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(Department of Education.)

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(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1941.

HONOURS.

MATHEMATICS

(GEOMETRY)

MONDAY, 16*th JUNE*.—MORNING, 10 A.M. TO 12.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. If X, Y, Z are points on the sides BC, CA, AB respectively of a triangle such that AX, BY, CZ are concurrent, prove that

$$\frac{BX}{CX} \cdot \frac{CY}{AY} \cdot \frac{AZ}{BZ} = -1.$$

Prove also that if YZ meets BC in X′, then {BC, XX′} is a harmonic range.

[40 marks.]

2. Prove that, if a straight line is drawn through any point to cut a circle, the line is divided harmonically by the circle, the point and the polar of the point with respect to the circle.

[40 marks]

3. Prove that the inverse of a circle with respect to a point not on its circumference is another circle. Show also that the centre of inversion is a centre of similitude of the two circles.

[40 marks.]

4. Solve the equations

(i) $\frac{1}{2}\tan^{-1}x=\cot^{-1}\frac{x}{2}$;

(ii) $\tan^{-1}(x+1)=3\tan^{-1}(x-1)$.

[40 marks.]

5. If α, β are the roots of the equation

$$a\cos x+b\sin x=c,$$

prove that $$\sin\alpha+\sin\beta=\frac{2bc}{a^2+b^2}$$

and that $$\cos\alpha+\cos\beta=\frac{2ac}{a^2+b^2}.$$

[40 marks.]

6. If D, E, F are the feet of the perpendiculars drawn from the vertices of an acute-angled triangle ABC to the opposite sides, prove that the sides of the triangle DEF are equal to $a\cos A$, $b\cos B$, $c\cos C$ respectively and that the angles are equal to $\pi-2A$, $\pi-2B$, $\pi-2C$ respectively. Prove also that the radius of the circumcircle of ABC is twice the radius of the circumcircle of DEF.

[42 marks.]

7. The co-ordinates of the vertices of a triangle are $(1, 0)$; $(-1, 2)$; $(-2, -\frac{1}{2})$. Find the co-ordinates of the orthocentre.

[42 marks.]

8. Find the equation of the inscribed circle of the triangle formed by the straight lines whose equations are

$$3x+4y-4=0,$$
$$12x-5y+5=0,$$
$$y=0.$$

[42 marks.]

9. Find the equation of the straight line which lies midway between the parallel straight lines $x-2y+1=0$ and $x-2y-4=0$.

Find also the equations of the two circles through the origin which touch both the given straight lines.

[42 marks.]

10. Show that the equation of any tangent to the parabola $y^2=4x$ can be put in the form $x-ay+a^2=0$.

Hence show that two tangents can be drawn to the parabola from the point (x_1, y_1), when $y_1^2-4x_1$ is positive.

Find the equation of each of the two tangents that can be drawn from the point (6, 5).

[42 marks.]