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LEAVING CERTIFICATE EXAMINATION, 1947.

MATHEMATICS—Algebra—Honours.

MONDAY, 16*th* *JUNE*.—MORNING, 10 TO 12.30.

Six questions may be answered, of which not more than *three* may be taken from Section B.

Mathematical Tables may be obtained from the Superintendent.

SECTION A.

1. Solve for x, y and z, the simultaneous equations

$$2xy+4x-y=8,$$
$$yz-2y+2z=12,$$
$$2zx-4x-z=25.$$

[40 marks].

2. Find A, B and C such that

$r^6 \equiv Ar^3[(r+1)^3+(r-1)^3]+Br^2[(r+1)^2+(r-1)^2]+Cr[(r+1)+(r-1)]$, where A, B and C are independent of r.

Hence, or otherwise, find the sum of the series

$$(2n)^6-(2n-1)^6+(2n-2)^6- \ldots -3^6+2^6-1^6.$$

[40 marks].

3. There are twenty-five points on a plane. How many different straight lines may be obtained by joining them in pairs when

(i) no three of the points are collinear;

(ii) six of the points lie on one straight line but, with the exception of these, no three or more of the points are collinear?

How many different triangles may be formed using the given points as vertices, in each of the above cases?

[40 marks].

4. Find all the factors of

(i) $(b-c)(b^3+c^3)+(c-a)(c^3+a^3)+(a-b)(a^3+b^3)$;

(ii) $(b-c)^5+(c-a)^5+(a-b)^5$.

[40 marks].

5. (i) Use the Binomial Theorem to evaluate $\sqrt[5]{0{\cdot}997}$ to six places of decimals.

(ii) If $(1+x)^n=c_0+c_1x+c_2x^2+ \ldots +c_nx^n$, where n is a positive integer, prove that

$$c_0+c_2+c_4+ \ldots =c_1+c_3+c_5+ \ldots =2^{n-1}.$$

[42 marks].

[P.T.O.

6. (i) Find, in its simplest form, the relationship between a, b and c in order that the three equations

$$x+y=a,\ x^2+y^2=b^2,\ x^3+y^3=c^3$$

may be simultaneously true.

(ii) If 1, ω, ω^2 are the three cube roots of unity prove that

$$(a+\omega b+\omega^2 c)^3+(a+\omega^2 b+\omega c)^3=(2a-b-c)(2b-c-a)(2c-a-b).$$

[42 marks].

Section B.

7. (i) Differentiate, from first principles, $\tan 2x$ with respect to x.

(ii) Differentiate with respect to x

$$\frac{x^n-1}{x-1}$$

and show that, if S_n is the sum to n terms of a geometric progression in which the first term is a and the common ratio r,

$$(r-1)\frac{dS_n}{dr}=(n-1)S_n-nS_{n-1},$$

where a and n are constant and r varies.

[42 marks].

8. A wire of given length a is cut into two portions which are bent into the shapes of a square and a circle respectively. If the sum of the areas thus enclosed is as small as possible show that the side of the square is twice the radius of the circle, and find the total area then enclosed.

[42 marks].

9. Evaluate

(i) $\int_0^1 (1-\sqrt{x})^3\,dx$; (ii) $\int_0^1 \frac{(x+1)dx}{\sqrt{(5x^2+10x+1)}}$; (iii) $\int_0^{\pi/4} \sec^4 x\,dx$.

[42 marks].

10. If A is the area enclosed between the curve $y=f(x)$, the x-axis, the y-axis and the ordinate at a variable point whose abscissa is x, prove that

$$\frac{dA}{dx}=f(x).$$

Find the area enclosed by the curves $y^2-4ax=0$ and $x^2-4ay=0$.

[42 marks].

11. In the case of the curve

$$y^2=x(x-3)(x-8)$$

(i) find the values of x for which real points on the curve exist; (ii) show that the curve is symmetrical with respect to the x-axis; (iii) prove that there are three points on the curve at which the tangents are parallel to the y-axis; (iv) find the points at which the tangents are parallel to the x-axis; (v) sketch the graph of the curve.

[42 marks].