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LEAVING CERTIFICATE EXAMINATION, 1933.

HONOURS.

MATHEMATICS
(GEOMETRY)

FRIDAY, 16*th* *JUNE*.—Morning, 10 a.m to 12.30 p.m.

Six questions may be answered. All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. Each of two equal circles passes through the centre of the other. Find the area common to both circles in terms of a, the radius of either circle.

2. A transversal LNM cuts the sides AB and AC of a triangle ABC in L and N respectively and the side BC produced in M; prove that

$$\frac{AL}{LB}\cdot\frac{BM}{MC}\cdot\frac{CN}{NA}=-1.$$

If AL : AB=CM : BC, prove that LN=NM.

3. Prove that the area of a triangle is $\sqrt{s(s-a)(s-b)(s-c)}$.

AD, the bisector of the angle A in a triangle ABC, meets BC in D; prove that $AD=\frac{2bc}{b+c}\cos\frac{A}{2}$

4. Prove that $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$ and that

$$2\tan^{-1}\frac{1}{2}=\frac{\pi}{4}+\sin^{-1}\frac{1}{5\sqrt{2}}$$

5. Illustrate graphically the variation of the function $\sin x+\sin 2x$, giving special attention to maximum and minimum and other notable values.

6. Prove that $\sin\theta$ is approximately equal to θ when θ is small.

A pole standing in a horizontal plane is 40 feet high. Find, to the nearest second, the angle it subtends at a point in the plane 3 miles away.

7. Find the area of a triangle in terms of the coordinates of its vertices.

The coordinates of three points A, B, C are (x_1, y_1), (x_2, y_2), (x_3, y_3) respectively. Find the locus of a point P which is such that the triangles ABP, BCP are equal in area. Interpret the result geometrically.

8. Find the equation of the locus of a point whose distances from the points (0, 0) and (1, 2) are in the ratio 1 : 3. Prove that the locus is a circle whose centre is collinear with (0, 0) and (1, 2).

9. A, B are two points on the rectangular axes OX, OY respectively. Straight lines are drawn, one bisecting AB at right angles and the other bisecting the angle XOY. Find the equations of these lines and prove that they intersect on the circle on AB as diameter.

10. What is the equation of the tangent at the point (x', y') on the parabola $y^2=4ax$?

Prove that the two parabolas $y^2=x$ and $x^2=8y$ cut one another at right angles and at an angle equal to $\tan^{-1}\frac{3}{5}$.