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LEAVING CERTIFICATE EXAMINATION, 1943.

MATHEMATICS—Geometry—Honours.

TUESDAY, 8th JUNE.—AFTERNOON 3 TO 5.30.

Seven questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. State the converse of Ceva's theorem.

(i) AD is the perpendicular from A on the side BC of an acute angled triangle ABC. Prove that $\frac{\text{BD}}{\text{DC}} = \frac{c\cos\text{B}}{b\cos\text{C}}$.

Hence deduce that the three altitudes of a triangle are concurrent.

(ii) Using a method similar to that employed in (i), prove that the internal bisectors of the angles of a triangle are concurrent.

[35 marks.]

2. If the polar of P passes through Q, prove that the polar of Q passes through P.

ABCD is a quadrilateral whose sides AB, BC, CD, DA touch a circle of centre O at P, Q, R, S respectively. PQ and SR meet at L: prove that OL is perpendicular to BD.

[35 marks.]

3. Given a circle and a point C outside the circle. Prove that the given circle can be inverted into itself by taking C as centre of inversion and by selecting a suitable radius of inversion.

Prove that two circles can be inverted into themselves by taking a point on their radical axis as centre of inversion. Also show how to invert three circles into themselves.

[35 marks.]

4. By using Ptolemy's theorem, or in any other way, prove that the quadrilateral whose vertices are $(1\frac{1}{2}, 1)$, $(5, 1)$, $(1, 2)$, $(1, 3)$ is cyclic.

[36 marks.]

5. Show that the line

$$ax+by+c+k(px+qy+r)=0$$

passes through the point of intersection of the lines $ax+by+c=0$, $px+qy+r=0$, for all values of k.

Find the equation of the line passing through the point of intersection of the lines $3x+y-5=0$, $x-2=0$, and perpendicular to the line $x-y+1=0$.

Find also the co-ordinates of the orthocentre of the triangle formed by those three lines.

[36 marks.]

6. Find the equation of the tangent to the circle $x^2+y^2=a^2$ at the point (x_1, y_1).

A triangle has two of its sides along the co-ordinates axes and its third side is a tangent to the circle $x^2+y^2=a^2$.

If the co-ordinates of the point of contact of the tangent are $(a\cos\theta, a\sin\theta)$ find the co-ordinates of the centroid of the triangle. Show that the locus of the centroid is $\frac{1}{x^2}+\frac{1}{y^2}=\frac{9}{a^2}$.

[36 marks.]

7. Show that the point $(at^2, 2at)$ lies on the parabola $y^2=4ax$ for all values of t.

Find the equation of the line joining the points P and Q whose co-ordinates are $(at^2_1, 2at_1)$ and $(at^2_2, 2at_2)$. If PQ passes through the focus of the parabola prove that $1+t_1t_2=0$.

Also prove that the tangents at P and Q cut one another at right angles and that their point of intersection lies on the directrix of the parabola.

[36 marks.]

8. Prove that

$a\cos\theta+b\sin\theta=\sqrt{(a^2+b^2)}\sin(\theta+\alpha)$, where $\tan\alpha=\frac{a}{b}$.

Deduce the maximum and minimum values of $a\cos\theta+b\sin\theta$ as θ varies.

Find the maximum and minimum values of

$$4\cos^2\theta+\sin^2\theta+4\sin\theta\cos\theta.$$

[36 marks].

9. Given that $\sin 18^\circ=\frac{1}{4}(\sqrt{5}-1)$,

deduce that $\cos 36^\circ=\frac{1}{4}(\sqrt{5}+1)$.

Find the general solutions of the equation

$2(\cos 2x-\cos 4x)=1$.

[35 marks].

10. With the usual notation, prove that in a triangle ABC,

(i) $r_1r_2+r_2r_3+r_3r_1=s^2$,

(ii) $r_1=s\tan\frac{1}{2}A$.

Calculate the least angle of the triangle in which $r_1=7$, $r_2=9$, $r_3=11$.

[35 marks].