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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1950.

MATHEMATICS—Algebra—Honours.

TUESDAY, 13th JUNE.—MORNING, 10 TO 12.30.

Not more than six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the equations

$$x(y+z)=\frac{9}{2}; \quad y(z+x)=5; \quad z(x+y)=2.$$

[40 marks.]

2. Factorize :

(i) $(b+c)^2(b-c) + (c+a)^2(c-a) + (a+b)^2(a-b)$;

(ii) $(b+c)^2(b^2-c^2) + (c+a)^2(c^2-a^2) + (a+b)^2(a^2-b^2)$;

(iii) $a(b^4-c^4) + b(c^4-a^4) + c(a^4-b^4)$.

[40 marks.]

3. (i) Prove that $\frac{2n}{[n]} = 1.3.5 \dots (2n-1) \times 2^n$.

(ii) Show that the number of combinations of n different

things r at a time is $\frac{[n]}{[r][n-r]}$.

(iii) In how many ways can a selection of 17 persons be made from 21 persons (a) so that a particular person is always included, (b) so that two particular persons are always excluded?

[40 marks.]

4. Write down the first three terms in the binomial expansions of $(1+2x)^{\frac{1}{2}}$ and $(4-5x)^{-\frac{1}{2}}$ in ascending powers of x .

If x is so small that its square and higher powers may be neglected, express in the form $l+mx$ the value of

$$\frac{(1+2x)^{\frac{1}{2}}}{(2+x)\sqrt{4-5x}}$$

[42 marks.]

5. Find the positive root of the equation $x^3 - 4x - 1 = 0$, correct to two places of decimals.

[42 marks.]

Or,

5. (i) Show that $3 + 5\sqrt{-1}$ is a root of the equation $x^4 + 3x^3 - 30x^2 + 366x - 340 = 0$ and find the other roots.

(ii) Express $(1 - 4\sqrt{-5})^{-\frac{1}{2}}$ in the form $x + y\sqrt{-1}$, where x and y are real numbers.

[42 marks.]

6. (i) Express $\frac{1}{(2r-1)(2r+1)}$ in the form $\frac{A}{2r-1} + \frac{B}{2r+1}$ and hence or otherwise, find the sum of the series

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)}$$

(ii) Find the n th term and the sum of n terms of the series $2^2 + 5^2 + 8^2 + \dots$.

[42 marks.]

7. Find from first principles the differential coefficient of (a) x^n , where n is a positive integer; (b) $\sin x$. Differentiate, with respect to x , (i) $x\sqrt{x^2+a^2}$; (ii) $x^2 \sin x \cos x$.

[42 marks.]

Or

7. Trace the curve $y = (x-2)(x-3)(x-4)$, paying special attention to maximum and minimum points and to the point of inflection.

[42 marks.]

8. Evaluate

(i) $\int_0^1 x(x^2+1)dx;$

(ii) $\int_0^a \sqrt{a^2-x^2} dx;$

(iii) $\int_0^{\frac{\pi}{2}} \sin^3 x dx;$

(iv) $\int_0^{\frac{\pi}{4}} \cos^2 2x dx.$

[42 marks.]