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(Department of Education).

BRAINNSE AN MHEADHON-OIDEACHAIS
(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1936.

HONOURS.

MATHEMATICS
(Algebra).

MONDAY, *22nd JUNE*.—Afternoon, 3.30 to 6 p.m.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the equations

(i) $x^2+x=2y$,
$x^3+1=3y$.

(ii) $x^2+y^2=x+y+18$,
$xy=6$.

[40 marks.]

2. Express $\sqrt[3]{122}$ in the form $a(1-x)^n$, where x is a small fraction, and find the value of $\sqrt[3]{122}$ to 7 places of decimals.

[40 marks.]

3. Find the sum of n terms of the series whose nth term is $n(n+1)(n+3)$.

Find the sum to infinity of the series $1+r+(1+a)r^2+(1+a+a^2)r^3+(1+a+a^2+a^3)r^4+\ldots$
$[-1<ar<1$ and $-1<r<1]$.

[40 marks.]

4. Draw a rough sketch of the curve
$$y=x^3+2x^2-4x+2.$$

Find, to one decimal place, the real root of the equation
$$x^3+2x^2-4x+2=0.$$

[40 marks.]

5. Prove that ${}_nC_r={}_nC_{n-r}=\frac{n!}{r!\,(n-r)!}$

How many different football teams of 15 boys can be chosen out of 18 boys so that at least one of two particular boys must be always included ?

[40 marks.]

6. Express $\sqrt{3-4i}$ in the form $a+bi$. $[i=\sqrt{-1}]$.

Prove that $a^3+b^3+c^3=(a+b+c)(a+wb+w^2c)(a+w^2b+wc)$, where w is one of the imaginary cube roots of unity.

[42 marks.]

7. Differentiate (i) a^2+x^2; (ii) $\sqrt{a^2+x^2}$; (iii) $(a^2-x^2)\sqrt{a^2+x^2}$; (iv) $\dfrac{\tan x}{x+\tan x}$

[42 marks.]

8. Find the value of

(i) $\displaystyle\int_1^4 \left(1+3\sqrt{x}-\frac{1}{x^2}\right)dx$; (ii) $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \cos x\, dx$;

(iii) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$; (iv) $\displaystyle\int_0^1 \frac{dx}{1+x^2}$

[42 marks.]

9. Trace the curve $y^2=(x-1)(2x-5)^2$.

Find the volume generated by the revolution of the loop about the axis of x.

[42 marks.]

10. Find the area enclosed by the parabola $y^2=3ax$ and the circle $x^2+y^2=4a^2$.

[42 marks.]