# AN ROINN OIDEACHAIS.

## LEAVING CERTIFICATE EXAMINATION, 1962.

## MATHEMATICS—Geometry—Honours.

*FRIDAY, 8th JUNE.*—MORNING, 10 TO 12.30.

Not more than *seven* questions may be answered.
Mathematical Tables may be obtained from the Superintendent.

1. O{ACBD} is a harmonic pencil.
If a line through B parallel to OA cuts OC (produced) at P and OD at Q, prove that PB = BQ.
If AOB is a right angle, prove that OB and OA are the internal and external bisectors, respectively, of the angle COD.
(35 marks.)

2. PQ is a common tangent to two circles which cut at A and B, the points P and Q being on the circles. If AB (produced) cuts the circle through P,A,Q at C, prove that PQ bisects BC.
(35 marks.)

3. (*a*) P is a point such that the tangents from P to two given non-intersecting circles are of equal length. Show that the locus of P is a straight line perpendicular to the line of centres.

(*b*) Given two circles of a non-intersecting system of coaxal circles, show how to construct (i) the radical axis of the system, (ii) a circle of the system such that its centre is a given point on the line of centres.
(35 marks.)

4. In a parallelogram OABC, the equations of OA and OC are $3y = x$ and $y = 3x$, respectively, and B is the point (4, 3). Find the equation of the line AB. Find, also, the angle AOC, the equation of the perpendicular from B to OC and the length of the perpendicular from B to OC.
(36 marks.)

5. Find the equation of the circle which has as a diameter the common chord of the two circles $x^2 + y^2 + 6x - 8y - 1 = 0$ and $x^2 + y^2 + 2x - 5y = 0$. Show that the required circle touches the axes of coordinates.
(36 marks.)

6. (*a*) The tangent to a parabola at a point P meets the axis of the parabola at T, and N is the foot of the perpendicular from P to the directrix. If S is the focus of the parabola, prove that NT is parallel to PS.

(*b*) Find the equation of the parabola with focus (1, 5) and vertex (−4, 0).
(36 marks.)

7. In a triangle ABC, using the usual notation, prove that

(i) $$\frac{b-c}{r_1} + \frac{c-a}{r_2} + \frac{a-b}{r_3} = 0,$$

(ii) $$b^2\sin 2C + c^2\sin 2B = 2bc\sin A.$$
(36 marks.)

8. (*a*) Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

(*b*) Show that $x = \frac{\pi}{10}$ (i.e. $x = 18°$) is a particular solution of the equation $\cos 3x = \sin 2x$ and find the general solution of the equation. Hence, or otherwise, find in surd form the value of $\sin 18°$.
(36 marks.)