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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1953.

MATHEMATICS—ALGEBRA—Honours.

MONDAY, 15*th JUNE*.—Morning, 10 to 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Solve the simultaneous equations :

(i) $2yz=3(2z-5y)$,
$5zx=3(x+2z)$,
$6xy=7y+2x$.

(ii) $x^{\frac{1}{6}}+y^{\frac{1}{5}}=5$,
$x^{\frac{1}{2}}+y^{\frac{3}{5}}=35$.

[35 marks.]

2. (i) Find the value of k so that the expression

$$2x^2-7xy+6y^2+3x-4y+k.$$

may be resolved into two linear factors. Find the factors.

(ii) Factorise

$$x^2(y^3-z^3)+y^2(z^3-x^3)+z^2(x^3-y^3).$$

[35 marks.]

3. Write down the first four terms in each of the following binomial expansions :

(i) $(1-x)^{-1}$; (ii) $(1+3x)^{\frac{1}{2}}$; (iii) $(4+x)^{-\frac{3}{2}}$.

If x is so small that its square and higher powers may be neglected, write

$$\frac{\sqrt{1+3x}}{(1-x)\sqrt{(4+x)^3}}$$

in the form $a+bx$, where a and b are constants.

[35 marks.]

4. (i) Find the sum of the first n terms of the series

$$(a+1)^2+(a+2)^2+(a+3)^2+ \ . \ . \ .$$

(ii) Write down the nth term of the series

$$\frac{1}{3\times4}+\frac{1}{4\times5}+\frac{1}{5\times6}+ \ . \ . \ .$$

and find the sum of the first n terms.

[36 marks.]

5. Trace the curve

$$y=(x-1)^3(x-2).$$

[36 marks.]

6. (*a*) Prove that as x tends to zero the limit of $\frac{\sin x}{x}$ is 1.

(*b*) Find from first principles the differential coefficient of $\tan x$.

(*c*) Differentiate $\left(\frac{x \tan x}{1-x}\right)^2$, with respect to x.

[36 marks.]

7. A wire, of length l, is cut into two portions. The two portions are bent so that one of them forms the sides of a square and the other forms the circumference of a circle. Find the ratio between the lengths of the two portions if the sum of the areas of the square and circle is the least possible.

[36 marks.]

8. Evaluate :

(i) $\int_0^{\frac{\pi}{8}} \sec^2 2\theta d\theta$;

(ii) $\int_0^{\frac{\pi}{4}} \sin 4\theta \cos 2\theta d\theta$; [Hint : Put $y=\cos 2\theta$]

(iii) $\int_0^1 \frac{x dx}{(1+x^2)^2}$;

(iv) $\int_0^2 \sqrt{4-x^2} \ dx$.

[36 marks.]