AN ROINN OIDEACHAIS

(Department of Education)

LEAVING CERTIFICATE EXAMINATION, 1961.

MATHEMATICS—Algebra—Honours.

TUESDAY, 13*th* *JUNE*.—MORNING, 10 TO 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. (*a*) Factorise $x^2-xy-2y^2-5x+y+6$;

   (*b*) If ω and ω^2 are the imaginary cube roots of unity, show that $1+\omega+\omega^2=0$, and that
   $a^2+b^2+c^2-ab-bc-ca=(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$.

   [35 marks.]

2. Solve fully the following simultaneous equations:

$$x^2+2xy-y^2+14=0,$$
$$x^2+3xy+2y^2+2=0.$$

   [35 marks.]

3. (*a*) Show that ${}_nC_r=\frac{n!}{(n-r)!\,r!}$,

   where ${}_nC_r$ is the number of combinations of n things taken r at a time.

   (*b*) A committee of *five* is to be chosen from *ten* candidates of whom *seven* are men and *three* are women. In how many ways can this be done

   (i) if all the candidates are eligible for election,

   (ii) if only one woman is to be on the committee,

   (iii) if one woman at least is to be on the committee?

   (*c*) Apply the binomial theorem to find the value of $(1{\cdot}01)^{10}+(0{\cdot}99)^{10}$ correct to *eight* places of decimals.

   [35 marks.]

4. (*a*) Show that the sum to n terms of the series $1^2+2^2+3^2+\ldots$ is $\frac{1}{6}n(n+1)(2n+1)$.

(*b*) Find the sum of the following series to n terms where n is *even*: $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\ldots$.

(*c*) If $f(r)=\frac{1}{r^2}$, prove that $f(r)-f(r+1)=\frac{2r+1}{r^2(r+1)^2}$ and deduce the sum to n terms of the series $\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+\ldots$

[36 marks.]

5. (*a*) Prove from first principles that $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$, where u and v are functions of x.

(*b*) Differentiate with respect to x
(i) $(1+x^2)^2$, (ii) $x(1+x^2)^{\frac{1}{2}}$.

(*c*) If $y=x^{-1}\sin x$, find $\frac{dy}{dx}$ and prove that

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+xy=0.$$

[36 marks.]

6. A piece of countryside is in the form of a square ABCD of side 60 furlongs and a path runs along the side AB.

In a motor-cycle test a competitor has to go from the corner A to the opposite corner C. If he can travel along the path at the rate of 5 furlongs per minute and if he can travel across country at the rate of 3 furlongs per minute, what is the shortest time in which he can go from A to C?

[36 marks.]

7. Find the value of

(i) $\int_0^1 x^2(1+2x)dx$; (ii) $\int_0^1 \frac{xdx}{(1+x^2)^3}$;

(iii) $\int_0^{\frac{\pi}{2}} \sin^2\theta d\theta$; (iv) $\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^2\theta d\theta$.

[36 marks.

8. Sketch the curve $y^2=x^2(9-x^2)$.
Find the total area enclosed by the curve.

[36 marks.]