AN ROINN OIDEACHAIS

(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1952.

MATHEMATICS—Geometry—Honours.

FRIDAY, *13th JUNE*.—MORNING, 10 TO 12.30.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent

1. A transversal cuts the sides AB, BC (produced), and CA of a triangle ABC at the points L, M, N respectively. Prove that

$$\frac{AL}{LB} \cdot \frac{BM}{MC} \cdot \frac{CN}{NA} = -1.$$

[40 marks.]

2. If A, B, C, D is a harmonic range and O is the middle point of AC, prove that $OC^2 = OB.OD$.

Prove that if a chord of a circle passes through a fixed point P, it is divided harmonically by P and the polar of P.

[40 marks.]

3. In the triangle ABC the equations of the sides AB, BC, CA are $x+3y=0$, $3x+2y+7=0$, $2x-y=0$ respectively. Find

(i) the co-ordinates of the vertices,

(ii) the co-ordinates of the middle-point of AB,

(iii) the equation of the median through C,

(iv) the equation of the perpendicular from A on BC.

[40 marks.]

4. With the point (2, 1) as centre a circle is drawn which touches the straight line $x+2y+1=0$. Find the radius of the circle, and write down the equation of the circle.

Find the equations of the tangents to the circle which are perpendicular to the straight line $x+2y+1=0$.

[42 marks.]

5. The straight line $4x+y-2=0$ cuts the circle

$$x^2+y^2-4x+6y+8=0$$

in the points A, B. Show that the equation

$$x^2+y^2-4x+6y+8+\lambda(4x+y-2)=0$$

represents a circle which passes through A and B, whatever the value of λ.

For what values of λ does the equation represent

(i) a circle which passes through the origin,

(ii) a circle which touches the x-axis,

(iii) a circle which has its centre on the straight line $x+y-1=0$?

[42 marks.]

6. Prove that the equation of the tangent at the point (x_1, y_1) on the parabola $y^2=4ax$ is $yy_1=2a(x+x_1)$.

P and Q are any two points on the parabola $y^2=4ax$. The tangents at P and Q intersect at R. Prove that the straight line through R parallel to the axis of the parabola bisects the chord PQ.

[42 marks.]

7. In a triangle ABC prove that

(i) $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$;

(ii) $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$,

where $2s=a+b+c$.

[42 marks.]

Or,

7. In a triangle ABC, using the usual notation, prove that

(i) $r\cos\frac{A}{2}=a\sin\frac{B}{2}\sin\frac{C}{2}$;

(ii) $a\cot A+b\cot B+c\cot C=2R+2r$.

[42 marks.]

8. (*a*) Prove that $2\tan^{-1}\frac{1}{3}-\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{11}{27}=\frac{\pi}{4}$;

(*b*) Solve the equation $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$;

(*c*) Find the general solution of the equation

$$\sin 6\theta-\sin 4\theta=\sin\theta.$$

[42 marks.]