



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2012

**Mathematics
(Project Maths – Phase 1)**

Paper 2

Higher Level

Monday 11 June Morning 9:30 – 12:00

300 marks

Examination number

Centre stamp

Running total	
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For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	2 questions

Answer **all eight** questions, as follows:

In Section A, answer:

Questions 1 to 5 and
either Question 6A **or** Question 6B.

In Section B, answer Question 7 and Question 8.

Write your answers in the spaces provided in this booklet. You will lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

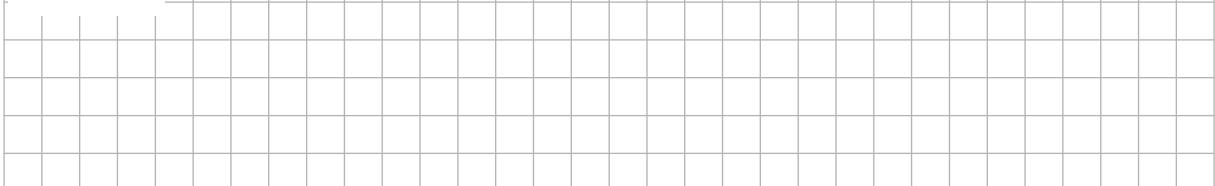
Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

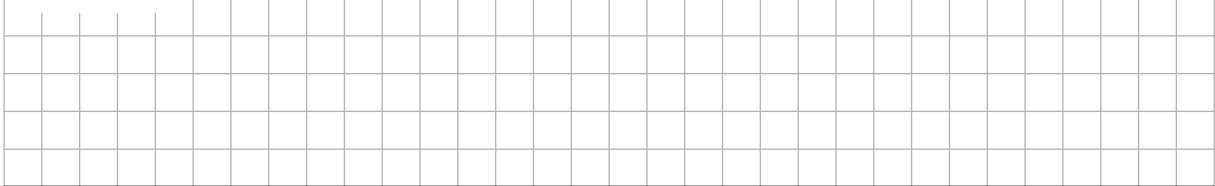
Question 1**(25 marks)**

- (a) Given the co-ordinates of the vertices of a quadrilateral $ABCD$, describe **three** different ways to determine, using co-ordinate geometry techniques, whether the quadrilateral is a parallelogram.

method 1:



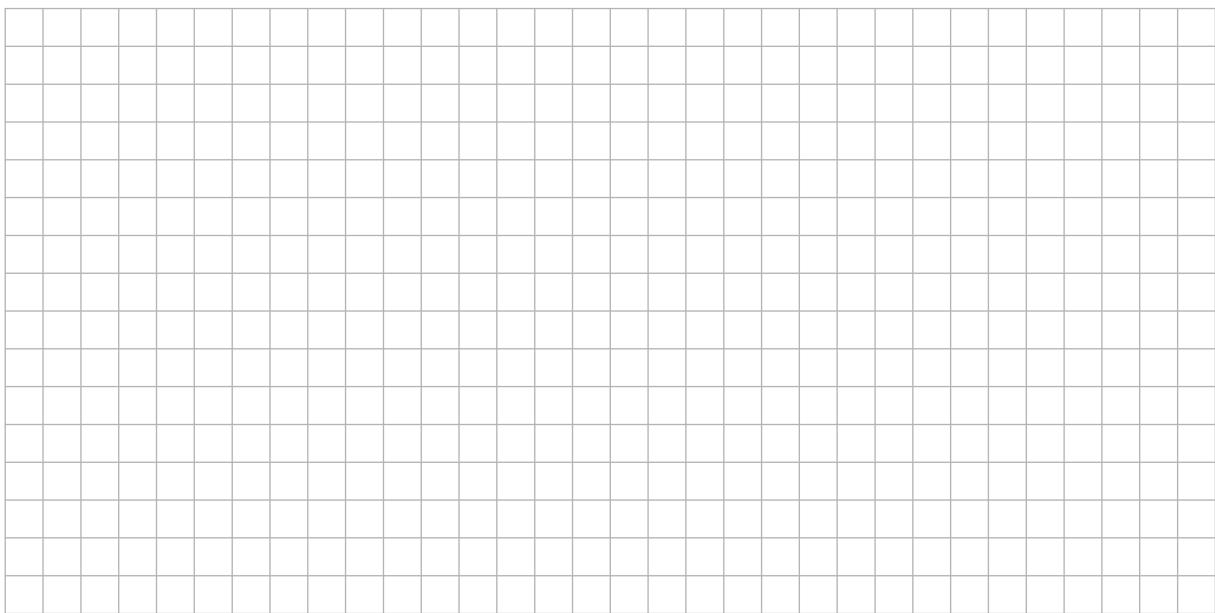
method 2:



method 3:



- (b) Using **one** of the methods you described, determine whether the quadrilateral with vertices $(-4, -2)$, $(21, -5)$, $(8, 7)$ and $(-17, 10)$ is a parallelogram.



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Question 2

(25 marks)

The equations of two circles are:

$$c_1 : x^2 + y^2 - 6x - 10y + 29 = 0$$

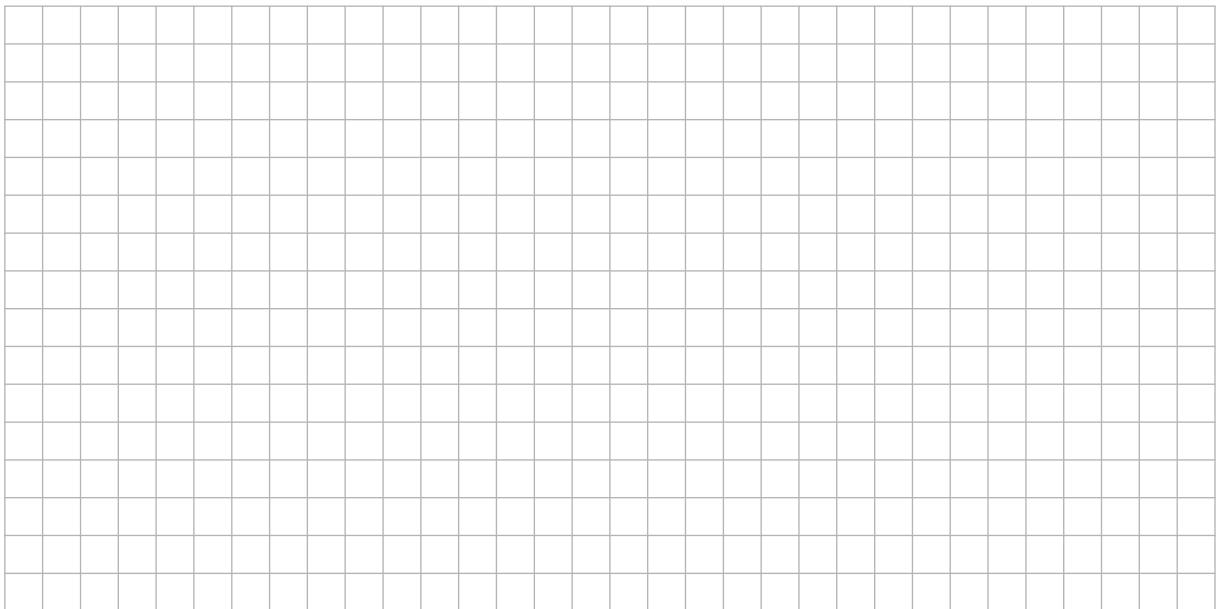
$$c_2 : x^2 + y^2 - 2x - 2y - 43 = 0$$

- (a) Write down the centre and radius-length of each circle.

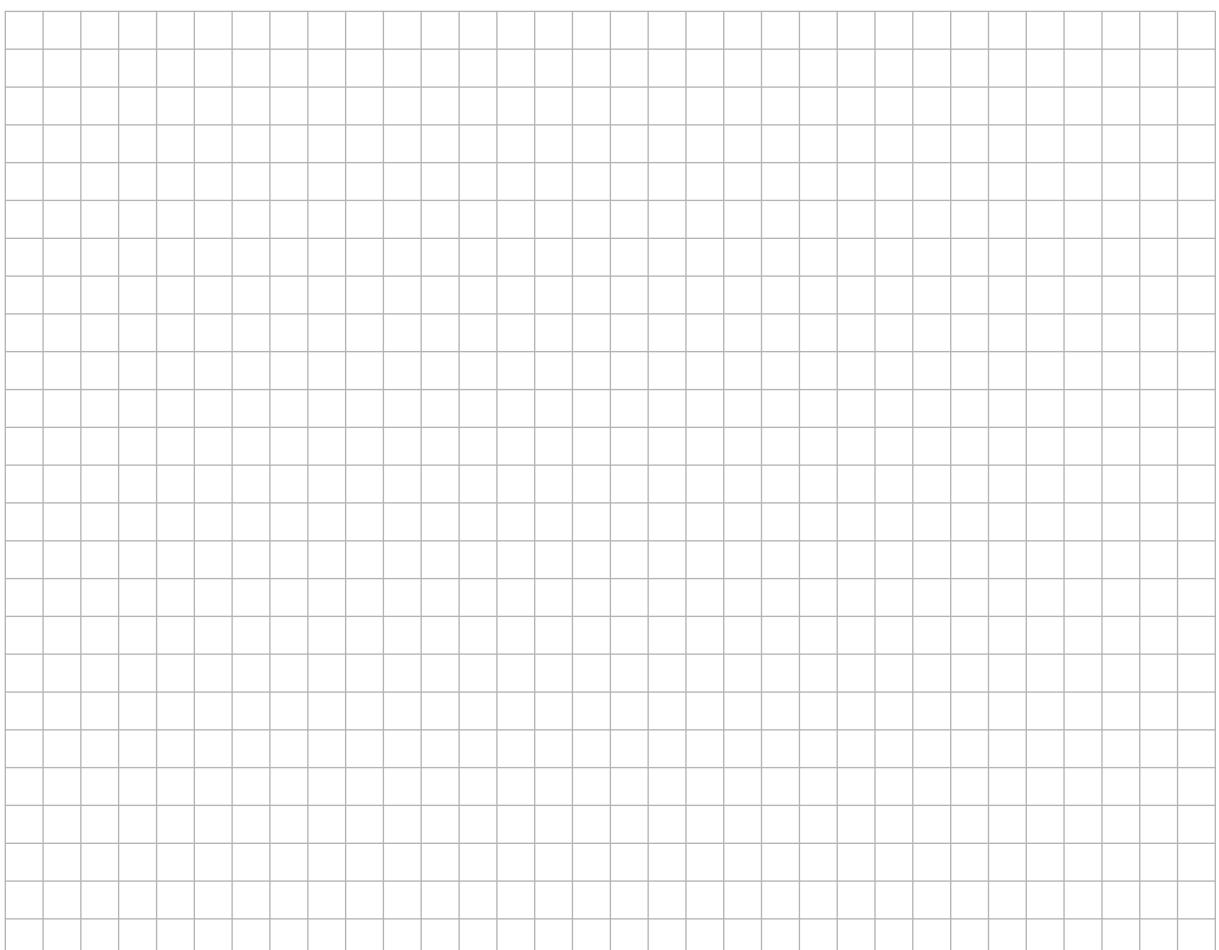
<p>centre of c_1: _____</p> <p>radius-length of c_1: _____</p>	<p>centre of c_2: _____</p> <p>radius-length of c_2: _____</p>
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- (b) Prove that the circles are touching.

- (c) Verify that $(4, 7)$ is the point that they have in common.



- (d) Find the equation of the common tangent.

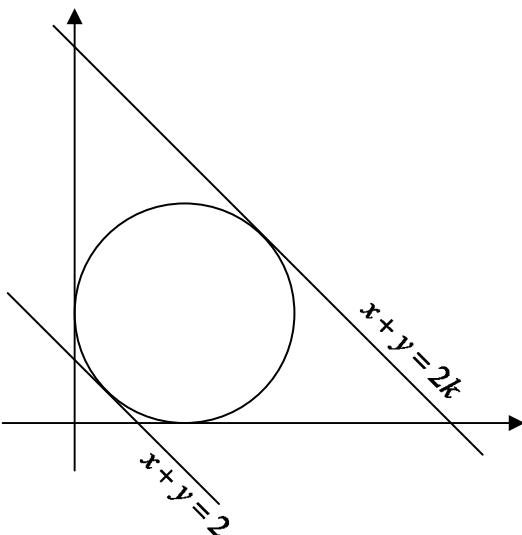


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Question 3**(25 marks)**

The circle shown in the diagram has, as tangents, the x -axis, the y -axis, the line $x + y = 2$ and the line $x + y = 2k$, where $k > 1$.

Find the value of k .



Question 4

(25 marks)

A certain basketball player scores 60% of the free-throw shots she attempts. During a particular game, she gets six free throws.

- (a) What assumption(s) must be made in order to regard this as a sequence of Bernoulli trials?

- (b) Based on such assumption(s), find, correct to three decimal places, the probability that:

- (i) she scores on exactly four of the six shots

- (ii) she scores for the second time on the fifth shot.

Question 5

(25 marks)

A company produces calculator batteries. The diameter of the batteries is supposed to be 20 mm. The tolerance is 0·25 mm. Any batteries outside this tolerance are rejected. You may assume that this is the only reason for rejecting the batteries.

- (a)** The company has a machine that produces batteries with diameters that are normally distributed with mean 20 mm and standard deviation 0·1 mm. Out of every 10 000 batteries produced by this machine, how many, on average, are rejected?



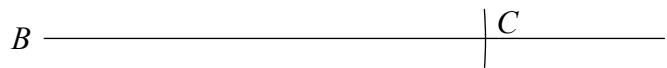
- (b) A setting on the machine slips, so that the mean diameter of the batteries increases to 20.05 mm, while the standard deviation remains unchanged. Find the percentage increase in the rejection rate for batteries from this machine.

Question 6**(25 marks)**

Answer either 6A or 6B.

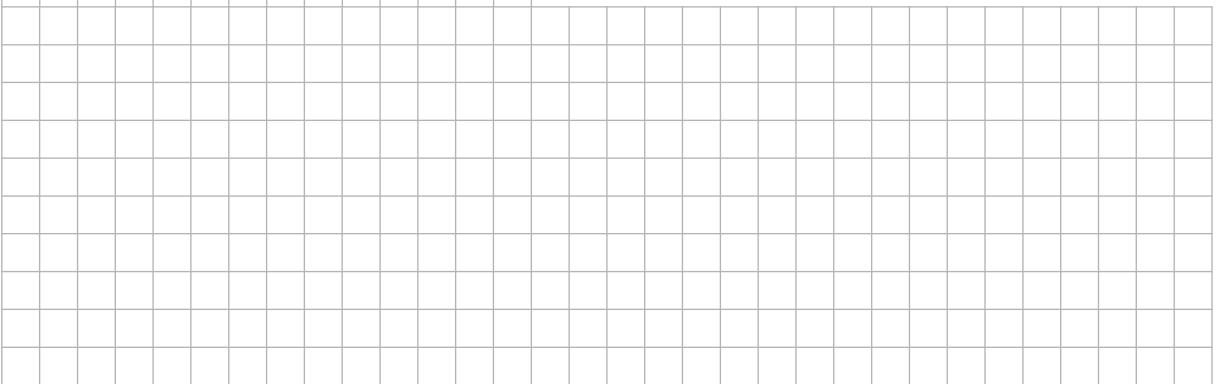
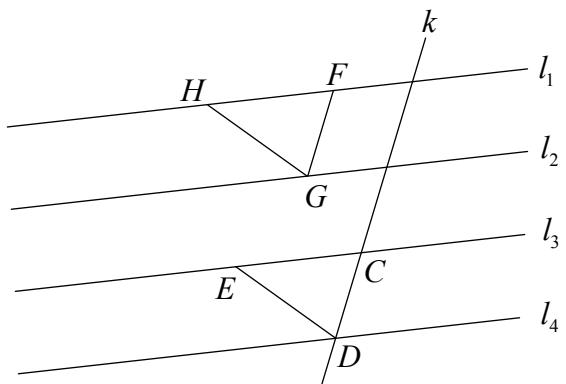
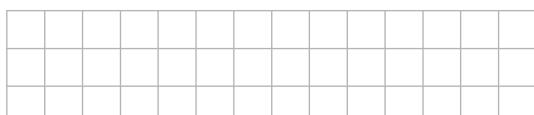
Question 6A

- (a) (i) Given the points B and C below, construct, without using a protractor or setsquare, a point A such that $|\angle ABC| = 60^\circ$.

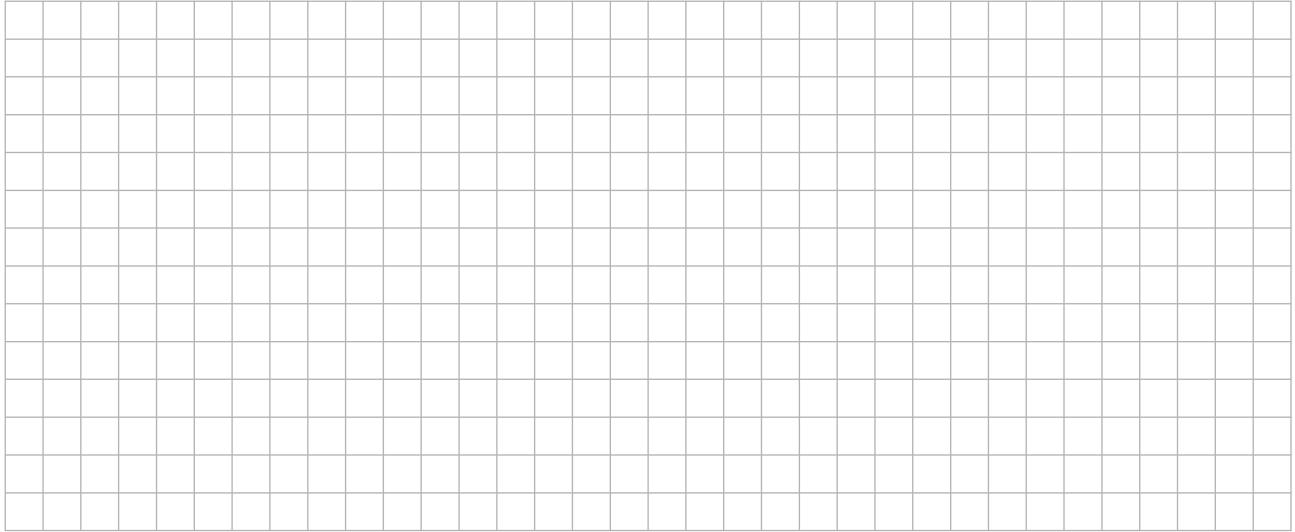


- (ii) Hence construct, on the same diagram above, and using a compass and straight edge only, an angle of 15° .

- (b) In the diagram, l_1, l_2, l_3 , and l_4 are parallel lines that make intercepts of equal length on the transversal k . FG is parallel to k , and HG is parallel to ED .
Prove that the triangles ΔCDE and ΔFGH are congruent.

*There is space to continue your work on the next page.*

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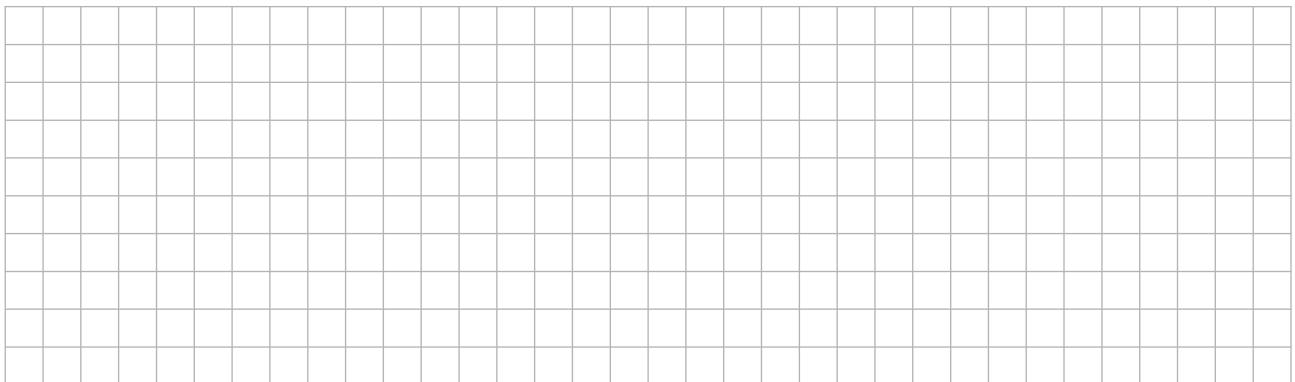
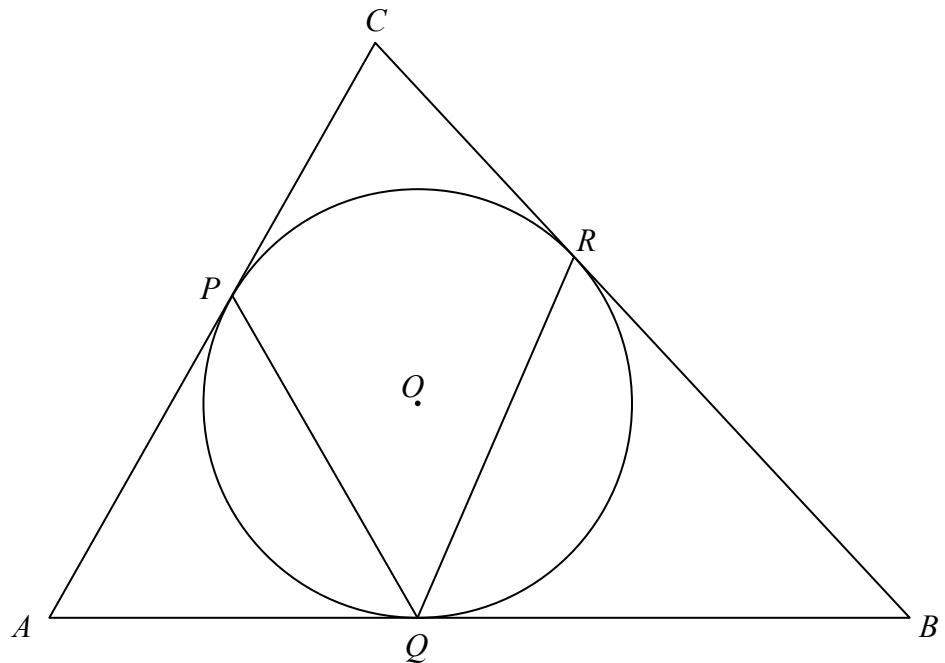


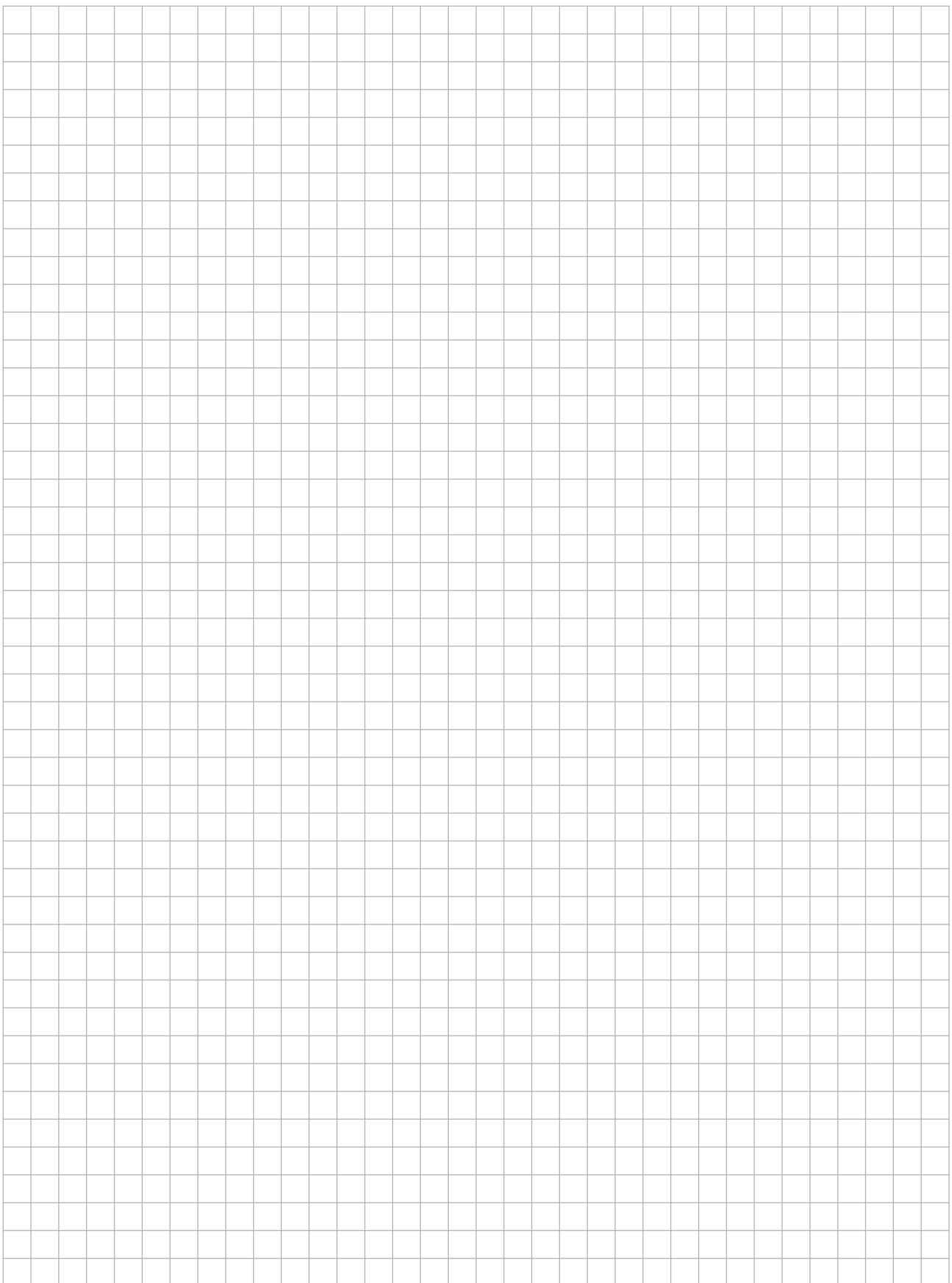
OR

Question 6B

The incircle of the triangle ABC has centre O and touches the sides at P , Q and R , as shown.

$$\text{Prove that } |\angle PQR| = \frac{1}{2}(|\angle CAB| + |\angle CBA|).$$





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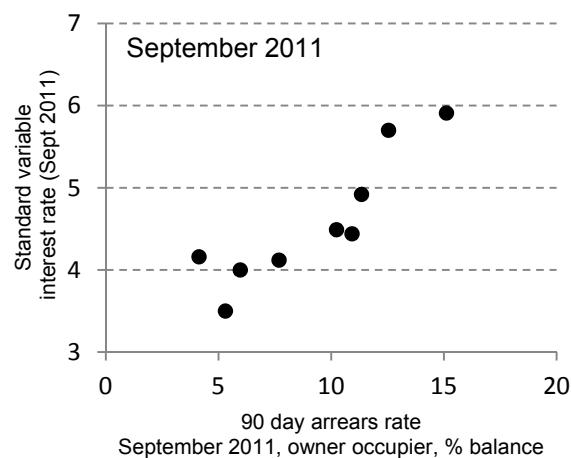
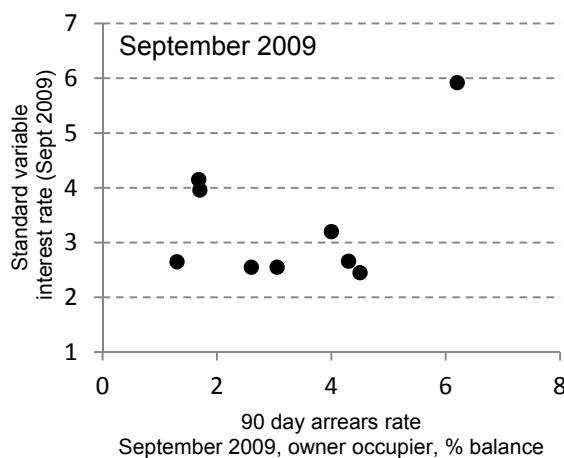
Answer Question 7 and Question 8.

Question 7

(75 marks)

To buy a home, people usually take out loans called *mortgages*. If one of the repayments is not made on time, the mortgage is said to be *in arrears*. One way of considering how much difficulty the borrowers in a country are having with their mortgages is to look at the percentage of all mortgages that are in arrears for 90 days or more. For the rest of this question, the term *in arrears* means in arrears for 90 days or more.

The two charts below are from a report about mortgages in Ireland. The charts are intended to illustrate the connection, if any, between the percentage of mortgages that are in arrears and the interest rates being charged for mortgages. Each dot on the charts represents a group of people paying a particular interest rate to a particular lender. The arrears rate is the percentage in arrears.



(Source: Goggin et al.*Variable Mortgage Rate Pricing in Ireland*, Central Bank of Ireland, 2012)

- (a) Paying close attention to the scales on the charts, what can you say about the change from September 2009 to September 2011 with regard to:

- (i) the arrears rates?

- (ii) the rates of interest being paid?

- (iii) the relationship between the arrears rate and the interest rate?

- (b) What additional information would you need before you could estimate the median interest rate being paid by mortgage holders in September 2011?

- (c) Regarding the relationship between the arrears rate and the interest rate for September 2011, the authors of the report state: “The direction of causality ... is important” and they go on to discuss this.

Explain what is meant by the “direction of causality” in this context.

- (d) A property is said to be in “negative equity” if the person owes more on the mortgage than the property is worth. A report about mortgaged properties in Ireland in December 2010 has the following information:

- Of the 475 136 properties examined, 145 414 of them were in negative equity.
 - Of the ones in negative equity, 11 644 were in arrears.
 - There were 317 355 properties that were neither in arrears nor in negative equity.

- (i) What is the probability that a property selected at random (from all those examined) will be in negative equity?

Give your answer correct to two decimal places.

- (ii) What is the probability that a property selected at random from all those in negative equity will also be in arrears?

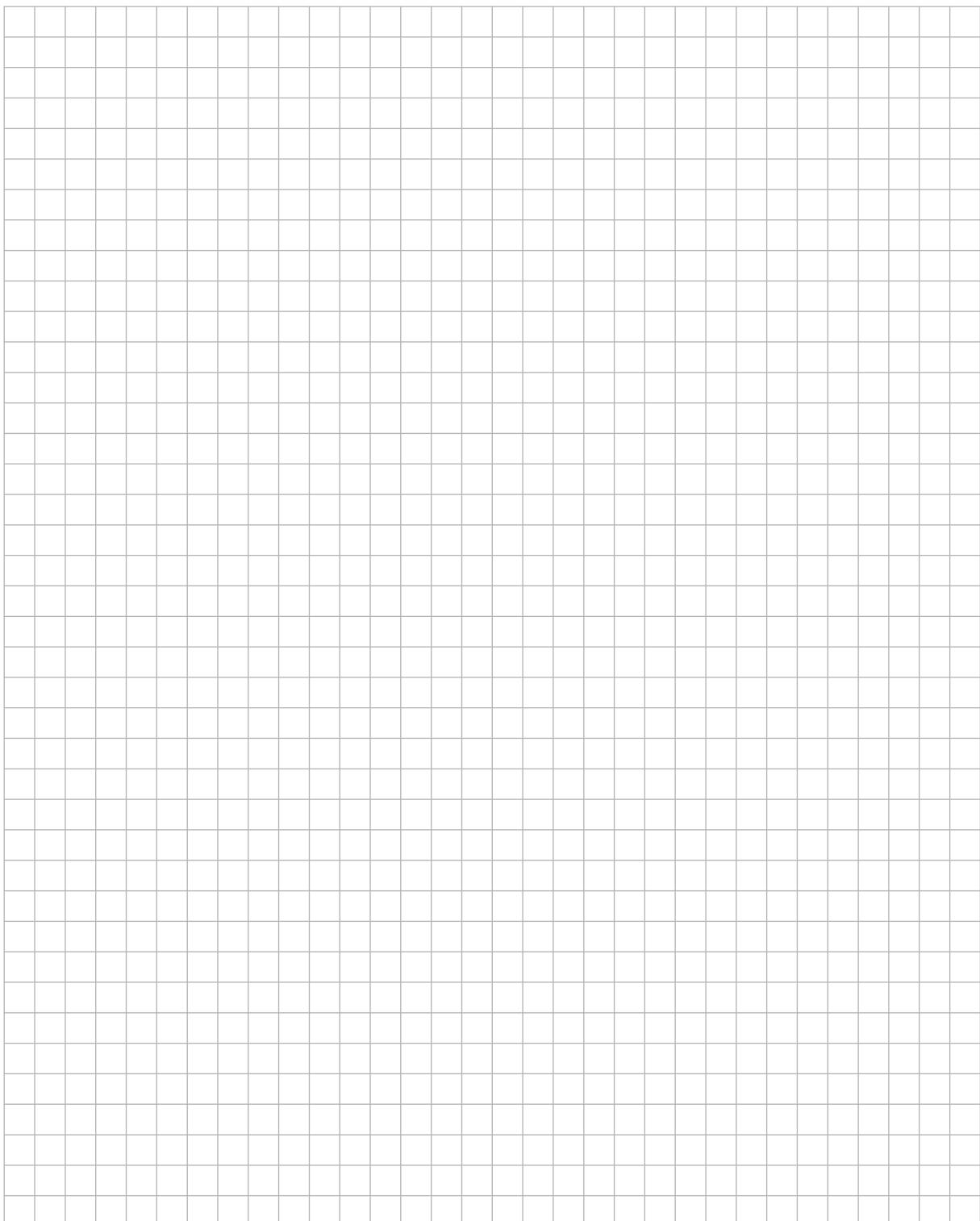
Give your answer correct to two decimal places.

- (iii) Find the probability that a property selected at random from all those in arrears will also be in negative equity.

Give your answer correct to two decimal places.

- (e) The study described in part (d) was so large that it can be assumed to represent the population. Suppose that, in early 2012, researchers want to know whether the proportion of properties in negative equity has changed. They analyse 2000 randomly selected properties with mortgages. They discover that 552 of them are in negative equity. Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to conclude that the situation has changed since December 2010.

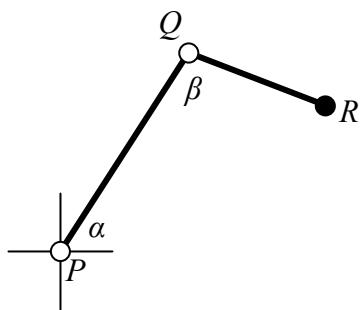
Be sure to state the null hypothesis clearly, and to state the conclusion clearly.

A large grid of squares, approximately 20 columns by 25 rows, intended for考生 to work out their calculations.

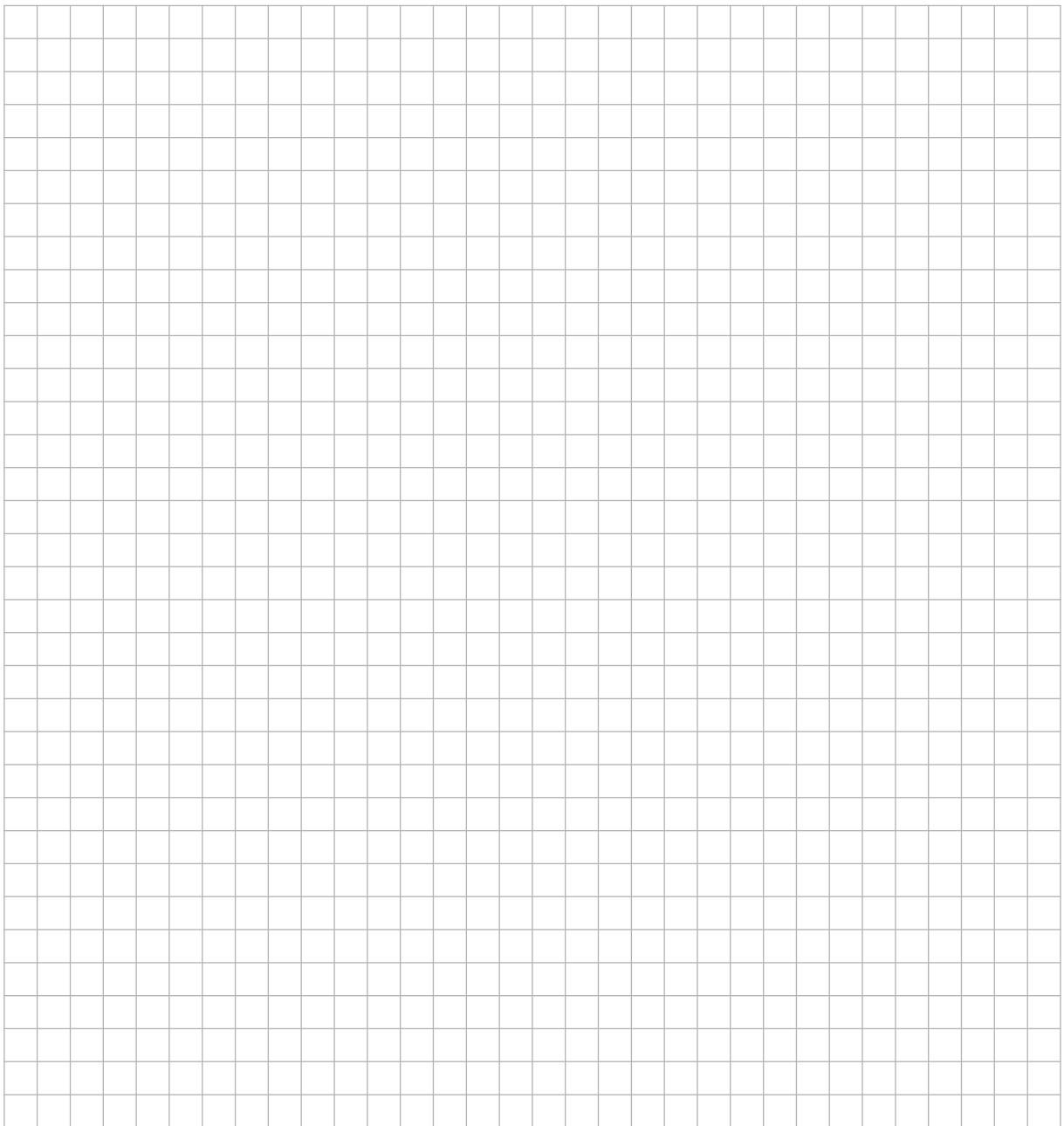
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Question 8**(75 marks)**

The diagram is a representation of a robotic arm that can move in a vertical plane. The point P is fixed, and so are the lengths of the two segments of the arm. The controller can vary the angles α and β from 0° to 180° .



- (a) Given that $|PQ| = 20$ cm and $|QR| = 12$ cm, determine the values of the angles α and β so as to locate R , the tip of the arm, at a point that is 24 cm to the right of P , and 7 cm higher than P . Give your answers correct to the nearest degree.



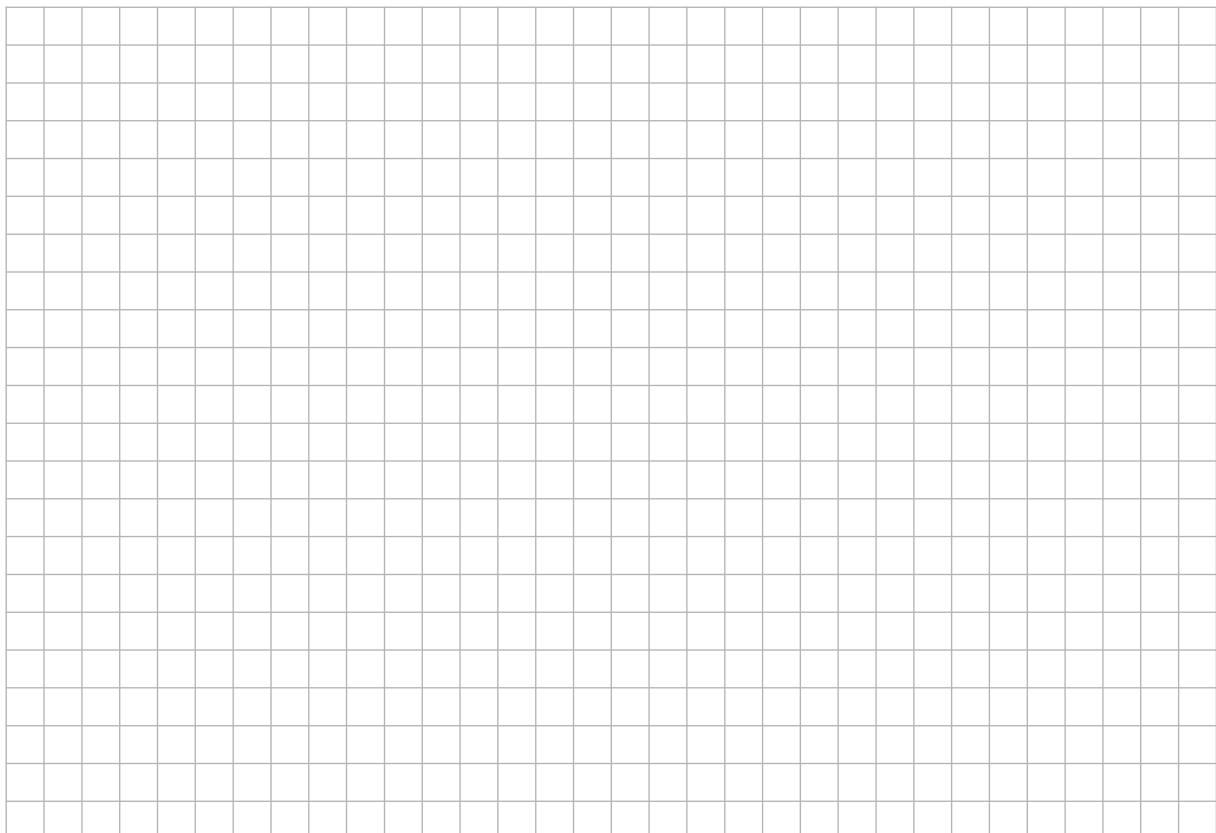
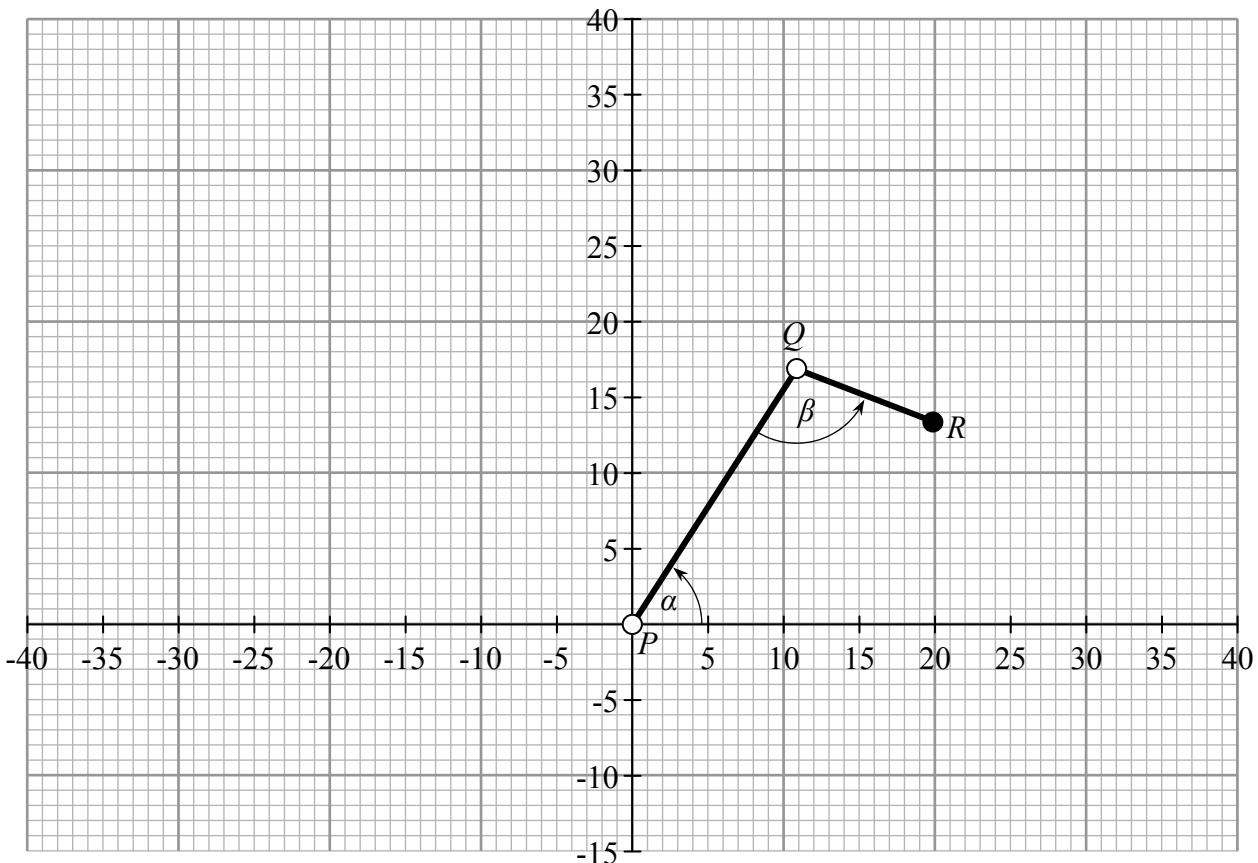
- (b) In setting the arm to the position described in part (a), which will cause the greater error in the location of R : an error of 1° in the value of α or an error of 1° in the value of β ?

Justify your answer. You may assume that if a point moves along a circle through a small angle, then its distance from its starting point is equal to the length of the arc travelled.

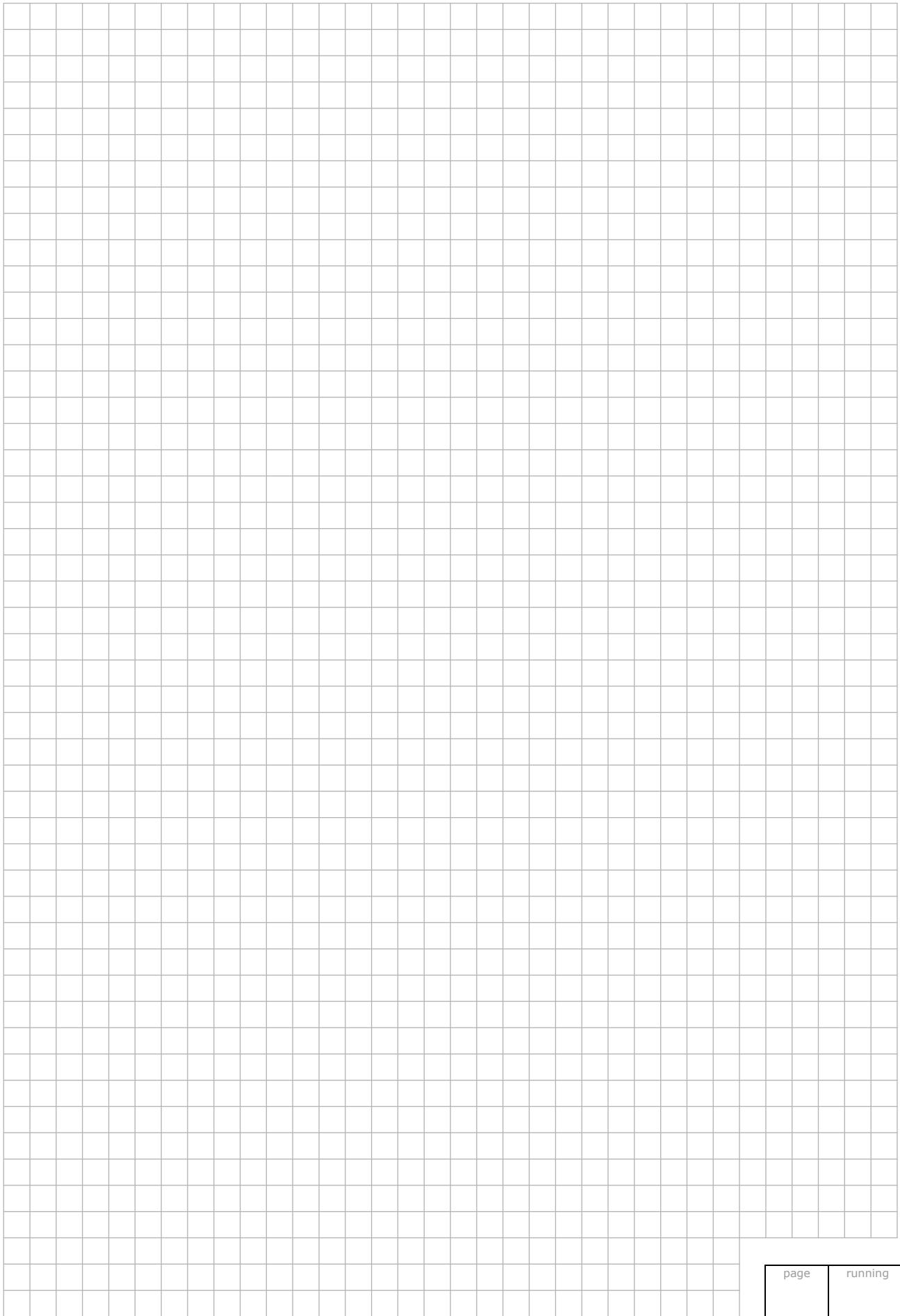
- (c) The answer to part (b) above depends on the particular position of the arm. That is, in certain positions, the location of R is more sensitive to small errors in α than to small errors in β , while in other positions, the reverse is true. Describe, with justification, the conditions under which each of these two situations arises.

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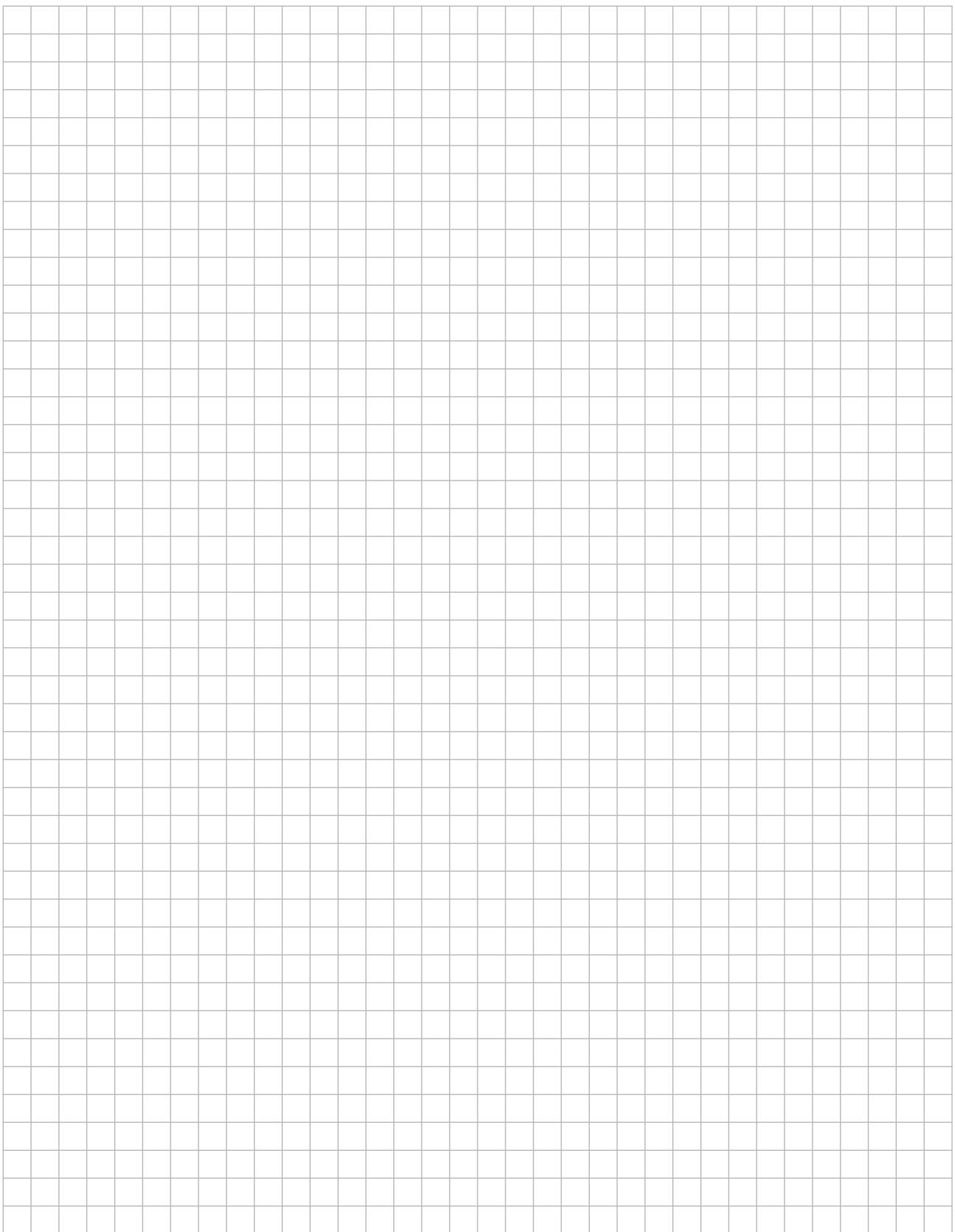
- (d) Illustrate the set of all possible locations of the point R on the coordinate diagram below. Take P as the origin and take each unit in the diagram to represent a centimetre in reality. Note that α and β can vary only from 0° to 180° .



You may use this page for extra work.

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Leaving Certificate 2012 – Higher Level

Mathematics (Project Maths – Phase 1) – Paper 2

Monday 11 June

Morning 9:30 – 12:00