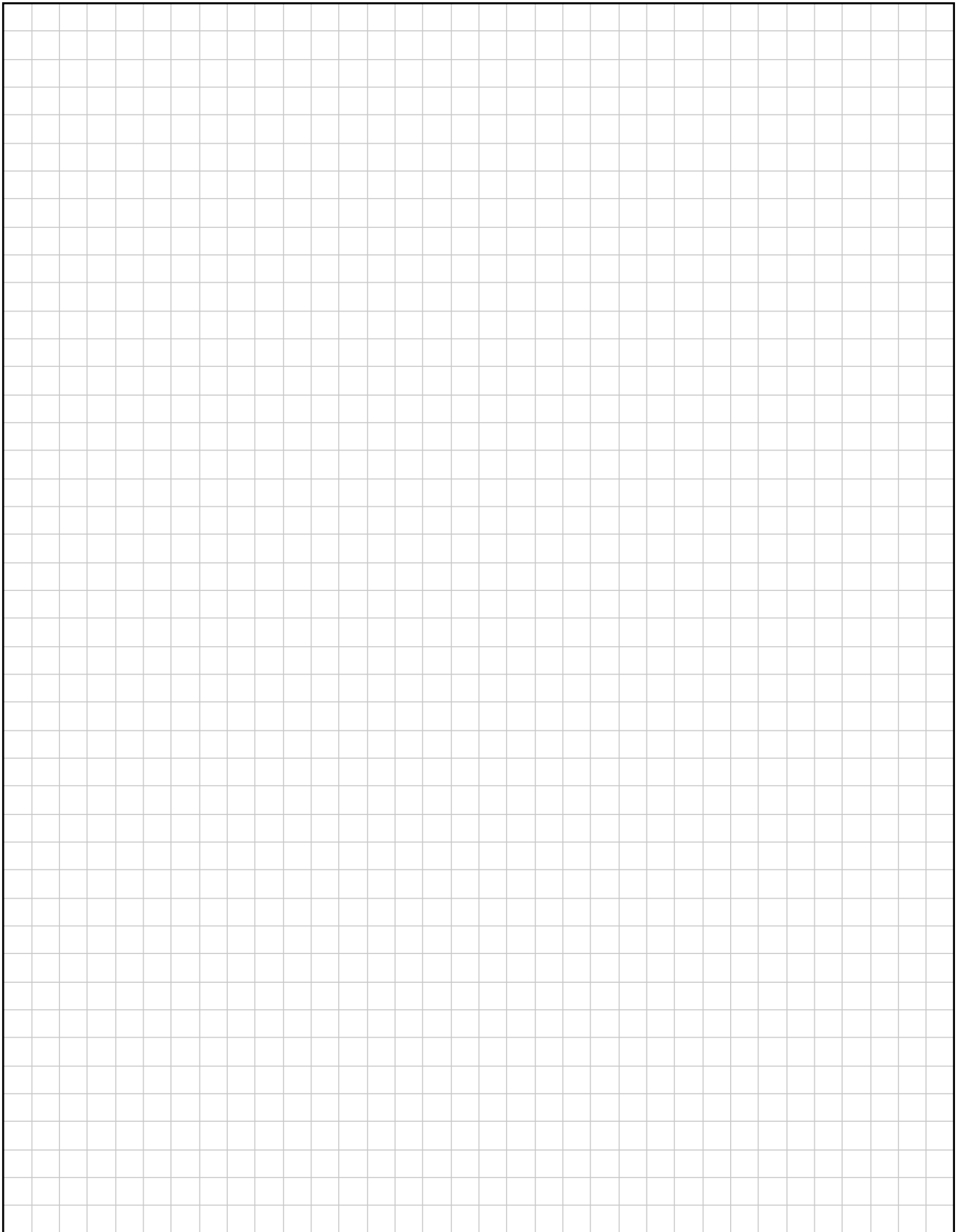
Page for extra work.

Label any extra work clearly with the question number and part.



Page for extra work.

Label any extra work clearly with the question number and part.



Do not write on this page

Copyright notice

This examination paper may contain text or images for which the State Examinations Commission is not the copyright owner, and which may have been adapted, for the purpose of assessment, without the authors' prior consent. This examination paper has been prepared in accordance with Section 53(5) of the *Copyright and Related Rights Act, 2000*. Any subsequent use for a purpose other than the intended purpose is not authorised. The Commission does not accept liability for any infringement of third-party rights arising from unauthorised distribution or use of this examination paper.

Leaving Certificate – Higher Level

Mathematics – Paper 1

2 hours 30 minutes