Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. ABC is a triangle in which A is an obtuse-angle. Show how to draw a straight line AD from A to BC so that (i) $AD^2 = BD \cdot DC$; (ii) $AD^2 = 2BD \cdot DC$.  

2. Prove that the inverse of a straight line is a circle through the centre of inversion and that the line is the radical axis of its inverse and the circle of inversion. 

3. If the points of section of a pencil of four rays by a transversal form a harmonic range, prove that the points of section by each other transversal form a harmonic range.

4. Prove that 

$$5 \tan^{-1} \frac{1}{7} + 2 \tan^{-1} \frac{3}{79} = 3 \tan^{-1} \frac{1}{7} + 2 \tan^{-1} \frac{2}{11} = \frac{\pi}{4}.$$  

5. Find the maximum and minimum values of

(i) $a\cos \theta + b\sin \theta$; 

(ii) $\sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$. 

49
6. In a triangle prove that

(i) \( \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}; \)

(ii) \( 4R = r_1 + r_2 + r_3 - r. \) [42 marks.]

7. Find the angle between the two straight lines given by

\[ 3x^2 - 20xy + 12y^2 + 22x + 12y - 45 = 0, \]

To which line is (5, 8) the nearer? [42 marks.]

8. Find the coordinates of the middle point of the straight line joining \((x_1, y_1)\) and \((x_2, y_2)\).

Three of the vertices of a parallelogram are \((0, 1); (-2, 3); (-3, -1)\). Find the coordinates of the fourth vertex in all its possible positions. Find also the area of the parallelogram. [42 marks.]

9. Show that the tangents from the point \((6, -1)\) to the circle

\[ x^2 + y^2 - 4x + 2y + 1 = 0 \]

are inclined at an angle of 60°, and find the length of the chord of contact. [42 marks.]

10. TA, TB are tangents to a circle and any secant TCD meets the circle at C, D. Q is the middle point of CD. Prove that TQ bisects the angle AQB and that TQ \( \propto \) AQ + BQ. [42 marks.]