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LEAVING CERTIFICATE EXAMINATION, 1930.

HONOURS.

MATHEMATICS (I).

FRIDAY, 13*th JUNE*.—10 A.M. TO 12.30 P.M.

Six questions may be answered.

All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the equations :

(i) $x(y+z)=a$
$y(z+x)=b$
$z(x+y)=c.$

(ii) $(x+\sqrt{x^2-a})(x+\sqrt{x^2-b})=ab.$

2. What conditions must p, q, r satisfy in order that px^2+qx+r may have a negative maximum value when x is positive ?

Find the limits of the function $\dfrac{(x-2)(x+5)}{2x-7}$ for real values of x and draw a rough graph of the function.

3. Explain how (i) the sum, (ii) the difference of two complex numbers may be represented by means of an Argand diagram.

If $\dfrac{1}{x+iy}+\dfrac{1}{u+iv}=1$, where x, y, u, v are real numbers and $i\equiv\sqrt{-1}$ express x and y in their simplest form in terms of u and v.

4. What is meant by *convergency* of a series ? Give any simple test of convergency.

Investigate the sum to infinity of :

(i) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+ - - -$.

(ii) $\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+ - - -$.

and determine the sum of n terms of series (ii).

5. If $(1+a)^{-3}=1+c_1a+c_2a^2+c_3a^3+\ .\ .\ .$ where c is independent of a, prove that

$$\frac{1-7a+a^2}{(1+a)^3}=1+c_3a+c_6a^2+c_9a^3+ - - - .$$

6. The centre of a regular hexagon is the point $(2,-1)$ and one of its vertices is $(-1,1)$: find the co-ordinates of the remaining vertices.

7. The area of a triangle and the length of the base being given, find the locus of the orthocentre. Make a rough sketch of the locus.

8. Show that the equation $x^2+y^2+2gx+2fy+c=0$ represents a circle and find under what conditions that circle will touch the y-axis and intercept on the x-axis a chord of length a. Deduce the equations of the circles which touch the y-axis at the point $y=10$ and intercept on the x-axis a chord of length 12.

9. Show that, in general, three normals can be drawn from a given point to a parabola.

P is a point inside the parabola $y^2=4ax$ such that two of the normals drawn to the curve through P are perpendicular to each other : show that the locus of P is a parabola.