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LEAVING CERTIFICATE EXAMINATION, 1934.

HONOURS.

MATHEMATICS
(GEOMETRY)

THURSDAY, 14*th JUNE*.—MORNING, 10 A.M. TO 12.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Construct an equilateral triangle equal in area to a square of side 2 inches. No proof is required, but every step in the construction should be stated.

[40 marks.]

2. Show that the locus of a point which moves so that its distance from a fixed point is equal to the length of its tangent to a given circle is a straight line perpendicular to the line joining the fixed point to the centre of the circle.

[40 marks.]

3. Draw the graph of $\tan\theta + \cot\theta$ from $\theta = 0°$ to $\theta = 180°$.

[40 marks.]

4. Prove that in a triangle

$$a = b\cos C + c\cos B.$$

Hence, or otherwise, prove that

$$\cos^2 A + \cos^2 B + \cos^2 C + 2\cos A\cos B\cos C = 1.$$

[40 marks.]

5. (i) Prove that $\tan 2\alpha = 2\tan\{\alpha + \tan^{-1}(\tan^3\alpha)\}$.

(ii) Solve the equation $\cos^{-1}x + 2\sin^{-1}x = \dfrac{2\pi}{3}$.

[40 marks.]

6. If α and β are the roots of $a \cos\theta + b \sin\theta = c$, prove that

$$\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{2b}{c+a} \quad \text{and that} \quad \tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{c-a.}{c+a.}.$$

Deduce that

$$\frac{\sin\frac{1}{2}(\alpha+\beta)}{b} = \frac{\cos\frac{1}{2}(\alpha+\beta)}{a} = \frac{\cos\frac{1}{2}(\alpha-\beta)}{c}.$$

[42 marks.]

7. Find the length of the perpendicular from the point (x', y') to the straight line $ax+by+c=0$.
The distance of a point from the line $3x-4y=1$ is twice its distance from the line $12x+5y=0$; find its locus.

[42 marks.]

8. The co-ordinates of the vertices of a triangle are (0,0); (4,0); (1, 3). Find the equation of the line joining the orthocentre to the circum-centre, and show that the centroid lies on this line.

[42 marks.]

9. Prove that the straight line $ax+by+c+\lambda(a'x+b'y+c')=0$ passes through a fixed point for all values of λ.
Show also that the circle $x^2+y^2-25+\lambda(5x+3y-3)=0$ passes through two fixed points, and find their co-ordinates.

[42 marks.]

10. Show that the equation of the parabola $(x-y)^2+3x-y=0$ can be put into the form $(x-y+a)^2+b\,(x+y+c)=0$, where $x-y+a=0$ and $x+y+c=0$ represent the axis and the tangent at the vertex respectively. Find the co-ordinates of the vertex.

[42 marks.]