AN ROINN OIDEACHAIS

M.40

LEAVING CERTIFICATE EXAMINATION, 1968

MATHEMATICS (HONOURS) – PAPER I – (300 marks)

WEDNESDAY, 12th JUNE – Morning, 10 to 12.30

Six questions to be answered.
All questions carry equal marks.
Mathematical Tables may be had from the Superintendent.
R is the set of real numbers.

1. A solid cylinder of diameter 2 feet and of length 4 feet lies embedded with its axis horizontal in level ground to a depth of 6 inches. Find the volume of that portion of the cylinder which is above ground level. Give your answer correct to 4 significant figures.

2. P and Q are two points outside a given circle and Q is on the polar of P with respect to the circle. Prove that P is on the polar of Q.
Three straight lines intersect in a point. Prove that the poles of the lines with respect to a given circle are collinear.

3. (a) The coordinates of the three vertices A, B, C of a triangle are respectively (2,5), (–4,2), (1,–8). Show that

$$x - 2y + 8 + \lambda(13x - y - 21) = 0, \ (\lambda \epsilon R)$$

is the equation of a line through A for each value of $\lambda$.
Find that value of $\lambda$ for which the line is (i) the line AB, (ii) a line parallel to the $x$-axis, (iii) a line perpendicular to BC.

(b) Plot the set of points whose coordinates are $(x,y)$ given that

$$x = 2t + 1, \quad y = t - 1 \text{ and } t = 0, 1, 2, 3.$$

Prove that the points are on a straight line and by eliminating $t$, or otherwise, find its equation in terms of $x$ and $y$.
Find the area contained by the $y$-axis and the two lines

$$x = \frac{2t+1}{t+1}, \quad y = \frac{t}{t+1} \ (t\epsilon R) \quad \text{and} \quad x = \frac{1+t}{1-t}, \quad y = \frac{3t}{1-t} \ (t\epsilon R).$$

4. (a) What loci are represented by the equations (i) $x^2 - y^2 = 0$, (ii) $xy = 0$, (iii) $y = x + k$ ($k = 0, 1, 2, 3$), (iv) $y = mx + 1$ ($m = 0, 1, 2, 3$) ?

(b) Find the equation of a circle of radius $p$ whose centre is in the first quadrant and which touches the $y$-axis at a point distant $k$ from the origin.
Prove that the equation of the other tangent which contains the origin is

$$(p^2 - k^2)x + 2pky = 0.$$

5. Show that $(at^2, 2at)$ is a point on the parabola $y^2 = 4ax$ and hence show that the equation of the tangent to the parabola at this point is $x - ty + at^2 = 0$.
P, Q, S are three points on a parabola such that the ordinates of the points are in geometric progression. Prove that the tangents at P and S meet on the ordinate through Q.

6. (a) If $\theta$ is the angular measure from the initial line $x = y$ and the origin is pole, fine in terms of $r$ and $\theta$

(i) The polar coordinates of any two of the following points whose cartesian coordinates are $(\sqrt{2}, -\sqrt{2})$, $(\sqrt{3}, -1)$, $(\cos\phi, \sin\phi)$.

(ii) The polar equation of the line $y = mx$ where $m = \tan\alpha$.

(b) Sketch any two of the following curves:

(i) $r = \theta, \ 0 \leqslant \theta \leqslant \pi$;

(ii) $r = \sin\theta$;

(iii) $r^2 = \sin^2\theta$.

OVER →

7. A set of statistical data is represented (i) by the mean, (ii) by the median, of the set. Point out some advantages and some disadvantages which each representative may have.

The following table gives the number of holdings in a district with the rateable valuation specified:

| Rateable Valuation in £ | 0 – 5 | 5 – 10 | 10 – 20 | 20 – 30 | 30 – 50 |
|---|---|---|---|---|---|
| Number of holdings | 25 | 45 | 81 | 65 | 24 |

Represent the distribution by a histogram.

Calculate the standard deviation of the distribution assuming the values are concentrated at the midpoints of the class intervals.

(Note: 0 – 5 means "less than 5", 5 – 10 means "at least 5 but less than 10" etc.)

8. (a) The length of a vector $\vec{x}$ is denoted by $|\vec{x}|$ and the vectors $i$, $j$ are orthonormal (i.e. $|i| = 1 = |j|$ and $i.j = 0$).

If $\vec{u} = 3i - 4j$ and $\vec{v} = 2i + j$, answer any two of the following:

(i) Find $\vec{u} \cdot \vec{v}$;

(ii) Show that $|\vec{u} + \vec{v}| < |\vec{u}| + |\vec{v}|$; (Diagram not sufficient)

(iii) Find the angle between $\vec{u}$ and $\vec{v}$.

(b) (i) If $\vec{u} = 3i - 4j$ and $\vec{w} = \alpha i + 2j$, find $\alpha$ such that $\vec{u} \perp \vec{w}$.

(ii) If $\vec{a}$ and $\vec{b}$ are any two non-parallel vectors and

$$\vec{z} = \vec{b} - \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}, \text{ show that } \vec{a} \perp \vec{z}.$$

9. (a) State the range of any two of the following functions, the domains being as given:

(i) $f(x) = e^x, \quad x \in R$;

(ii) $g(x) = \frac{1}{\log x}, \quad 0 < x < 1$;

(iii) $h(x) = x \sin kx, \quad \frac{-\pi}{2k} \leqslant x \leqslant \frac{\pi}{2k}, \quad k > 0$.

(b) Let $f(x) = \sin tx$ ($x \in R$, $t \in R$ and $t$ independent of $x$) be a periodic function of period $l$.

(i) Show that $f(l) = 0$ and hence express $t$ in terms of $l$.

(ii) If $f$ and $g$ are two periodic functions defined on the same domain D and having the same period $l$, show that $F = f - g$ (i.e. $F(x) = f(x) - g(x)$ for all $x \in D$) is also a periodic function.

10. (a) Find the general solution of the equations;

(i) $\sin\theta = \frac{\sqrt{3}}{2}$;

(ii) $1 + \cos\theta = \sin^2\theta$.

(b) (i) Use De Moivres theorem to express $\sin 3\theta$ as a polynomial in $\sin\theta$.

(ii) Given that $\frac{d}{dx}(e^{ix}) = ie^{ix}$, show that for all $\theta \in R$,

$$e^{i\theta}(\cos\theta - i \sin\theta) = \text{constant}$$

and hence deduce that the value of this constant is 1.

Deduce further that $e^{i\theta} = \cos\theta + i \sin\theta$.