Not more than seven questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. A transversal cuts the sides PQ, QR (produced) and RP of a triangle PQR at the points L, M, N respectively. Prove that
   \[ \frac{PL}{QM} \cdot \frac{RN}{LQ} \cdot \frac{MR}{NP} = -1 \]

   [35 marks.]

2. Prove that the rectangle contained by the diagonals of a cyclic quadrilateral is equal to the sum of the two rectangles contained by its opposite sides.
   P is any point on the circumcircle of an equilateral triangle ABC. Prove that \( PA^2 + PB^2 + PC^2 = 2AB^2 \).

   [35 marks.]

3. A pencil of four rays is cut by a transversal. If the points of section form an harmonic range, prove that the points in which any transversal cuts the rays will form an harmonic range.

   [35 marks.]

4. A triangle is bounded by the three straight lines \( x + 2y = 8 \), \( x - y = 2 \), \( 8x + y = -11 \). Find the co-ordinates of the orthocentre of the triangle.
   Find also the area of the triangle.

   [36 marks.]

5. A point P moves in such a way that its distance from the point (-2,1) is always twice its distance from the point (1,4). Find the equation of the locus of P and show that the locus is a circle.
   Find the radius of the circle and the co-ordinates of the centre.

   [36 marks.]

6. Find the equation of the circle which passes through the three points (4, 1), (-2, 3), (0, -1).
   Tangents are drawn to the circle from the point (5, 0). Find the equations of the tangents and the co-ordinates of the points of contact.

   [36 marks.]
7. Prove that the equation of the tangent at a point \((x_1, y_1)\) on the parabola \(y^2 = 4ax\) is \(yy_1 = 2a(x + x_1)\).

\(P\) is a point on a parabola of which \(S\) is the focus. The tangent at \(P\) cuts the directrix at \(B\) and the axis of the parabola at \(Q\). Prove that the tangent at the vertex bisects \(PQ\) and that the angle \(PSB\) is a right angle.

[36 marks.]

8. (a) In a triangle, using the usual notation, prove that

\[ r_1 + r_2 - r = 4R. \]

(b) Prove that \(\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{2} - \tan^{-1} \frac{3}{11} = \frac{\pi}{4} \).

(c) Find the general solution of the equation

\[ \cos \theta + \cos 3\theta + \cos 7\theta = 0. \]

[36 marks.]