2024.M30K

(deferred examination)

2024L003A2EK

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2024

Mathematics

Paper 2

Higher Level

2 hours 30 minutes

300 marks

Examination number					

Centre stamp

For the Examiner only				
		Section	Question	Mark
		A	1	
			2	
Disallowed			3	
A			4	
B			5	
Total Disall.			6	
		B	7	
Cumulative Check			8	
Running Total			9	
− Total Disall.			10	
= Total		↔	Total	

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- **any five** questions from Section A – Concepts and Skills
- **any three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

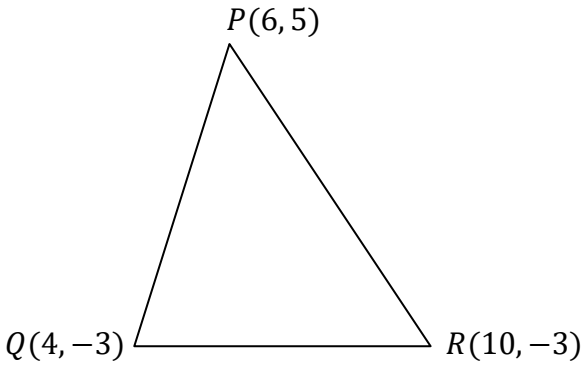
Write the make and model of your calculator(s) here: | |

Section A — Concepts and Skills — 150 marks

Answer **any five** questions from this section.

Question 1 **(30 marks)**

(a) The vertices of the triangle PQR are $P(6,5)$, $Q(4,-3)$, and $R(10,-3)$.



(i) Write down the midpoint of $[PQ]$.

(ii) Hence, or otherwise, find the equation of the perpendicular bisector of $[PQ]$.

(iii) Hence, find the co-ordinates of the **circumcentre** of the triangle PQR (that is, the point where the perpendicular bisectors of the sides meet).

(b) The line AB intersects the x-axis at A and the y-axis at B.
The point $(-6, 2)$ is the midpoint of $[AB]$.

Find the co-ordinates of A and of B.

$A = (\quad , \quad)$ $B = (\quad , \quad)$

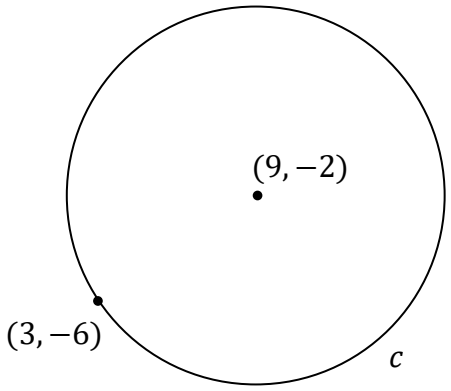
Question 2 **(30 marks)**

(a) The circle s has centre $(2, 4)$.
The point $(-1, -3)$ lies on the circle s.

Find the equation of the circle s.

(b) The circle c has centre $(9, -2)$.
The point $(3, -6)$ lies on the circle c.

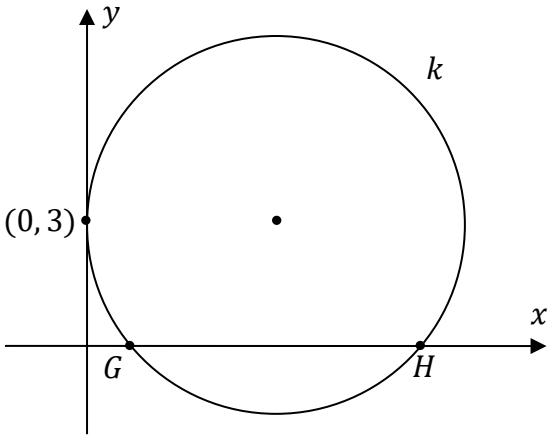
Using the diagram on the right, or otherwise, write down the co-ordinates of two other points that lie on the circle c.



Answer: (,) and (,)

(c) The circle k has its centre in the first quadrant.
The y-axis is a tangent to the circle k at the point $(0, 3)$.
The circle k intersects the x-axis at the points G and H, with $|GH| = 8$.

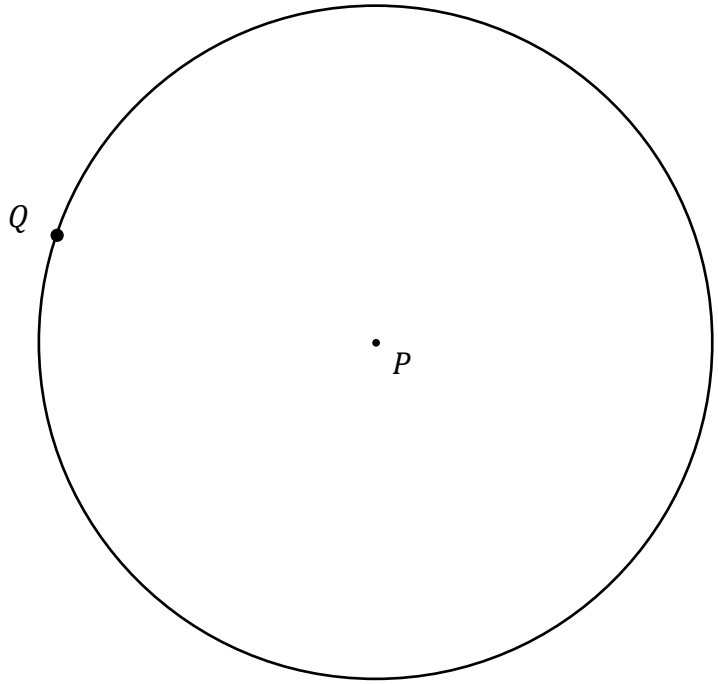
Find the equation of the circle k.



Question 3 **(30 marks)**

(a) The diagram below shows a circle.
The centre is marked P. The point Q is on the circle.

Construct the tangent to the circle at the point Q, using only a compass and straight edge.
Show all of your construction lines clearly.



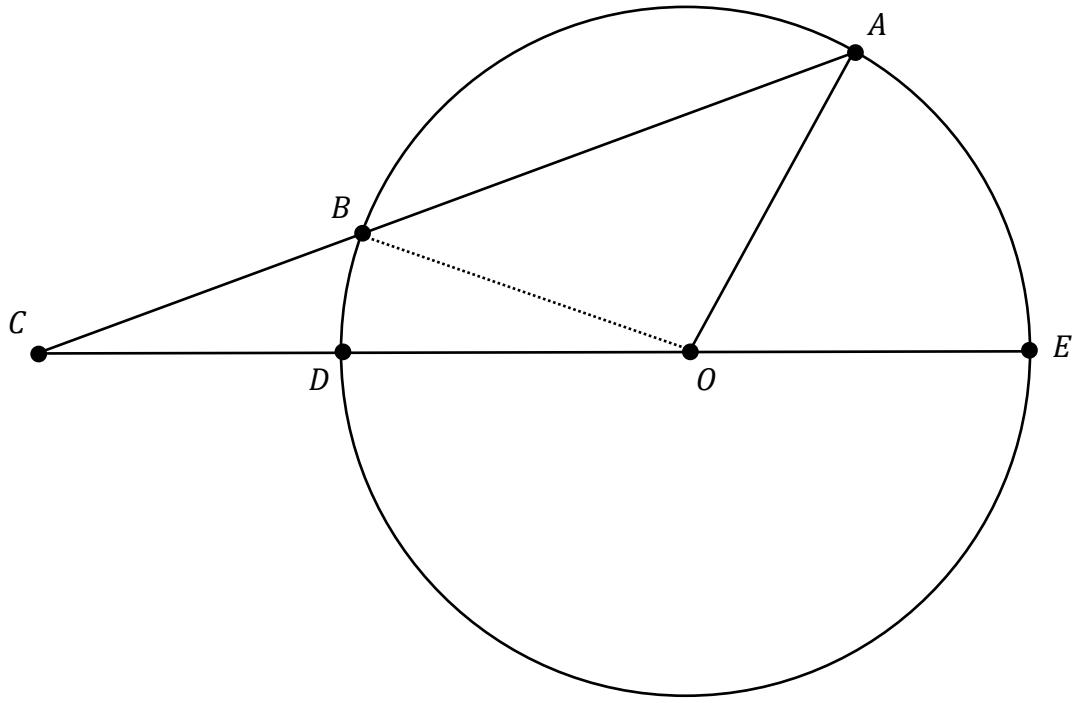
(b) Another circle has a circumference of 30 cm.
Work out the **area** of this circle. Give your answer in cm^2, in terms of π.

(c) O is the centre of a different circle.
A and B are points on this circle, as shown in the diagram below.

The chord $[AB]$ is extended through B to C, so that $|BC|$ is equal to the radius of the circle.
CO intersects the circle at D and at E, with $D \in [CO]$.

Let $|\angle BCO| = x$.
Write $|\angle AOE|$ in terms of x.

Show your working out.



Question 4 **(30 marks)**

(a) Find the value of $\cos 105°$ without using a calculator.

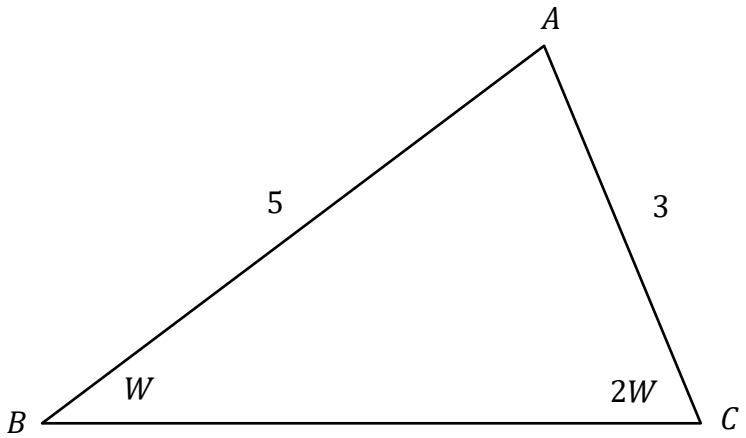
Give your answer in the form $\frac{1-\sqrt{a}}{b\sqrt{b}}$, where $a, b \in \mathbb{N}$. Show your working out.

(b) **(i)** Prove that $\sin 2A = 2 \sin A \cos A$.

(ii) The triangle ABC is shown below.
$|AB| = 5,\ |AC| = 3, |\angle ABC| = W$, and $|\angle ACB| = 2W$, where $0° < W < 45°$.

Find the value of $\cos W$, in the form $\frac{p}{q}$, where $p, q \in \mathbb{N}$.

Use the result from part **(b)(i)** and the Sine Rule.



$\cos W =$ _____

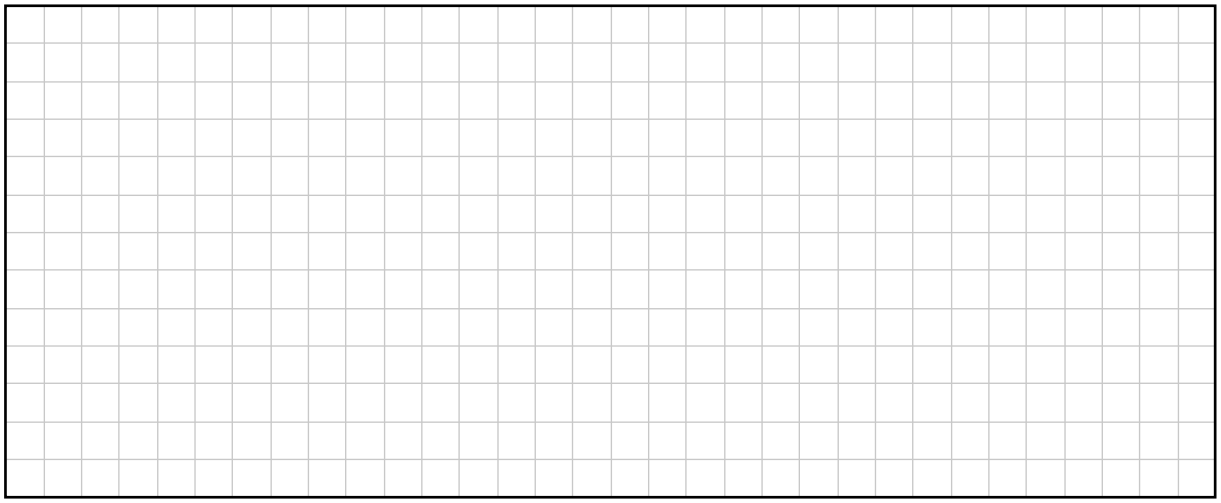
Question 5 **(30 marks)**

(a) Assume that the margin of error for a population proportion is $\frac{1}{\sqrt{n}}$, where n is the sample size.

(i) A survey is carried out on a random sample of 1600 people.

Find the margin of error for a population proportion, based on this sample.
Give your answer as a percentage.

(ii) What sample size would be needed to give a margin of error of exactly 1%?



(b) A, B, C, D, E and F are six events.

(i) A and B are independent. $P(A) = 0{\cdot}2$ and $P(B) = 0{\cdot}3$.
Find the value of $P(A \cap B)$.

(ii) $P(C) = 0{\cdot}4$, $P(D) = 0{\cdot}5$, and $P(C \cap D) = 0{\cdot}08$.
Find the value of $P(C \cup D)$.

(iii) $P(E) = 0{\cdot}6$, $P(F) = x + 0{\cdot}4$, and $P(E \cap F) = x$, where $x \in \mathbb{R}$ is a constant.

Find the value of x for which:

$$P(E|F) = \frac{1}{4}$$

Question 6 **(30 marks)**

(a) A 7-letter code is made using the letters A, B, C, D, E, F, and G.
Each letter is used exactly once in the code.

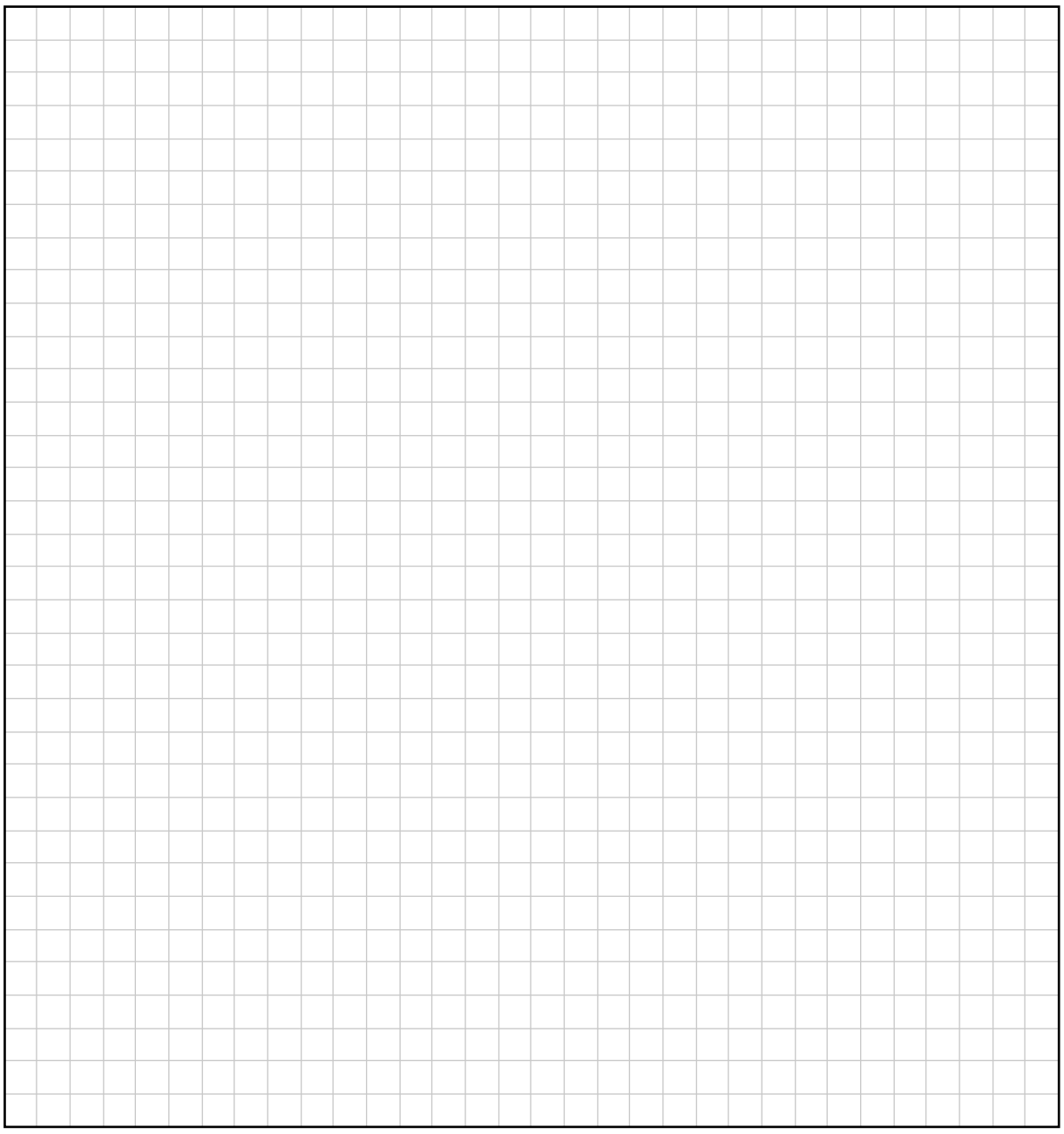
(i) How many such codes are there?

(ii) In how many of these codes are the two vowels (A and E) together?

(b) The mode of 6 positive whole numbers is 2.
The median of the 6 numbers is $5{\cdot}5$.

Find the 6 such numbers that have the **smallest** possible mean.

Answer: ☐ , ☐ , ☐ , ☐ , ☐ , ☐



Section B — Contexts and Applications — 150 marks

Answer **any three** questions from this section.

Question 7 **(50 marks)**

Fiadh owns a forest.

(a) The heights of trees in a forest of this type are usually normally distributed, with a mean height of 360 cm and a standard deviation of 27 cm.

Use this information to answer parts **(a)(i)**, **(a)(ii)**, and **(b)**.

(i) One tree is picked at random from this forest.
Find the probability that the height of this tree is between 330 cm and 390 cm.

(ii) The shortest (smallest) 5% of the trees in the forest are **not** sent to the factory when the forest is harvested, as they are not tall enough.

What is the minimum height that a tree needs to be in order for it to be sent to the factory when the forest is harvested?

(b) Fiadh claims that the mean height of the trees in her forest is different to the usual height, because they have become infected with a disease. She picks 100 trees at random from her forest and measures their height. The mean height of this sample is 350 cm.

Test the hypothesis, at the 5% level of significance, that the mean height of the trees in the forest is different to the usual height of 360 cm.

The null hypothesis is given below. State your alternative hypothesis, state your conclusion, and give a reason for your conclusion.

Null Hypothesis:

"The mean height of the trees in Fiadh's forest is 360 *cm."*

Alternative Hypothesis:

Calculations:

Conclusion:

Reason for your conclusion:

This question continues on the next page.

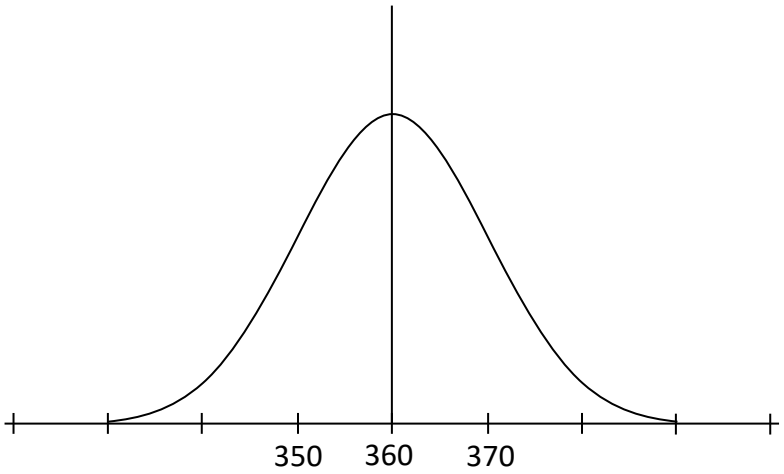
(c) Fiadh wants to estimate the number of trees that have become infected with the disease. She picks a random sample of 160 trees and examines them.
48 of these trees are infected with the disease.

(i) Find the 95% confidence interval for the proportion of trees in the forest that are infected. Give each value in your answer correct to 3 significant figures.

(ii) Explain what this 95% confidence interval means, in the context of the question.

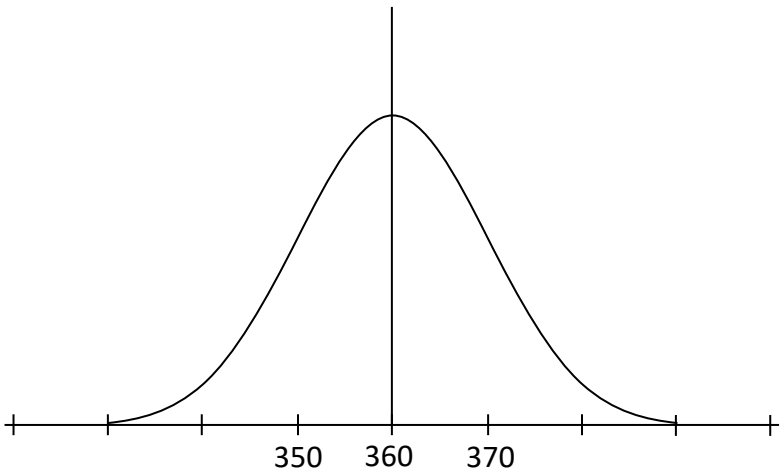
(d) **(i)** The diagram below shows a normal curve with a mean of 360.

On the same diagram, draw a new curve that will have the same standard deviation as the given curve, but with a mean of 350.
Use the same axes and scales.



(ii) The diagram below shows the same normal curve as above, with a mean of 360.

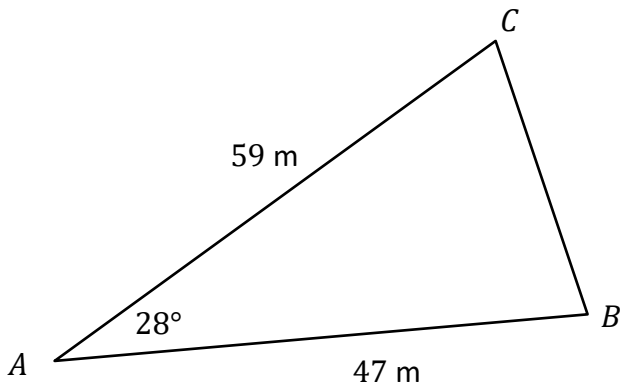
On the diagram below, draw a new curve that will also have a mean of 360, but that will have a smaller standard deviation than the given curve.
Use the same axes and scales.



Question 8 **(50 marks)**

Henryk has a farm.

(a) The diagram below shows one of Henryk's fields.
It is in the shape of a triangle ABC.
$|AC| = 59$ m, $|AB| = 47$ m, and $|\angle CAB| = 28°$, as shown.



(i) Find the area of the field, correct to the nearest m^2.

(ii) Work out the length of the side $[BC]$, correct to 2 decimal places.

Henryk has two balers: a square baler and a round baler.
These are machines that pick up straw from a field and make it into bales.

(b) Henryk's square baler travels at an average speed of 8 km/hour while it is baling.
For one of his fields, this baler travels 4800 m while it is baling.

Work out how many minutes this will take.

(c) This baler makes bales in the shape of **rectangular** solids.

A bale has a volume of $0{\cdot}415$ m^3, correct to 3 decimal places.
The ratio of the dimensions of this bale is:

$$\text{height : width : length} = 2 : 3 : 5.$$

Find the dimensions of this bale. Give each value correct to the nearest cm.

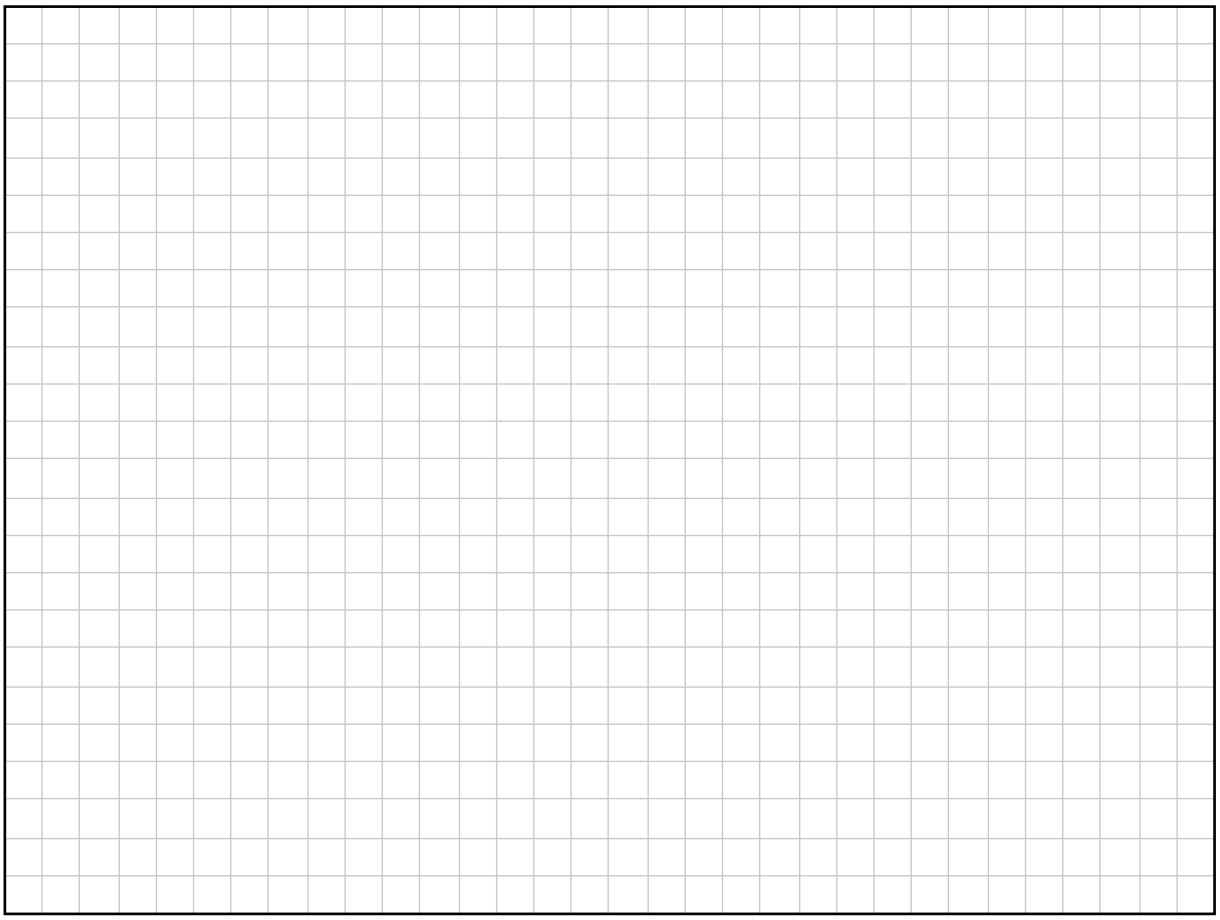
height = _____ width = _____ length = _____

This question continues on the next page.

(d) Henryk's round baler makes bales in the shape of **cylinders**.
The height of each cylinder is equal to its diameter.

The volume of a cylindrical bale is $2{\cdot}780$ m^3, correct to 3 decimal places.

Work out the radius of a cylindrical bale.
Give your answer in metres, correct to the nearest cm.

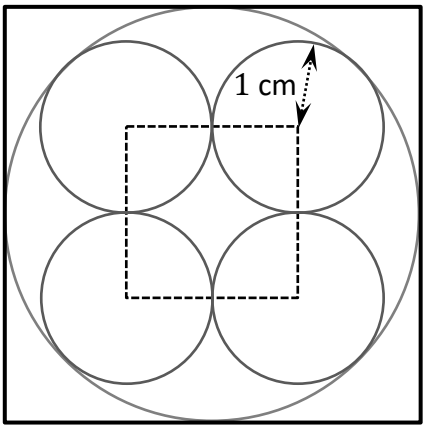


(e) Henryk is tiling his patio. The tiles are square, with a design made of circles.

The design consists of a large circle inscribed inside the square tile.
There are 4 smaller circles of equal size inside the large circle.
The centres of these 4 circles form a square, and each small circle touches two other small circles and the large circle, as shown in the diagram below.

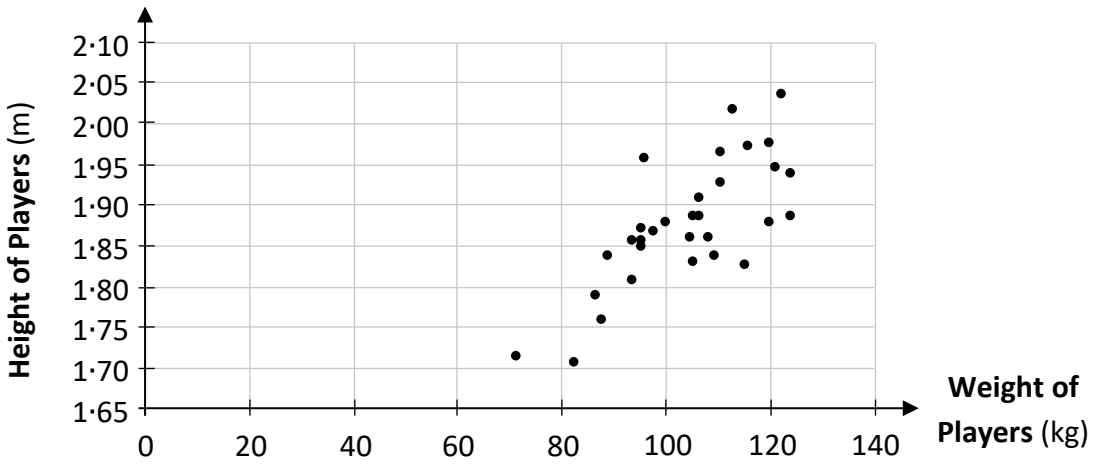
The radius of each small circle is 1 cm.

Work out the area of the square tile (the large square in the diagram below).
Give your answer in the form $\left(a + b\sqrt{c}\right)$ cm^2, where $a, b, c \in \mathbb{Z}$.



Question 9 **(50 marks)**

(a) The scatterplot below shows the height and the weight of each player on a rugby team, in metres and in kilograms, respectively.



(i) One of the numbers below is the correct correlation coefficient for the data above. Put a tick (✓) in the correct box to show which one is the correct value.
Tick **one** box only. Justify your answer.

$0{\cdot}4$	$-0{\cdot}3$	$0{\cdot}9$	$-0{\cdot}8$
☐	☐	☐	☐

Justification:

(ii) On the scatterplot above, draw the line of best fit for the data, by eye, **and** use this line to estimate the weight of a rugby player that is $1{\cdot}8$ m tall.

Weight: _____

(b) This particular rugby team usually miss 8% of the tackles that they attempt.
Use this fact to answer parts **(b)(i)** and **(b)(ii)**. Assume that all tackles are independent.
Give each answer correct to 4 decimal places.

(i) Find the probability that the 4th tackle in a game is the first one that is missed.

(ii) Find the probability that the team miss **at least 2** tackles out of the first 20 tackles that they attempt in a game.

This question continues on the next page.

(c) At a particular stage of the game, this rugby team will score either 7 points, 5 points, or 0 points. The probability of each of these is x, y, and z, respectively, where $x, y, z \in \mathbb{R}$.

Points	7	5	0
Probability	x	y	z

The probability of getting 7 points is four times the probability of getting 5 points.
The expected value of the number of points is $1{\cdot}155$.

Work out the value of z, the probability of scoring 0 points.

(d) The coach of the rugby team must pick 3 forwards from a total of 7 possible forwards.

(i) How many different combinations of 3 forwards can the coach pick?

(ii) On Saturday, the coach picks 3 forwards at random from the 7 forwards.
On Sunday, the coach again picks 3 forwards at random from the same 7 forwards.

Find the probability that **no forward** is picked on **both** Saturday and Sunday.

Question 10 **(50 marks)**

Chloë is cycling her bike.

Chloë cycles over a small nail.
Afterwards, she keeps cycling at the same speed, with the nail stuck in her front wheel.
As the front wheel rotates, the height of the nail increases and decreases, and is given by:

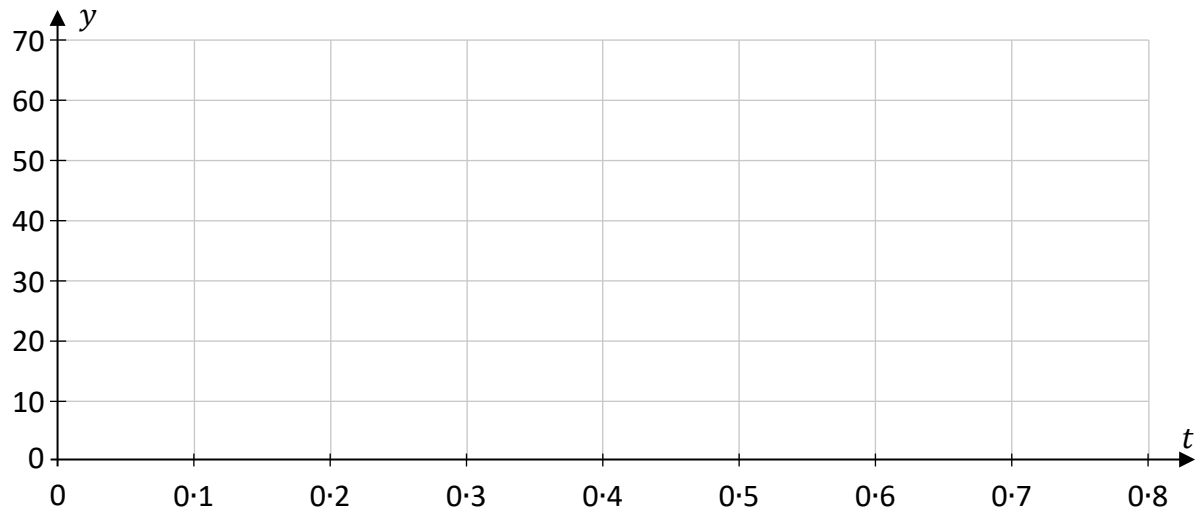
$$h(t) = 30 - 30\cos(5\pi t),$$

where h is the height of the nail, in cm, t seconds after the nail gets stuck in the wheel, $t \in \mathbb{R}$, $t \geq 0$, and $5\pi t$ is in **radians**.

(a) **(i)** Complete the table below. The value of $h(0{\cdot}2)$ is given.

t	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8
$h(t)$			60						

(ii) Draw the graph of $y = h(t)$ for $0 \leq t \leq 0{\cdot}8$, $t \in \mathbb{R}$.



(b) The period of $h(t)$ is $0{\cdot}4$ seconds.
Explain what this means in the context of Chloë's front wheel.

(c) The radius of the wheel is 30 cm.

(i) Work out the distance that Chloë travels in one full revolution (rotation) of the wheel. Give your answer correct to the nearest millimetre.

(ii) Using your answer to **(c)(i)**, and the period of $h(t)$, work out the speed that Chloë is cycling at. Give your answer correct to the nearest **km/hour**.

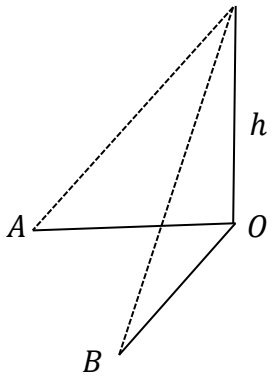
This question continues on the next page.

(d) The vertical height of a building is h metres.
The base of the building is at the point O.

A and B are two other points.
O, A, and B are all on horizontal ground.

From A, the angle of elevation to the top of the building is $45°$
(that is, the angle from O to A to the top of the building is $45°$).

As a result, $|OA| = h$ metres.

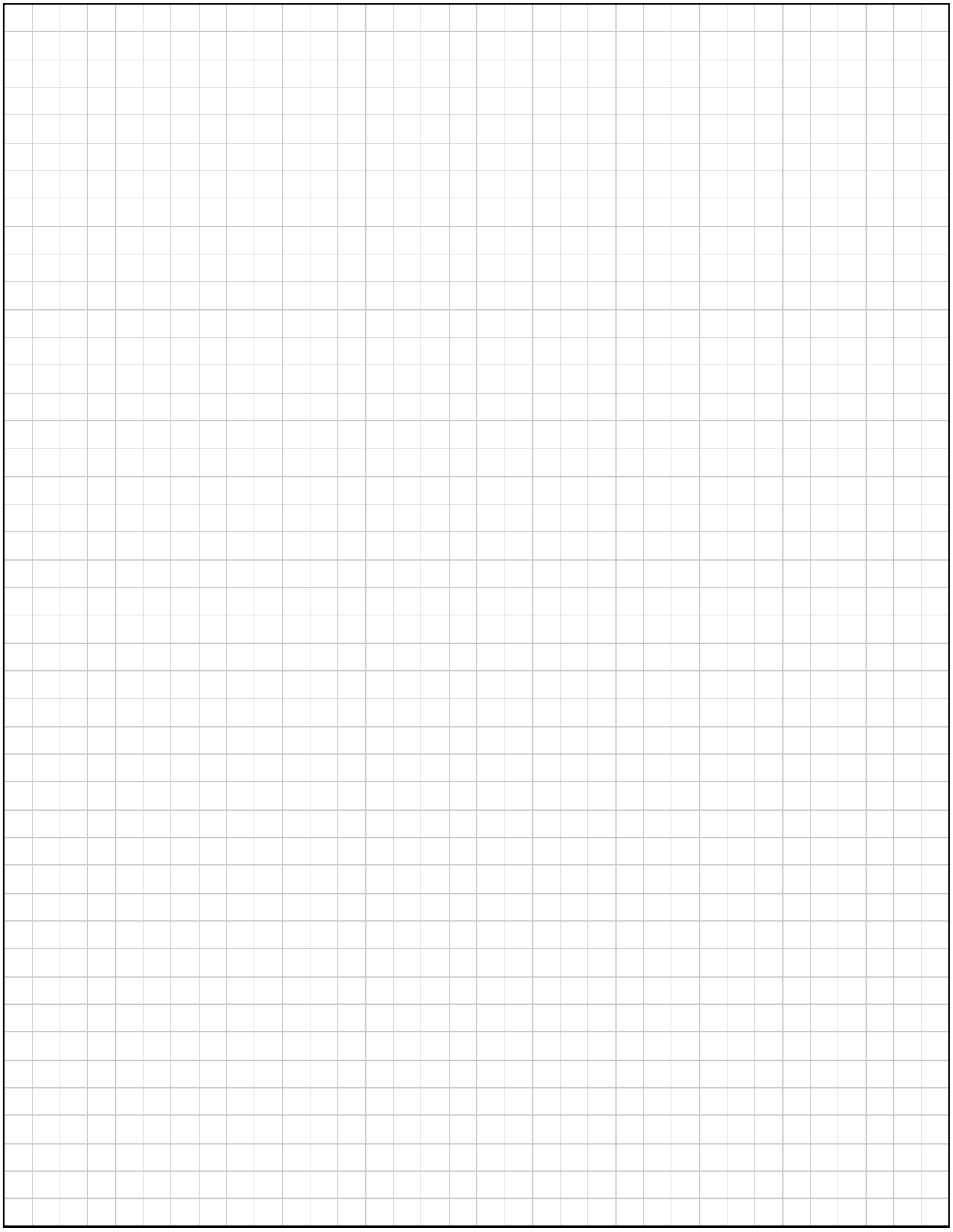


(i) From B, the angle of elevation to the top of the building is $30°$.
Write $|OB|$ in terms of h.

(ii) The distance $|AB| = 120$ metres and $|\angle AOB| = 60°$.

Given this information, use the triangle ABO to work out the value of h, the height of the building. Give your answer in metres, correct to 1 decimal place.

Page for extra work.
Label any extra work clearly with the question number and part.



Leaving Certificate 2024 – Higher Level

Mathematics – Paper 2

2 hours 30 minutes