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(Department of Education).

BRAINSE AN MHEÁN-OIDEACHAIS
(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1930.

HONOURS.

MATHEMATICS (II).

TUESDAY, 17th JUNE.—Afternoon, 3.30 to 6 p.m.

Six questions may be answered. All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. Show from first principles how to express the derivative of u/v in terms of the derivitives of u and v where u and v are functions of x.

Assuming that $\frac{1-x^{n+1}}{1-x}=1+x+x^2+\ .\ .\ .\ +x^n$, deduce by differentiation, the sum of the series $1+2x+3x^2+\ .\ .\ .\ +nx^{n-1}$, and hence show that, if $1>x>-1$, $1+2x+3x^2+\ .\ .\ .$ ad inf. $=(1-x)^{-2}$

2. Explain how the turning and inflexional points of a function are determined without drawing the graph of the function, and show how a maximum value is distinguished from a minimum.

Find the values of x that make the function $2x^3-7x^2-4x+19$ a maximum or a minimum and sketch a rough graph of the function from $x=-1$ to $x=3$.

3. Determine the greatest cone which can be inscribed in a sphere of radius r, and find its volume.

4. Deduce from first principles the derivitive of $\sin x$. Calculate the area enclosed by the x-axis and the curve $y=\sin x$ from $x=0$ to $x=\pi$, and determine the volume of the solid of revolution generated by rotating that area round the x-axis.

5. AC is the diameter of a semi-circle of centre O and radius R. A straight line OB meets the circumference at B and makes an angle 2θ with OC. Show that the length of the line joining the centres of the inscribed circles of triangles OAB and OCB is

$$R\left[\frac{2-\sin 2\theta}{(1+\sin\theta)(1+\cos\theta)}\right]^{\frac{1}{2}}$$

6. A′, B′ are the mid-points of the sides a, b of triangle ABC; D, E are the feet of the perpendiculars from A, B on the opposite sides; A′D, B′E are bisected in P, Q: prove that

$$PQ=\tfrac{1}{4}(a^2+b^2-2ab\cos 3C)^{\frac{1}{2}}.$$

7. Two circles of radii a, b touch externally and a circle of radius x touches both circles and one of their direct common tangents: prove that $\dfrac{1}{\sqrt{x}}$ is equal to the sum or difference of $\dfrac{1}{\sqrt{a}}$ and $\dfrac{1}{\sqrt{b}}$.

8. On the side BC of triangle ABC of area Δ an equilateral triangle BCD is described. Through E, the point of intersection of AD (produced if necessary) and BC, lines are drawn parallel to DC, DB and meeting AC, AB in G, F respectively. Prove that the triangle EFG is equilateral and show that the length of its side is $2a\Delta/(a^2\sin 60^\circ\mp 2\Delta)$ according as A and D are on the same or on opposite sides of BC.

9. From X, Y, Z the vertices of a triangle perpendiculars p, q, r are drawn to the opposite sides x, y, z; show that $\dfrac{1}{p}$ is equal to the sum or difference of $\left(\dfrac{1}{r^2}-\dfrac{1}{x^2}\right)^{\frac{1}{2}}$ and $\left(\dfrac{1}{q^2}-\dfrac{1}{x^2}\right)^{\frac{1}{2}}$ according as neither or one of the angles Y, Z is obtuse. Show that, if q and r are known and $q>r$, p must lie between $\dfrac{qr}{q+r}$ and $\dfrac{qr}{q-r}$.