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LEAVING CERTIFICATE EXAMINATION, 1968

MATHEMATICS (HONOURS) – PAPER II (300 marks)

MONDAY, 17th JUNE – MORNING 10 to 12.30

Six questions to be answered. All questions are of equal value. Mathematical Tables may be obtained from the Superintendent.

N is the set of natural numbers
R is the set of real numbers.

1. Find the positive root of the equation $2x^3 + 10x^2 + 3x - 30 = 0$ correct to two places of decimals.

2. (a) If $z = x + iy$, $(x, y \in R)$, find the value of x and the value of y when

$$\frac{z+i}{z-i} = \frac{z+1}{z-3}.$$

(b) Write down the factors of $x^2 + y^2$, $(x, y \in R)$ and hence express $(x^2 + y^2)^3$ as the sum of two squares.

3. The information in the following table is the result of a survey of houses that have amenities as listed;

Amenity	Central Heating	Phone	Television	Garage
Percentage of houses	65	75	80	85

At least what percentage of houses have

(a) Central heating and phone ?
(b) Central heating, phone and television ?
(c) All four amenities ?

4. (a) Show that $1 + \frac{1}{2} + (\frac{1}{3} + \frac{1}{4}) + (\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}) > 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ and use this result to show that the series

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots \text{ is not convergent.}$$

The series $\sum_{n=1}^{\infty} u_n = u_1 + u_2 + u_3 + \dots + u_n + \dots$ is convergent. Show that

$$u_n = S_n - S_{n-1}$$

where S_n is the sum to n terms of the series and hence deduce that $\lim_{n\to\infty} u_n = 0$.

Give an example to show that the statement "$\lim_{n\to\infty} u_n = 0$ implies $\sum_{n=1}^{\infty} u_n$ is convergent" is false.

(b) State the ratio test for the convergence of a series of positive terms.
Use the ratio test to prove that the series

$$\sum_{n=1}^{\infty} n^2 x^n$$

converges for $0<x<1$.
Does this series converge for $0<x\leq1$? Give your reasons.

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5. (a) Differentiate from first principles $\sin x$ with respect to x.
Differentiate with respect to x:

$$\frac{x}{\sqrt{1+x}}, \quad \sin^2\sqrt{1+x}, \quad \log \sin^2 x.$$

(b) If $y = e^{-\alpha x}\cos nx$, (α, n independent of x) show that $\frac{d^2y}{dx^2} + 2\alpha\frac{dy}{dx} + (\alpha^2 + n^2)y = 0$.

6. Two sources of heat A and B are connected by a straight iron rod 20 ft. in length and the rod conducts the heat from source to source. The source B gives out 10 times as much heat as the source A. Assuming that the quantity of heat arriving at any point of the rod from a given source varies inversely as the square of the distance of the point from the source, find the distance from A of that point of the rod which receives least heat.

(Hint: $\frac{k}{x^2}$ is the quantity of heat arriving from A at a point of the rod x ft. from A, where k is a constant.)

7. (a) (i) Evaluate $\int_0^1 (x+1)(2x+3)dx$.

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin 2t\, dt}{\sqrt{1-\cos 2t}}$.

(iii) Use the approximation $e^t = 1 + t + \frac{1}{2!}t^2$ to evaluate $\frac{1}{2\cdot 5}\int_0^1 e^{-\frac{x^2}{2}}dx$.

(b) The area bounded by the curve $y = x^2$ and the line $y = 4$ is divided into two equal areas by the line $y = a$. Show that $a^3 = 16$.

8. Find the number of ways it is possible to arrange a, a, a, b, b in a row taking the five letters all at a time.
An unbiassed penny is tossed 40 times. In how many ways is it possible to get exactly 25 heads ?
Write a short note on the Binomial Distribution mentioning its use and its formula.
In a football match the probability that a player scores from a free kick is $\frac{1}{4}$. From four free kicks find the probability that the player gets (i) one score (ii) at least one score.

9. (i) If $\frac{1}{\sqrt[3]{1+x}} = a_0 + a_1x + a_2x^2 + \dots$ find the values of the constants a_0, a_1, a_2.
Hence calculate the value of $\frac{1}{\sqrt[3]{65}}$ approximately.

(ii) If $x^{t+1} - y^{t+1} = A(x^t - y^t) + B(x - y)$, where $t \in N$, express A and B as polynomials in x or y.
Hence prove by induction that $17^n - 13^n (n \in N$ and $n \geq 1)$ is divisible by 4.

10. (i) If A and B are sets show that $(A' \cup B') \subset (A \cap B)'$.
Given also that $(A \cap B)' \subset A' \cup B'$, deduce that $(A \cap B)' = A' \cup B'$. Illustrate your answer by a diagram. (Note: A' means the complement of A).

(ii) Trace the graph $y^2 = \frac{x^2}{1 - x^2}$ paying special attention to the point of inflexion and to the shape of the graph as y tends to infinity.

or

10. In a given period a factory making chocolate and cheese can produce at most 1,500 cwt. A wholesaler who lives 15 miles away must be supplied with at least 300 cwt. of chocolate and a supermarket which is 20 miles away must be supplied with at least 200 cwt. of cheese. The goods are delivered by lorries and the factory has only enough lorries for 26,000 cwt. miles (note: p cwt. carried q miles is pq cwt. miles). If the profit on chocolate is twice the profit on cheese, find the number of cwt. of each commodity that should be distributed to the wholesaler and to the supermarket so as to maximize the profit.