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LEAVING CERTIFICATE EXAMINATION, 1947.

MATHEMATICS—Geometry—Honours.

WEDNESDAY, 11th JUNE.—MORNING, 10 TO 12.30.

Six questions may be answered, of which not more than *four* may be selected from Section A.

All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

SECTION A.

1. State and prove Ceva's theorem.

An excircle of a triangle ABC touches BC at X and touches AB, AC produced at Z, Y, respectively. Show that AX, BY, CZ are concurrent.

2. Prove that any circle passing through the limiting points of a non-intersecting coaxal system cuts every circle of the system orthogonally.

Given two circles of a non-intersecting coaxal system, show how to find the limiting points and how to construct the circle of the system which passes through a given point.

3. If A, B, C, D, be a harmonic range, prove that the inverse points A′, B′, C′, D′ with regard to any point O on the range, also form a harmonic range. (Hint: Take the radius of the circle of inversion to be unity.)

4. Find the coordinates of P, the point of intersection of the lines

$$3x-4y+1=0, \quad 5x-8y-1=0.$$

O is the origin, and a straight line is drawn through P perpendicular to OP to cut the axes of coordinates in A and B; find the ratio of AP : PB.

5. Show that the polar of (x_1, y_1) with respect to the circle

$$x^2+y^2+2gx+2fy+c=0$$

is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$.

Hence, or otherwise, find the coordinates of the points of contact of the tangents from the point $(2,-7)$ to the circle

$$x^2+y^2-2x+8y+12=0.$$

6. Prove the following properties of a parabola :

(i) Any tangent makes equal angles with the axis and with the line joining its point of contact to the focus.

(ii) The foot of the perpendicular from the focus on a tangent lies on the tangent at the vertex of the parabola.

Section B.

7. With the usual notation for a triangle ABC, prove that

$$r = 4R \sin \tfrac{1}{2}A \sin \tfrac{1}{2}B \sin \tfrac{1}{2}C.$$

Factorize $\cos A + \cos B + \cos C - 1$ and hence, or otherwise, show that $\cos A + \cos B + \cos C$ is greater than 1, but not greater than $1\frac{1}{2}$.

8. Find the maximum and minimum values of $p\cos\theta + q\sin\theta$ as θ varies.

Hence show that, if x and y are the maximum and minimum values respectively, of

$a\cos^2\theta + 2h\sin\theta\cos\theta + b\sin^2\theta$, as θ varies, then $x+y=a+b$, $xy=ab-h^2$.

9. Express the values of (i) $\sin\frac{\pi}{8}$, (ii) $\sin\frac{\pi}{16}$, in surd form.

Hence find expressions for the side and area of a regular sixteen-sided polygon inscribed in a circle of radius r.

10. (i) Solve the equation

$$2\tan^{-1}x + \cot^{-1}x = \frac{2\pi}{3}.$$

(ii) Find the general solution of the equation

$$\sin\theta + \sin 4\theta + \sin 7\theta = 0.$$